Determination of nuclear symmetry energy in the Cornwall-Jackiw-Tomboulis approach

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Within the Cornwall-Jackiw-Tomboulis (CJT) approach a general formalism is established for the study of asymmetric nuclear matter (ANM) described by the four-nucleon interactions. Restricting ourselves to the double-bubble approximation (DBA), we determine the bulk properties of ANM, in particular, the density dependence of the nuclear symmetry energy, which is in good agreement with data of recent analyses.

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I. INTRODUCTION

In recent years heavy-ion collisions and, especially, nuclear reactions induced by radioactive beams have offered a new opportunity to consider the isospin degree of freedom in nuclear physics, namely, to extract from experimental measurements useful information on isospin-dependent issues of asymmetric nuclear matter (ANM) such as the equation of state (EOS), in-medium nucleon-nucleon potentials, etc. It is known that the binding energy per nucleon of ANM in the parabolic approximation can be written as

$$\epsilon_{\text{bind}}(\rho_B, \alpha) = \epsilon_{\text{bind}}(\rho_B, 0) + E_{\text{sym}}(\rho_B)\alpha^2 + 0(\alpha^4), \tag{1.1}$$

where ρ_B is the baryon density, $\rho_B = \rho_p + \rho_n$, and $\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the isospin asymmetry with ρ_p and ρ_n being proton and neutron densities, respectively. In Eq. (1.1) the so-called nuclear symmetry energy (NSE)

$$E_{\text{sym}}(\rho_B) = \frac{1}{2}\left(\frac{\partial^2 \epsilon_{\text{bind}}(\rho_B, \alpha)}{\partial \alpha^2}\right)_{\alpha=0} \tag{1.2}$$

and its related quantities play a very important role in understanding a lot of interesting astrophysical problems [1–3], dynamics of heavy-ion reactions at intermediate energies [4–7], the structure of neutron-rich nuclei, and the nuclei close to the drip-line [8–13]. Unfortunately, the density dependencies of NSE predicted by various models [14] are quite different from each other at both low and high densities, although nowadays there has been significant progress, both theoretical and experimental, in determining NSE at subnormal densities [15–17]. In addition, it is interesting to mention that the liquid-gas phase transitions [18] were predicted for nuclear matter produced in heavy-ion collisions. In the present work we study the EOS of ANM starting from the Cornwall-Jackiw-Tomboulis (CJT) effective action method [19,20] and the four-nucleon model given by the Lagrangian density:

$$\mathcal{L} = \bar{\psi}(i\hat{\partial} - M)\psi + \frac{G_\sigma}{2}(\bar{\psi}\psi)^2 - \frac{G_\omega}{2}(\bar{\psi}\gamma^\mu\psi)^2 + \frac{G_\rho}{2}(\bar{\psi}\vec{\tau}\gamma^\mu\psi)^2. \tag{1.3}$$

Here $\psi(x)$ is the nucleon field, M is the nucleon mass, $\vec{\tau}$ denotes the isospin matrices, and $G_{\sigma,\omega,\rho}$ are coupling constants.

The article is organized as follows. In Sec. II we derive the CJT effective potential corresponding to Eq. (1.3) and then establish the expression for binding energy per nucleon. The numerical computation is performed in Sec. III. After we fix the model parameters, we determine the density dependence of NSE. Section IV is devoted to the conclusions and outlook.

II. CJT EFFECTIVE POTENTIAL

By bosonization

$$\check{\sigma} = \frac{g_\sigma}{m_\sigma^2}\bar{\psi}\psi,$$
$$\check{\omega}_\mu = \frac{g_\omega}{m_\omega^2}\bar{\psi}\gamma_\mu\psi,$$
$$\vec{\check{\rho}}_\mu = \frac{g_\rho}{m_\rho^2}\bar{\psi}\vec{\tau}\gamma_\mu\psi,$$

Eq. (1.3) takes the form

$$\mathcal{L} = \bar{\psi}(i\hat{\partial} - M)\psi + g_\sigma\bar{\psi}\check{\sigma}\psi - g_\omega\bar{\psi}\gamma^\mu\check{\omega}_\mu\psi + g_\rho\bar{\psi}\gamma^\mu\vec{\tau}.\vec{\check{\rho}}_\mu\psi - \frac{m_\sigma^2}{2}\check{\sigma}^2 + \frac{m_\omega^2}{2}\check{\omega}^\mu\check{\omega}_\mu - \frac{m_\rho^2}{2}\vec{\check{\rho}}^\mu\vec{\check{\rho}}_\mu,$$

in which $G_{\sigma,\omega,\rho} = g^2_{\sigma,\omega,\rho}/m^2_{\sigma,\omega,\rho}$.

It is worth mentioning that the preceding bosonization established the nucleon-antinucleon bound states that have the quantum numbers (spin, isospin, and charges) of the σ, ω, and ρ mesons. At the energies of interest in this work, the only relevant degrees of freedom are hadrons. In this respect, it is reasonable to identify these bound states to corresponding mesons, the quark content of which had to be taken into account when they were created in high-energy $N\bar{N}$ collision.

According to Refs. [19] and [20], we obtain the expression for the CJT effective action

$$V = \frac{m_\sigma^2}{2}\sigma^2 - \frac{m_\omega^2}{2}\omega^2 + \frac{m_\rho^2}{2}\rho^2 - i\int\frac{d^4q}{(2\pi)^4} \times \text{tr}\left[\ln S_0^{-1}(q)S^p(q) - S_0^{p-1}(q;\sigma,\omega,\rho)S^p(q) + 1\right] - i\int\frac{d^4q}{(2\pi)^4}\text{tr}\left[\ln S_0^{-1}(q)S^n(q) - S_0^{n-1}(q;\sigma,\omega,\rho)S^n(q) + 1\right]$$

