Classes of β-γ mixing and $E0$ transitions in deformed nuclei

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We investigate the effects of β-γ mixing using branching ratios between the 2^+_γ level and the ground state band in well-deformed nuclei. We find that the deviations from the well-known Alaga rules vary as a function of the energy separation between the 2^+_γ and 2^+_β levels. For nuclei where these two intrinsic excitations are nearly degenerate, we find two classes of behavior. For one of those, the systematics can be reproduced in a simple bandmixing formalism but only with an anomalously strong interaction strength between the 2^+_γ and 2^+_β levels, on the order of tens of keV. This result is supported by $X(E0/E2; 2^+_\gamma \to 2^+_1)$ values that are large for these nuclei (where measured), indicating significant $K = 0$ components in the 2^+_γ levels. In the other class, there is virtually no mixing. These nuclei have previously been associated with near-SU(3) structure.

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The deformed axially symmetric rotor [1] has been a benchmark for describing structure for over 50 years. To first order, it provides a starting point for describing and understanding energies and electromagnetic transition strengths. $E2$ transition strengths, in particular, can provide sensitive and important information on collective structural effects. In the case of a deformed axially symmetric rotor, the rotational and intrinsic excitations can be decoupled, at least at low spin. This allows the $E2$ transition matrix element between members of two collective bands to be expressed as a product of a Clebsch-Gordon coefficient and a reduced matrix element, the latter being independent of the spins of the levels involved. A ratio of reduced $E2$ transition probabilities between members of such bands is then just a ratio of the squares of the appropriate Clebsch-Gordan coefficients, as given by the well-known Alaga rules [2].

It is a well-studied phenomenon that, especially for the γ band, the simple predictions given by the Alaga rules are not completely fulfilled due to the small breakdown of the adiabatic assumption. Nonadiabatic coupling is usually described through bandmixing between the ground band and γ and β bands. We note that, within this article, we use the term "β" band simply as a label for the first excited $K = 0^+$ band, because microscopically, such excitations are forbidden within a major shell [1]. The formalism for bandmixing was discussed by Lipas [3], followed by its application to a number of well-deformed nuclei [4–6]. Usually, bandmixing is constrained to an analysis in terms of $\Delta K = 2$ mixing between the γ band and ground band. Including higher order mixing with the β band has been attempted, but is usually less successful.

It is the purpose of the present article to investigate deviations from the Alaga rules systematically for a wide range of nuclei in the well-deformed rare-earth region. We analyze deviations in branching ratios in terms of a simple bandmixing formalism incorporating mixing between the γ, β, and ground bands. Contrary to previous studies, we find that for a significant number of nuclei, where the energies of the 2^+_γ and 2^+_β levels are close, a quite large mixing matrix element between β and γ is required to reproduce the systematics. In a second class of nuclei where these energies are also nearly degenerate, there is little or no mixing. These results are further supported by an analysis of $X(E0/E2; 2^+_\gamma \to 2^+_1)$ values in these nuclei. The key to this analysis will be to view the mixing as a function of the *energy difference* between the β and γ bands.

In Fig. 1(a), the branching ratio, $B(E2; 2^+_\gamma \to 0^+_1)/B(E2; 2^+_\gamma \to 4^+_1)$ is plotted as a function of mass number, A. In the Alaga limit, this ratio takes on a value of 14.3. We constrain the data in Fig. 1 to the rare-earth region with $60 \leqslant Z \leqslant 76$ and $R_{4/2} \equiv E(4^+_1)/E(2^+_1) > 2.90$ to consider only those nuclei close to deformed or well-deformed. This branching ratio was chosen as a starting point, because the transitions can be taken as pure $E2$ and therefore knowledge of an $E2/M1$ mixing ratio is not required. The data points show complete scatter with no obvious correlation.

In Fig. 1(b), the same branching ratio is plotted now as a function of the energy difference between the 2^+_γ and 2^+_β levels. Plotted in this way, several distinct patterns now emerge from the data. First, there is a group of nuclei where the energies of the 2^+_γ and 2^+_β levels are nearly equal and the branching ratio is close to the Alaga predictions. These two features are precisely those of the SU(3) limit of the Interacting Boson Approximation (IBA) model [8]. The nuclei in this group include ^{156,158}Gd, ^{170,176}Yb, and ^{176,178,180}Hf. These nuclei are indicated by the open circles in Fig. 1(b). The Gd nuclei were one of the first proposed [9] examples of SU(3) behavior and the heavy Yb and Hf nuclei near $N = 104$ have also been discussed [10] in terms of SU(3) characteristics. The focus of the present discussion will be on a second pattern observed in Fig. 1(b). There is another group of nuclei that also have nearly equal 2^+_γ and 2^+_β energies, but the branching ratios have very small values, ∼0. Third, in nuclei where the 2^+_γ and 2^+_β levels become further apart in energy, there is a systematic trend that, with increasing energy separation between 2^+_γ and 2^+_β, the branching ratio increases toward the Alaga ratio.

In Fig. 2, the branching ratio $B(E2; 2^+_\gamma \to 2^+_1)/B(E2; 2^+_\gamma \to 0^+_1)$ is plotted, again as a function of the energy separation between the 2^+_γ and 2^+_β levels. In the Alaga limit,

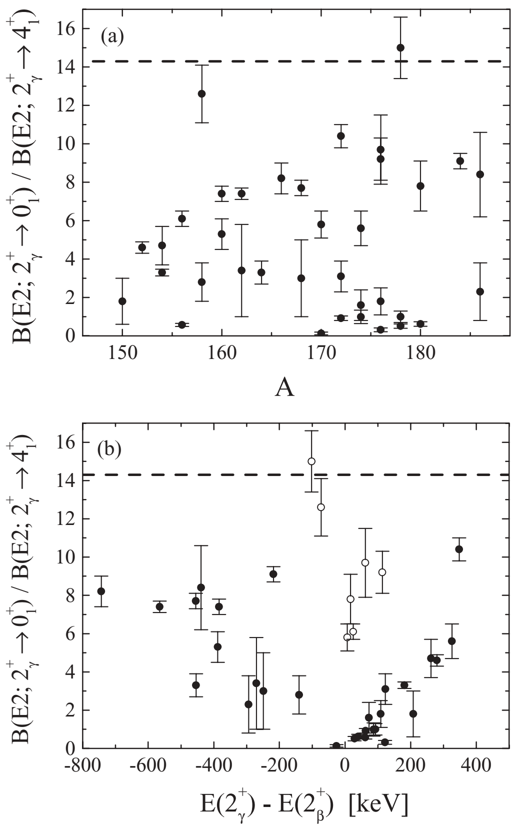

FIG. 1. Branching ratio, $B(E2;2^+_\gamma \to 0^+_1)/B(E2;2^+_\gamma \to 4^+_1)$, plotted as (a) a function of mass number, A, and (b) a function of the energy difference between the 2^+_γ and 2^+_β levels. Nuclei included in the figure are the Nd-Os isotopes that have $R_{4/2} > 2.90$. Data are from Ref. [7]. The dashed line indicates the value obtained in the Alaga limit. In the case of panel (b), open symbols denote the SU(3)-like nuclei described in the text.

this ratio takes on a value of 1.43. For this branching ratio, the $E2/M1$ mixing ratio for the $2^+_\gamma \to 2^+_1$ transition has not been measured for some of the nuclei. The corresponding branching ratios are included in Fig. 2 (denoted by triangles with downward arrows) assuming pure $E2$ character for the transitions and can be taken as upper limits. The SU(3)-like nuclei described above are again denoted by open circles.

The SU(3)-like nuclei mentioned above all have values of $B(E2;2^+_\gamma \to 2^+_1)/B(E2;2^+_\gamma \to 0^+_1)$ close to the Alaga limits. For most nuclei, this branching ratio lies above the Alaga predictions, with the exception of a few nuclei, again with the 2^+_β and 2^+_γ levels close in energy, which have ratios below the Alaga limits.