$$
\begin{aligned}
&+\frac{i}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\big[\ln C_0^{-1}C(q)-C_0^{-1}C(q)+1\big]\\
&+\frac{i}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\big[\ln D_0^{\mu\nu-1}D_{\mu\nu}(q)-D_0^{\mu\nu-1}D_{\mu\nu}(q)+1\big]\\
&+\frac{i}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\big[\ln R_0^{33\mu\nu-1}R_{33\mu\nu}(q)\\
&-R_0^{33\mu\nu-1}R_{33\mu\nu}(q)+1\big]\\
&-\frac{i}{2}g_\sigma\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\mathrm{tr}\{[S^p(q)\Gamma^p(q,k-q)S^p(k)\\
&+S^n(q)\Gamma^n(q,k-q)S^n(k)]C(k-q)\}\\
&+\frac{i}{2}g_\omega\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\mathrm{tr}\{\gamma^\mu[S^p(q)\Gamma^{p\nu}(q,k-q)S^p(k)\\
&+S^n(q)\Gamma^{n\nu}(q,k-q)S^n(k)]D_{\mu\nu}(k-q)\}\\
&-\frac{i}{4}g_\rho\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\mathrm{tr}\{\gamma^\mu[S^p(q)\Gamma^{p3\nu}(q,k-q)S^p(k)\\
&-S^n(q)\Gamma^{n3\nu}(q,k-q)S^n(k)]R_{33\mu\nu}(k-q)\},\qquad(2.1)
\end{aligned}
$$

where Γ, Γ^μ, and $\Gamma^{3\mu}$ are the effective vertices taking into account all higher loop contributions;

$$
\begin{aligned}
iS_0^{-1}(k)&=\hat{k}-M,\\
iS_0^{p-1}(k;\sigma,\omega,\rho)&=iS_0^{-1}(k)+g_\sigma\sigma-g_\omega\gamma^0\omega+\frac{g_\rho}{2}\gamma^0\rho,\\
iS_0^{n-1}(k;\sigma,\omega,\rho)&=iS_0^{-1}(k)+g_\sigma\sigma-g_\omega\gamma^0\omega-\frac{g_\rho}{2}\gamma^0\rho,\\
iC_0^{-1}&=-m_\sigma^2,\\
iD_{0\mu\nu}^{-1}&=g_{\mu\nu}m_\omega^2,\\
iR_{033\mu\nu}^{-1}&=-\delta_{33}g_{\mu\nu}m_\rho^2,
\end{aligned}
$$

S, C, $D_{\mu\nu}$, and $R_{33\mu\nu}$ are the propagators of nucleon, σ, ω, and ρ mesons, respectively; σ, ω, and ρ are expectation values of the σ, ω, and ρ fields in the ground state of ANM,

$$\sigma=\langle\breve{\sigma}\rangle=\text{const.},\quad\langle\breve{\omega}\rangle=\omega\delta_{0\mu},\quad\langle\breve{\rho}\rangle=\rho\delta_{3a}\delta_{0\mu}.$$

The ground state corresponds to the solution of

$$\frac{\delta V}{\delta\phi}=0,\qquad(2.2)$$

$$\frac{\delta V}{\delta G}=0.\qquad(2.3)$$

Eq. (2.2) is the gap equation and Eq. (2.3) is the Schwinger-Dyson (SD) equation for propagators G.

Substituting Eq. (2.1) into Eq. (2.2) we get the gap equations for σ, ω, and ρ, and the SD equation for S, C, $D_{\mu\nu}$, and $R_{33\mu\nu}$, accordingly,

$$\sigma=-\frac{g_\sigma}{m_\sigma^2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\{S^p(q)+S^n(q)\},\qquad(2.4a)$$

$$\omega=-\frac{g_\omega}{m_\omega^2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\{\gamma_0[S^p(q)+S^n(q)]\},\qquad(2.4b)$$

$$\rho=-\frac{g_\rho}{2m_\rho^2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\{\gamma_0[S^p(q)-S^n(q)]\},\qquad(2.4c)$$

$$
\begin{aligned}
iS^{p-1}(k)&=iS_0^{-1}(k)-\Sigma^p(k),\qquad(2.5)\\
\Sigma^p(k)&=-g_\sigma\sigma+g_\omega\gamma^0\omega-\frac{g_\rho}{2}\gamma^0\rho\\
&\quad+ig_\sigma\int\frac{d^4q}{(2\pi)^4}[S^p(q)\Gamma^p(q,q+k)C(q+k)]\\
&\quad-ig_\omega\int\frac{d^4q}{(2\pi)^4}[\gamma^\mu S^p(q)\Gamma^{p\nu}(q,q+k)\\
&\quad\times D_{\mu\nu}(q+k)]\\
&\quad+i\frac{g_\rho}{2}\int\frac{d^4q}{(2\pi)^4}[\gamma^\mu S^p(q)\Gamma^{p3\nu}(q,q+k)\\
&\quad\times R_{33\mu\nu}(q+k)],\\
iS^{n-1}(k)&=iS_0^{-1}(k)-\Sigma^n(k),\qquad(2.6)\\
\Sigma^n(k)&=-g_\sigma\sigma+g_\omega\gamma^0\omega+\frac{g_\rho}{2}\gamma^0\rho\\
&\quad+ig_\sigma\int\frac{d^4q}{(2\pi)^4}[S^n(q)\Gamma^n(q,q+k)C(q+k)]\\
&\quad-ig_\omega\int\frac{d^4q}{(2\pi)^4}[\gamma^\mu S^n(q)\Gamma^{n\nu}(q,q+k)\\
&\quad\times D_{\mu\nu}(q+k)]\\
&\quad-i\frac{g_\rho}{2}\int\frac{d^4q}{(2\pi)^4}[\gamma^\mu S^n(q)\Gamma^{n3\nu}(q,q+k)\\
&\quad\times R_{33\mu\nu}(q+k)],\\
iC^{-1}(k)&=-m_\sigma^2-\Pi_\sigma(k),\\
\Pi_\sigma(k)&=-ig_\sigma\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\{S^p(q)\Gamma^p(q,q+k)S^p(q+k)\\
&\quad+S^n(q)\Gamma^n(q,q+k)S^n(q+k)\},\\
iD_{\mu\nu}^{-1}(k)&=m_\omega^2g_{\mu\nu}+\Pi_{\omega\mu\nu}(k)\\
\Pi_{\omega,\mu\nu}(k)&=-ig_\omega\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\big\{\gamma_\mu\big[S^p(q)\Gamma_\nu^p(q,q+k)\\
&\quad\times S^p(q+k)+S^n(q)\Gamma_\nu^n(q,q+k)S^n(q+k)\big]\big\},\\
iR_{33\mu\nu}^{-1}(k)&=-m_\rho^2g_{\mu\nu}-\Pi_{\rho\mu\nu}(k)\\
\Pi_{\rho,\mu\nu}(k)&=-i\frac{g_\rho}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\big\{\gamma_\mu\big[S^p(q)\Gamma_\nu^{p3}(q,q+k)\\
&\quad\times S^p(q+k)-S^n(q)\Gamma_\nu^{n3}(q,q+k)S^n(q+k)\big]\big\}.
\end{aligned}
$$

Σ, Π_σ, $\Pi_{\omega,\mu\nu}$, and $\Pi_{\rho,\mu\nu}$ are self-energies of nucleon, σ, ω, and ρ mesons, respectively.