We now attempt to describe the overall features of Figs. 1 and 2 using simple mixing of the γ, β, and ground state bands. Since this approach has been well described in previous work [3], we omit a detailed description of the formalism. We follow the convention given in Ref. [3] and for convenience present the final results for the first- and second-order effects of mixing of the γ band into the ground band and the mixing between the γ band and β band, respectively, in Table I. The parameters

TABLE I. Correction factors for the reduced $E2$ transition strengths between states of the γ band and the ground band. Adopted from Ref. [3].

I_i	I_f	$\frac{B(E2;I_i\to I_f)}{B_o(E2;I_i\to I_f)}$
$I-2$	I	$[1+(2I+1)Z_\gamma + I(I-1)Z_{\beta\gamma}]^2$
$I-1$	I	$[1+(I+2)Z_\gamma]^2$
I	I	$[1+2Z_\gamma - \frac{1}{3}I(I+1)Z_{\beta\gamma}]^2$
$I+1$	I	$[1-(I-2)Z_\gamma]^2$
$I+2$	I	$[1-(2I+1)Z_\gamma + (I+1)(I+2)Z_{\beta\gamma}]^2$

Z_γ and $Z_{\beta\gamma}$ are measures of the mixing amplitude between the γ and ground band and the γ and β bands, respectively.

We first consider if the deviations from the Alaga rules shown in Figs. 1 and 2 can be explained by simple $\Delta K = 2$ mixing between the γ band and ground band, effectively setting $Z_{\beta\gamma} = 0$, using the corrections given in Table I. Figure 3(a) gives the branching ratios from the γ band to the ground state band, $B(E2;2^+_\gamma \to 0^+_1)/B(E2;2^+_\gamma \to 4^+_1)$ and $B(E2;2^+_\gamma \to 2^+_1)/B(E2;2^+_\gamma \to 0^+_1)$, as a function of only the γ-ground mixing term, Z_γ. We consider here only positive values of Z_γ, consistent with previous determinations. To obtain small values of the branching ratio $B(E2;2^+_\gamma \to 0^+_1)/B(E2;2^+_\gamma \to 4^+_1)$, significantly large values of Z_γ (>0.1) are required. For the ratio $B(E2;2^+_\gamma \to 2^+_1)/B(E2;2^+_\gamma \to 0^+_1)$, all values are 1.43 or greater if just the Z_γ term is used to describe the mixing.

By comparing the γ to ground mixing calculations in Fig. 3(a) with the data in Figs. 1(b) and 2, it becomes evident that for over half of the nuclei in Fig. 1, the deviations from the Alaga rules cannot be explained through simple mixing of the γ band with the ground band. In particular, those nuclei in Fig. 1(b) in which the branching ratio is less than 3 would require very large values of Z_γ.

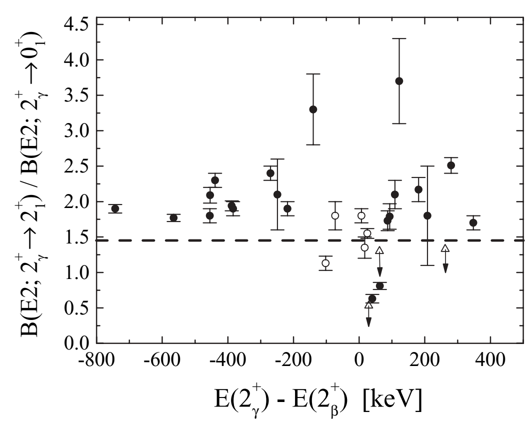

FIG. 2. Branching ratio, $B(E2;2^+_\gamma \to 2^+_1)/B(E2;2^+_\gamma \to 0^+_1)$, plotted as a function of the energy difference between the 2^+_γ and 2^+_β levels. Nuclei included in the figure are the Nd-Os isotopes that have $R_{4/2} > 2.90$. Data are from Ref. [7]. The dashed line indicates the value obtained in the Alaga limit. Open symbols denote the SU(3)-like nuclei described in the text. Open triangles with downward arrows represent upper limits for those nuclei where the $E2/M1$ mixing ratio has not been measured.