Next let us truncate Eq. (2.1) in the two-loop approximation, in which case $\Gamma_\sigma=g_\sigma$, $\Gamma_{\omega;\mu}=g_\omega\gamma_\mu$, and $\Gamma_{\rho;\mu}=g_\rho\gamma_\mu\tau_3/2$ (see Fig. 1). At first we expand $\Sigma^a(k)$, $a=\{p,n\}$, in terms of its Dirac components

$$\Sigma^a(k)=\gamma_0\Sigma_0^a(k)-\vec{\gamma}\vec{k}\Sigma_v^a(k)-\Sigma_s^a(k),$$

which together with Eqs. (2.5) and (2.6) lead to

$$S^a(k)=\big(\hat{k}^{*a}+M_k^{*a}\big)G_k^a,\qquad(2.7a)$$

$$G_k^a=-2\pi\delta(k^{*a2}-M^{*a2})n_k,\qquad(2.7b)$$

$$k_0^{*a}=k_0-\Sigma_0^a(k),\qquad(2.7c)$$

$$\vec{k}^{*a}=\vec{k}\big[1-\Sigma_v^a(k)\big],\qquad(2.7d)$$

$$M_k^{*a}=M+\Sigma_s^a(k),\qquad(2.7e)$$

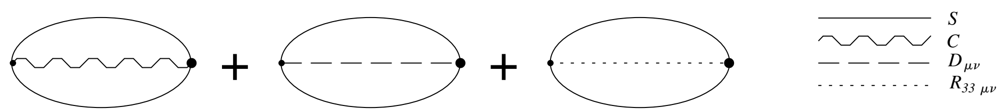

FIG. 1. The two-loop graphs that give contribution to the effective potential.

with n_k^{*a} being the Fermi distribution function,

$$n_k^{a*\pm} = \frac{1}{e^{(E_k^{a*}\pm\mu^{a*})/T}+1},$$
$$E_k^{*a} = \sqrt{k^{*2}+M^{a*2}}.$$

In Eq. (2.7a) we retain only the density dependent part of nucleon propagators, which is dominant at relatively low density [21–24]. Inserting respectively Eq. (2.7) into Eqs. (2.4), (2.5), and (2.6) we arrive at

$$\begin{aligned}
\sigma &= \frac{1}{\pi^2}\frac{g_\sigma}{m_\sigma^2}\int_0^\infty q^2dq\left[\frac{M_q^{p*}}{E_q^{p*}}\left(n_q^{p*-}+n_q^{p*+}\right)\right.\\
&\quad\left.+\frac{M_q^{n*}}{E_q^{n*}}\left(n_q^{n*-}+n_q^{n*+}\right)\right],\\
\omega &= \frac{1}{\pi^2}\frac{g_\omega}{m_\omega^2}\int_0^\infty q^2dq\big[\left(n_q^{p*-}-n_q^{p*+}\right)+\left(n_q^{n*-}-n_q^{n*+}\right)\big],\\
\rho &= \frac{1}{2\pi^2}\frac{g_\rho}{m_\rho^2}\int_0^\infty q^2dq\big[\left(n_q^{p*-}-n_q^{p*+}\right)-\left(n_q^{n*-}-n_q^{n*+}\right)\big],
\end{aligned}\tag{2.8}$$

and

$$\begin{aligned}
\Sigma_s^p(k) &= -\frac{1}{\pi^2}\frac{g_\sigma^2}{m_\sigma^2}\int_0^\infty q^2dq\left[\frac{M_q^{p*}}{E_q^{p*}}\left(n_q^{p*-}+n_q^{p*+}\right)\right.\\
&\quad\left.+\frac{M_q^{n*}}{E_q^{n*}}\left(n_q^{n*-}+n_q^{n*+}\right)\right]-\frac{g_\sigma^2}{8\pi^2}\int_0^\infty q^2dq\\
&\quad\times\int_{-1}^1 d\chi\frac{M_q^{p*}}{E_q^{p*}M_{q+k}^{\sigma2}}\left(n_q^{p*-}+n_q^{p*+}\right)\\
&\quad-\frac{g_\omega^2}{2\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{M_q^{p*}}{E_q^{p*}M_{q+k}^{\omega2}}\left(n_q^{p*-}+n_q^{p*+}\right)\\
&\quad-\frac{g_\rho^2}{8\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{M_q^{p*}}{E_q^{p*}M_{q+k}^{\rho2}}\left(n_q^{p*-}+n_q^{p*+}\right),\\
\Sigma_s^n(k) &= -\frac{1}{\pi^2}\frac{g_\sigma^2}{m_\sigma^2}\int_0^\infty q^2dq\left[\frac{M_q^{p*}}{E_q^{p*}}\left(n_q^{p*-}+n_q^{p*+}\right)\right.\\
&\quad\left.+\frac{M_q^{n*}}{E_q^{n*}}\left(n_q^{n*-}+n_q^{n*+}\right)\right]-\frac{g_\sigma^2}{8\pi^2}\int_0^\infty q^2dq\\
&\quad\times\int_{-1}^1 d\chi\frac{M_q^{n*}}{E_q^{n*}M_{q+k}^{\sigma2}}\left(n_q^{n*-}+n_q^{n*+}\right)\\
&\quad-\frac{g_\omega^2}{2\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{M_q^{n*}}{E_q^{n*}M_{q+k}^{\omega2}}\left(n_q^{n*-}+n_q^{n*+}\right)\\
&\quad+\frac{g_\rho^2}{8\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{M_q^{n*}}{E_q^{n*}M_{q+k}^{\rho2}}\left(n_q^{n*-}+n_q^{n*+}\right),\\
\Sigma_0^p(k) &= \frac{1}{\pi^2}\frac{g_\omega^2}{m_\omega^2}\int_0^\infty q^2dq\big[\left(n_q^{p*-}-n_q^{p*+}\right)+\left(n_q^{n*-}-n_q^{n*+}\right)\big]\\
&\quad-\frac{1}{4\pi^2}\frac{g_\rho^2}{m_\rho^2}\int_0^\infty q^2dq\big[\left(n_q^{p*-}-n_q^{p*+}\right)\\
&\quad-\left(n_q^{n*-}-n_q^{n*+}\right)\big]\\
&\quad-\frac{g_\sigma^2}{8\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{n_q^{p*-}-n_q^{p*+}}{M_{q+k}^{\sigma2}}\\
&\quad+\frac{g_\omega^2}{4\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{n_q^{p*-}-n_q^{p*+}}{M_{q+k}^{\omega2}}\\
&\quad+\frac{g_\rho^2}{16\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{n_q^{p*-}-n_q^{p*+}}{M_{q+k}^{\rho2}},\\
\Sigma_0^n(k) &= \frac{1}{\pi^2}\frac{g_\omega^2}{m_\omega^2}\int_0^\infty q^2dq\big[\left(n_q^{p*-}-n_q^{p*+}\right)\\
&\quad+\left(n_q^{n*-}-n_q^{n*+}\right)\big]+\frac{1}{4\pi^2}\frac{g_\rho^2}{m_\rho^2}\\
&\quad\times\int_0^\infty q^2dq\big[\left(n_q^{p*-}-n_q^{p*+}\right)-\left(n_q^{n*-}-n_q^{n*+}\right)\big]\\
&\quad-\frac{g_\sigma^2}{8\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{n_q^{n*-}-n_q^{n*+}}{M_{q+k}^{\sigma2}}\\
&\quad+\frac{g_\omega^2}{4\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{n_q^{n*-}-n_q^{n*+}}{M_{q+k}^{\omega2}}\\
&\quad-\frac{g_\rho^2}{16\pi^2}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{n_q^{n*-}-n_q^{n*+}}{M_{q+k}^{\rho2}},\\
\Sigma_v^p(k) &= \frac{g_\sigma^2}{8\pi^2k}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{q^{p*}\chi}{E_q^{p*}M_{q+k}^{\sigma2}}\left(n_q^{p*-}+n_q^{p*+}\right)\\
&\quad-\frac{g_\omega^2}{4\pi^2k}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{q^{p*}\chi}{E_q^{p*}M_{q+k}^{\omega2}}\left(n_q^{p*-}+n_q^{p*+}\right)\\
&\quad-\frac{g_\rho^2}{16\pi^2k}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{q^{p*}\chi}{E_q^{p*}M_{q+k}^{\rho2}}\left(n_q^{p*-}+n_q^{p*+}\right),\\
\Sigma_v^n(k) &= \frac{g_\sigma^2}{8\pi^2k}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{q^{n*}\chi}{E_q^{n*}M_{q+k}^{\sigma2}}\left(n_q^{n*-}+n_q^{n*+}\right)\\
&\quad-\frac{g_\omega^2}{4\pi^2k}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{q^{n*}\chi}{E_q^{n*}M_{q+k}^{\omega2}}\left(n_q^{n*-}+n_q^{n*+}\right)\\
&\quad+\frac{g_\rho^2}{16\pi^2k}\int_0^\infty q^2dq\int_{-1}^1 d\chi\frac{q^{n*}\chi}{E_q^{n*}M_{q+k}^{\rho2}}\left(n_q^{n*-}+n_q^{n*+}\right),
\end{aligned}\tag{2.9}$$

here

$$M_k^{\sigma 2} = m_\sigma^2 + \Pi_\sigma(k), \quad M_k^{\omega 2} = m_\omega^2 + \Pi_\omega(k),$$
$$M_k^{\rho 2} = m_\rho^2 + \Pi_\rho(k),$$
$$\chi = \cos(\vec{q}^*, \vec{k}^*), \ \Pi_{\omega;\mu\nu} = (g_{\mu\nu} - k_\mu k_\nu / k^2)\Pi_\omega,$$
$$\Pi_{\rho;\mu\nu} = (g_{\mu\nu} - k_\mu k_\nu / k^2)\Pi_\rho.$$

Finally, after some algebra we get the truncated expression for V,

$$\begin{aligned}
V =& \frac{m_\sigma^2}{2}\sigma^2 - \frac{m_\omega^2}{2}\omega^2 + \frac{m_\rho^2}{2}\rho^2 - \frac{i}{2}\int \frac{d^4q}{(2\pi)^4} \\
&\times \left\{ \ln M_q^{\sigma 2} + \ln M_q^{\omega 2} + \ln M_q^{\rho 2} + \left[M_q^\sigma\right]^{-2} + \left[M_q^\omega\right]^{-2} \right. \\
&\left. + \left[M_q^\rho\right]^{-2} \right\} + \frac{1}{\pi^2}\int_0^\infty q^2 dq \left[T \ln\left(n_q^{n*-} n_q^{n*+}\right) \right. \\
&\left. - T \ln\left(n_q^{n-} n_q^{n+}\right)\right] + \frac{1}{\pi^2}\int_0^\infty q^2 dq \left[T \ln(n_q^{p*-} n_q^{p*+}) \right. \\
&\left. - T \ln(n_q^{p-} n_q^{p+})\right] + 2\pi^2 g_\sigma^2 \int \frac{d^3q}{(2\pi)^4}\int \frac{d^3k}{(2\pi)^4} \\
&\times \int_{-1}^1 d\chi \left[\frac{\left(n_q^{p*-} - n_q^{p*+}\right)\left(n_k^{p*-} - n_k^{p*+}\right)}{M_{q-k}^{\sigma 2}} \right. \\
&\left. + \frac{\left(n_q^{n*-} - n_q^{n*+}\right)\left(n_k^{n*-} - n_k^{n*+}\right)}{M_{q-k}^{\sigma 2}} \right] \\
&+ 2\pi^2 g_\sigma^2 \int \frac{d^3q}{(2\pi)^4}\int \frac{d^3k}{(2\pi)^4} \\
&\times \int_{-1}^1 d\chi \left[\frac{M_q^{n*} M_k^{n*} - q^{n*} k^{n*}\chi}{E_q^{n*} E_k^{n*} M_{q-k}^{\sigma 2}} \right. \\
&\times \left(n_q^{n*-} + n_q^{n*+}\right)\left(n_k^{n*-} + n_k^{n*+}\right) + \frac{M_q^{p*} M_k^{p*} - q^{p*} k^{p*}\chi}{E_q^{p*} E_k^{p*} M_{q-k}^{\sigma 2}} \\
&\left. \times \left(n_q^{p*-} + n_q^{p*+}\right)\left(n_k^{p*-} + n_k^{p*+}\right) \right] \\
&- 4\pi^2 g_\omega^2 \int \frac{d^3q}{(2\pi)^4}\int \frac{d^3k}{(2\pi)^4} \\
&\times \int_{-1}^1 d\chi \left[\frac{\left(n_q^{n*-} - n_q^{n*+}\right)\left(n_k^{n*-} - n_k^{n*+}\right)}{M_{q-k}^{\omega 2}} \right. \\
&\left. + \frac{\left(n_q^{p*-} - n_q^{p*+}\right)\left(n_k^{p*-} - n_k^{p*+}\right)}{M_{q-k}^{\omega 2}} \right] \\
&+ 4\pi^2 g_\omega^2 \int \frac{d^3q}{(2\pi)^4}\int \frac{d^3k}{(2\pi)^4}\int_{-1}^1 d\chi \\
&\times \left[\frac{2M_q^{n*} M_k^{n*} + q^{n*} k^{n*}\chi}{E_q^{n*} E_k^{n*} M_{q-k}^{\omega 2}} \left(n_q^{n*-} + n_q^{n*+}\right)\left(n_k^{n*-} + n_k^{n*+}\right) \right. \\
&\left. + \frac{2M_q^{p*} M_k^{p*} + q^{p*} k^{p*}\chi}{E_q^{p*} E_k^{p*} M_{q-k}^{\omega 2}} \left(n_q^{p*-} + n_q^{p*+}\right)\left(n_k^{p*-} + n_k^{p*+}\right) \right] \\
&- \pi^2 g_\rho^2 \int \frac{d^3q}{(2\pi)^4}\int \frac{d^3k}{(2\pi)^4} \\
&\times \int_{-1}^1 d\chi \left[\frac{\left(n_q^{p*-} - n_q^{p*+}\right)\left(n_k^{p*-} - n_k^{p*+}\right)}{M_{q-k}^{\rho 2}} \right. \\
&\left. - \frac{\left(n_q^{n*-} - n_q^{n*+}\right)\left(n_k^{n*-} - n_k^{n*+}\right)}{M_{q-k}^{\rho 2}} \right] \\
&+ \pi^2 g_\rho^2 \int \frac{d^3q}{(2\pi)^4}\int \frac{d^3k}{(2\pi)^4} \\
&\times \int_{-1}^1 d\chi \left[\frac{2M_q^{p*} M_k^{p*} + q^{p*} k^{p*}\chi}{E_q^{p*} E_k^{p*} M_{q-k}^{\rho 2}} \left(n_q^{p*-} + n_q^{p*+}\right) \right. \\
&\times \left(n_k^{p*-} + n_k^{p*+}\right) - \frac{2M_q^{n*} M_k^{n*} + q^{n*} k^{n*}\chi}{E_q^{n*} E_k^{n*} M_{q-k}^{\rho 2}} \\
&\left. \times \left(n_q^{n*-} + n_q^{n*+}\right)\left(n_k^{n*-} + n_k^{n*+}\right) \right]. \qquad (2.10)
\end{aligned}$$

Starting from Eq. (2.10) we establish successively the expressions for the thermodynamical potential Ω, the energy density ϵ, and the binding energy per nucleon $\epsilon_{\rm bind}$:

$$a/\ \Omega = V - V_{\rm vac}, \qquad (2.11)$$
$$\text{with } V_{\rm vac} = V(M, \rho = 0, T = 0).$$
$$b/\ \epsilon = \Omega + \mu_p \rho_p + \mu_n \rho_n, \qquad (2.12)$$
$$c/\ \epsilon_{\rm bind} = -M + \epsilon/\rho_B. \qquad (2.13)$$

It is obvious that all necessary information on dynamics of our system is provided by Eqs. (2.10)–(2.13). However, in this article, let us restrict ourselves to make use of the double-bubble approximation (DBA), in which $\Pi_\sigma = \Pi_\omega = \Pi_\rho = 0$, or equivalently $M_\sigma = m_\sigma,\ M_\omega = m_\omega,\ M_\rho = m_\rho$ (see Fig. 2). Then the expressions in Eq. (2.9) turn out to be very simple and based on them we define

$$\mu^{p*} = \mu^p - \Sigma_0^p = \mu^p - \frac{1}{\pi^2}\left[G_\omega + \frac{G_\rho}{4}\right]$$
$$\times \int_0^\infty q^2 dq \left(n_q^{n*-} - n_q^{n*+}\right)$$

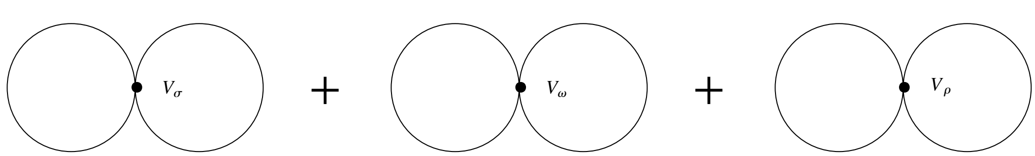

FIG. 2. The double-bubble graphs that give contribution to the effective potential. $V_\sigma \sim g_\sigma^2/m_\sigma^2$, $V_\omega \sim g_\omega^2/m_\omega^2$, $V_\rho \sim g_\rho^2/m_\rho^2$.

$$-\frac{1}{4\pi^2}\left[-G_\sigma+6G_\omega-\frac{G_\rho}{2}\right]\times\int_0^\infty q^2dq\left(n_q^{p*-}-n_q^{p*+}\right), \tag{2.14}$$

$$\mu^{n*}=\mu^n-\Sigma_0^n=\mu^n-\frac{1}{\pi^2}\left[G_\omega+\frac{G_\rho}{4}\right]\times\int_0^\infty q^2dq\left(n_q^{p*-}-n_q^{p*+}\right)-\frac{1}{4\pi^2}\left[-G_\sigma+6G_\omega-3\frac{G_\rho}{2}\right]\times\int_0^\infty q^2dq\left(n_q^{n*-}-n_q^{n*+}\right), \tag{2.15}$$

$$M^{p*}=M+\Sigma_s^p=M-\frac{1}{\pi^2}G_\sigma\times\int_0^\infty q^2dq\frac{M^{n*}}{E_q^{n*}}\left(n_q^{n*-}+n_q^{n*+}\right)-\frac{1}{4\pi^2}[5G_\sigma+4G_\omega+G_\rho]\times\int_0^\infty q^2dq\frac{M^{p*}}{E_q^{p*}}\left(n_q^{p*-}+n_q^{p*+}\right), \tag{2.16}$$

$$M^{n*}=M+\Sigma_s^n=M-\frac{1}{\pi^2}G_\sigma\times\int_0^\infty q^2dq\frac{M^{p*}}{E_q^{p*}}\left(n_q^{p*-}+n_q^{p*+}\right)-\frac{1}{4\pi^2}[5G_\sigma+4G_\omega-G_\rho]\times\int_0^\infty q^2dq\frac{M^{n*}}{E_q^{n*}}\left(n_q^{n*-}+n_q^{n*+}\right). \tag{2.17}$$

Ultimately we are led to

$$\begin{aligned}V(M^*,\mu,T)&=\frac{m_\sigma^2}{2}\sigma^2-\frac{m_\omega^2}{2}\omega^2+\frac{m_\rho^2}{2}\rho^2\\&+\frac{1}{\pi^2}\int_0^\infty q^2dq\left[T\ln\left(n_q^{p*-}n_q^{p*+}\right)+T\ln\left(n_q^{n*-}n_q^{n*+}\right)\right]\\&+\frac{G_\sigma-2G_\omega-G_\rho/2}{8\pi^4}\left[\int_0^\infty q^2dq\left(n_q^{p*-}-n_q^{p*+}\right)\right]^2\\&+\frac{G_\sigma-2G_\omega+G_\rho/2}{8\pi^4}\left[\int_0^\infty q^2dq\left(n_q^{n*-}-n_q^{n*+}\right)\right]^2\\&+\frac{G_\sigma+4G_\omega-G_\rho}{8\pi^4}\left[\int_0^\infty q^2dq\,\frac{M^{p*}}{E_q^{p*}}\left(n_q^{p*-}+n_q^{p*+}\right)\right]^2\\&+\frac{G_\sigma+4G_\omega+G_\rho}{8\pi^4}\left[\int_0^\infty q^2dq\,\frac{M^{n*}}{E_q^{n*}}\left(n_q^{n*-}+n_q^{n*+}\right)\right]^2.\end{aligned} \tag{2.18}$$

III. NUMERICAL COMPUTATIONS

At $T=0$ Eqs. (2.14)–(2.18) are respectively reduced to

$$\mu^{p*}=\mu^p-\frac{1}{4\pi^2}\left[6G_\omega-G_\sigma-\frac{G_\rho}{2}\right]\times\frac{(\mu^{p*2}-M^{p*2})^{3/2}}{3}-\frac{1}{\pi^2}\left[G_\omega+\frac{G_\rho}{4}\right]\frac{(\mu^{n*2}-M^{n*2})^{3/2}}{3}, \tag{3.1}$$

$$\mu^{n*}=\mu^n-\frac{1}{4\pi^2}\left[6G_\omega-G_\sigma-3\frac{G_\rho}{2}\right]\times\frac{(\mu^{n*2}-M^{n*2})^{3/2}}{3}-\frac{1}{\pi^2}\left[G_\omega+\frac{G_\rho}{4}\right]\frac{(\mu^{p*2}-M^{p*2})^{3/2}}{3}, \tag{3.2}$$

$$\begin{aligned}M^{p*}=M&-\frac{1}{8\pi^2}[5G_\sigma+4G_\omega+G_\rho]\\&\times M^{p*}\left(\mu^{p*}\sqrt{\mu^{p*2}-M^{p*2}}-M^{p*2}\times\ln\left|\frac{\mu^{p*}+\sqrt{\mu^{p*2}-M^{p*2}}}{M^{p*}}\right|\right)\\&-\frac{1}{2\pi^2}G_\sigma M^{n*}\left(\mu^{n*}\sqrt{\mu^{n*2}-M^{n*2}}-M^{n*2}\ln\left|\frac{\mu^{n*}+\sqrt{\mu^{n*2}-M^{n*2}}}{M^{n*}}\right|\right),\end{aligned} \tag{3.3}$$

$$\begin{aligned}M^{n*}=M&-\frac{1}{8\pi^2}[5G_\sigma+4G_\omega-G_\rho]M^{n*}\\&\times\left(\mu^{n*}\sqrt{\mu^{n*2}-M^{n*2}}-M^{n*2}\times\ln\left|\frac{\mu^{n*}+\sqrt{\mu^{n*2}-M^{n*2}}}{M^{n*}}\right|\right)\\&-\frac{1}{2\pi^2}G_\sigma M^{p*}\left(\mu^{p*}\sqrt{\mu^{p*2}-M^{p*2}}-M^{p*2}\times\ln\left|\frac{\mu^{p*}+\sqrt{\mu^{p*2}-M^{p*2}}}{M^{p*}}\right|\right),\end{aligned} \tag{3.4}$$

$$\begin{aligned}V(M^*,\mu,0)=\frac{1}{8\pi^4}G_\sigma\Bigg[&M^{p*}\left(\mu^{p*}\sqrt{\mu^{p*2}-M^{p*2}}-M^{p*2}\times\ln\left|\frac{\mu^{p*}+\sqrt{\mu^{p*2}-M^{p*2}}}{M^{p*}}\right|\right)\\&+M^{n*}\left(\mu^{n*}\sqrt{\mu^{n*2}-M^{n*2}}-M^{n*2}\times\ln\left|\frac{\mu^{n*}+\sqrt{\mu^{n*2}-M^{n*2}}}{M^{n*}}\right|\right)\Bigg]^2\\&+\frac{1}{18\pi^4}G_\omega\left(k_{F_p}^3+k_{F_n}^3\right)^2\\&-\frac{1}{72\pi^4}G_\rho\left(k_{F_p}^3-k_{F_n}^3\right)^2\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{72\pi^4}(G_\sigma - 2G_\omega)\left(k_{F_p}^6 + k_{F_n}^6\right) \\
&+\frac{1}{72\pi^4}\frac{G_\rho}{2}\left(k_{F_p}^6 - k_{F_n}^6\right) \\
&+\frac{1}{8\pi^2}\Bigg[\mu^{p*}(2\mu^{p*2} - M^{p*2})\sqrt{\mu^{p*2} - M^{p*2}} \\
&- M^{p*4}\ln\left|\frac{\mu^{p*} + \sqrt{\mu^{p*2} - M^{p*2}}}{M^{p*}}\right| \\
&+\mu^{n*}(2\mu^{n*2} - M^{n*2})\sqrt{\mu^{n*2} - M^{n*2}} \\
&- M^{n*4}\ln\left|\frac{\mu^{n*} + \sqrt{\mu^{n*2} - M^{n*2}}}{M^{n*}}\right|\Bigg] \\
&-\frac{1}{32\pi^4}(G_\sigma + 4G_\omega + G_\rho) \\
&\times\Bigg[M^{p*}\Bigg(\mu^{p*}\sqrt{\mu^{p*2} - M^{p*2}} - M^{p*2} \\
&\times\ln\left|\frac{\mu^{p*} + \sqrt{\mu^{p*2} - M^{p*2}}}{M^{p*}}\right|\Bigg)\Bigg]^2 \\
&-\frac{1}{32\pi^4}(G_\sigma + 4G_\omega - G_\rho) \\
&\times\Bigg[M^{n*}\Bigg(\mu^{n*}\sqrt{\mu^{n*2} - M^{n*2}} - M^{n*2} \\
&\times\ln\left|\frac{\mu^{n*} + \sqrt{\mu^{n*2} - M^{n*2}}}{M^{n*}}\right|\Bigg)\Bigg]^2 - \mu\rho_B,
\end{aligned}
\tag{3.5}
$$

with

$$
\begin{aligned}
\rho_B &= \frac{1}{3\pi^2}\left(k_{F_p}^3 + k_{F_n}^3\right), \\
\rho_3 &= \rho_p - \rho_n = \frac{1}{3\pi^2}\left(k_{F_p}^3 - k_{F_n}^3\right).
\end{aligned}
\tag{3.6}
$$

The masses of nucleon and mesons are chosen to be $M = 939$ MeV, $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV, and $m_\rho = 770$ MeV.

The numerical calculation therefore is ready to be carried out step by step as follows. We first fix the coupling constants G_σ and G_ω. To this end, Eq. (3.3) or Eq. (3.4) is solved numerically for symmetric nuclear matter ($G_\rho = 0$). Its solution is then substituted into the nuclear binding energy $\epsilon_{\rm bind}$ in Eq. (2.13) with V given in Eq. (3.5) and ρ_B and ρ_3 given in Eq. (3.6). Two parameters, g_σ and g_ω, are adjusted to yield the the binding energy $E_{\rm bind} = -15.8$ MeV at normal density $\rho_B = \rho_0 = 0.16\ {\rm fm}^{-3}$ as is shown in Fig. 3. The corresponding values for G_σ and G_ω are $G_\sigma = 195.6/M^2$ and $G_\omega = 1.21G_s$.

As to fixing G_ρ let us employ the expansion of NSE around ρ_0,

$$
E_{\rm sym} = a_4 + \frac{L}{3}\left(\frac{\rho_B - \rho_0}{\rho_0}\right) + \frac{K_{\rm sym}}{18}\left(\frac{\rho_B - \rho_0}{\rho_0}\right)^2 + \cdots,
$$

with a_4 being the bulk symmetry parameter of the Weiszaecker mass formula. Experimentally we know $a_4 = 30{-}35$ MeV.

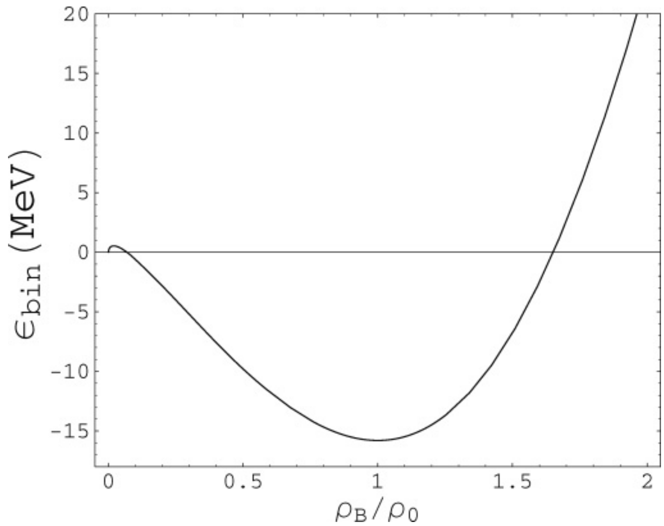

FIG. 3. The ρ_B dependence of $\epsilon_{\rm bind}$ in symmetric nuclear matter.

L and $K_{\rm sym}$ are related, respectively, to slope and curvature of NSE at ρ_0

$$
\begin{aligned}
L &= 3\rho_0\left(\frac{\partial E_{\rm sym}}{\partial\rho_B}\right)_{\rho_B=\rho_0}, \\
K_{\rm sym} &= 9\rho_0^2\left(\frac{\partial^2 E_{\rm sym}}{\partial\rho_B^2}\right)_{\rho_B=\rho_0}.
\end{aligned}
$$

Then G_ρ is fitted to give $a_4 = 32$ MeV; its value is $G_\rho = 0.972G_\sigma$. Thus, all of the model parameters are known. Let us now determine the density dependence of NSE. Taking into account Eqs. (1.2), (2.13), (3.5), and (3.6) altogether and then carrying out the numerical computation with the aid of *The Mathematica Book* [25], we obtain Fig. 4; here,

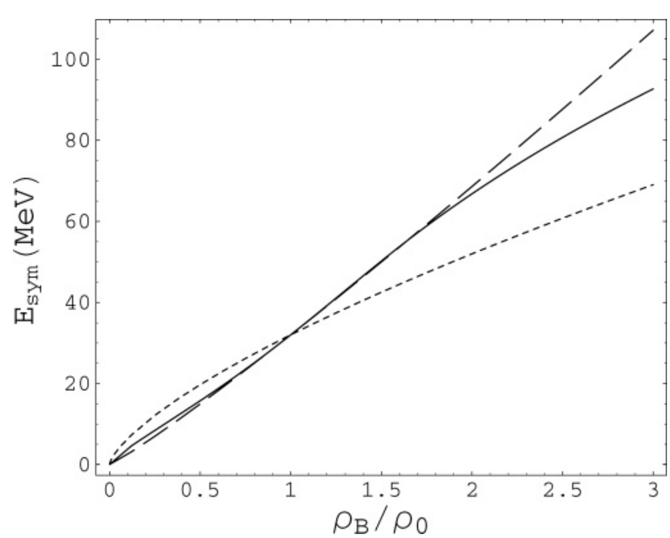

FIG. 4. The ρ_B/ρ_0 dependence of $E_{\rm sym}$ (solid line), E_1 (dotted line) and E_2 (dashed line).

for comparison, we also depict the graphs of the functions $E_1 = 32(\rho_B/\rho_0)^{0.7}$ and $E_2 = 32(\rho_B/\rho_0)^{1.1}$.

It is easily verified that $E_{\rm sym}(\rho_B)$ with the graph given in Fig. 4 can be approximated by the function

$$E_{\rm sym} \approx 32(\rho_B/\rho_0)^{1.05}.$$

The preceding expression for NSE is clearly in agreement with the analysis of Refs. [14,15,26], in which the experimental data [27] were processed within the isospin- and momentum-dependent IBUU04 transport model [28]

$$32(\rho_B/\rho_0)^{0.7} < E_{\rm sym}(\rho_B) < 32(\rho_B/\rho_0)^{1.1}.$$

To proceed further let us go to the isobaric incompressibility of ANM, which at saturation density can be expanded around $\alpha = 0$ to second order in α as [17,29]

$$K(\alpha) \approx K_0 + K_{\rm asy}\alpha^2.$$

with $K_{\rm asy}$ being the isospin-dependent part [30]

$$K_{\rm asy} \approx K_{\rm sym} - 6L.$$

$K_{\rm asy}$ can be extracted from experimental measurements of giant monopole resonances in neutron-rich nuclei. K_0 is the incompressibility of symmetric nuclear matter at ρ_0.

In the following are given, respectively, the computed values of parameters directly connected with NSE:

(i) The slop parameter $L = 105.997$ MeV, which is consistent with the result of Refs. [15] and [26].
(ii) The symmetry pressure $P_{\rm sym} = \rho_0 L/3 = 4.34\ \ 10^7\ \ {\rm MeV}^4 = 0.0286\ {\rm fm}^{-4}$, which is very useful for structure studies of nuclei.
(iii) $K_{\rm asy} = -549.79$ MeV. This value is in good agreement with the one determined from the use of in-medium NN cross section in the IBUU04 model, $K_{\rm asy} = -500 \pm 50$ MeV [13–15,26,31].
(iv) $K_0 = 547.56$ MeV.

Finally the shift of the nuclear saturation density with asymmetry at lowest order in α reads

$$\Delta\rho_0 = -\frac{3\rho_0 L}{K_0} = -713661\ {\rm MeV}^3 = -0.093\ {\rm fm}^{-3}.$$

IV. CONCLUSION AND OUTLOOK

Developing the previous work [19] we have carried out in this article a more realistic study concerning isospin degree of freedom of ANM. The EOS of ANM given in Eq. (1.1) and Eq. (2.13) is our principal result. The DBA was used to compute numerically the density dependence of NSE and other physical quantities of ANM. The obtained results are quite consistent with recent works, except for K_0, which is too large. This is the shortcoming of the present model. It is proved [32,33] that within the mean-field approximation the nonlinear point-coupling model yielded softer incompressibility of symmetric nuclear matter when higher-order interactions of nucleons were taken into consideration. In this respect, studying this model beyond the mean-field theory [34] is probably a very important problem for understanding many important nuclear properties. It is evident that EOS of ANM is a fundamental issue for both nuclear physics and astrophysics. It governs phase transitions in ANM. However, we should bear in mind the fact that phase transitions are basically nonperturbative phenomena. Therefore, in this research domain we really need a nonperturbative approach. It is our EOS that was obtained by means of the CJT effective action formalism, a famous nonperturbative method of quantum field theory, and, as a consequence, it could be most suitable for the study of phase transitions and other nuclear properties beyond mean-field approximation [35].

ACKNOWLEDGMENT

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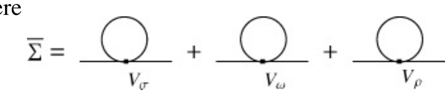
[34] The nucleon propagator $S(x)$ fulfills the equation

$$[i\partial\!\!\!/ - (M - g_\sigma\sigma) - g_\omega\gamma^0\omega + g_\rho\tau_3\gamma^0\rho - \bar{\Sigma}]S(x-y) = i\delta(x-y),$$

where



in the double-bubble approximation, which is also called the Hartree-Fock approximation.
[35] Tran Huu Phat, Nguyen Tuan Anh, and Nguyen Van Long, Phase transition of ANM in the CJT formalism (in preparation).