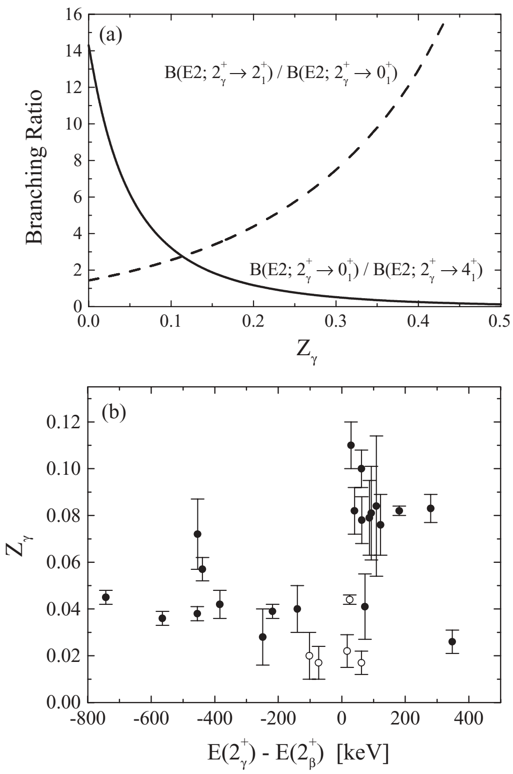

FIG. 3. (a) Branching ratios from the γ band to the ground state band as a function of Z_γ calculated using simple $\Delta K = 2$ bandmixing between the γ band and ground band. (b) Experimental Z_γ values extracted using the branching ratio from the 3^+_γ level, $B(E2; 3^+_\gamma \to 2^+_1)/B(E2; 3^+_\gamma \to 4^+_1)$ plotted as a function of the energy difference between the 2^+_γ and 2^+_β levels. Open symbols denote the group of SU(3)-like nuclei described in the text.

To proceed, we first extract values of the Z_γ parameter that are independent of mixing with the β band. That is, we determine the mixing parameter Z_γ from experimental data using the ratio $B(E2; 3^+_\gamma \to 2^+_1)/B(E2; 3^+_\gamma \to 4^+_1)$. Using the decay of the odd-spin member of the γ band allows for isolation of the just the Z_γ term, as given in Table I. Considering only those nuclei where the $E2/M1$ mixing ratio is known for the above transitions, the calculated Z_γ values are given in Fig. 3(b), plotted as a function of the energy difference between the 2^+_γ and 2^+_β levels. The extracted Z_γ values range from 0.02 to 0.10, in agreement with previous systematic studies [11]. Although some scatter is present, there is an overall systematic behavior that is observed. On average, nuclei with $|E(2^+_\gamma)$-$E(2^+_\beta)| <$ 300 keV have Z_γ values ~0.08–0.10, whereas for nuclei with $|E(2^+_\gamma) - E(2^+_\beta)| >$ 300 keV, the Z_γ values are mostly between 0.03 and 0.04.

We now determine if including mixing between the γ and β bands can account for the behavior in Figs. 1(b) and 2 and extract the necessary strength required. To do this, we effectively work "backward," assuming different matrix elements between the γ and β bands and calculate the corresponding effect on the branching ratios. In the limit of small mixing, the mixing amplitude (amplitude of unperturbed β state in γ state) can be written as

$$\epsilon'_{\beta\gamma}(J) \sim \frac{\langle \phi_\beta(J)|V_{\Delta K=2}|\phi_\gamma(J)\rangle}{[E_\gamma(J) - E_\beta(J)]_{\rm unp}} \equiv \frac{V_{\beta\gamma}}{\Delta E_{\gamma\beta}}, \qquad (1)$$

where, for the sake of simplicity, we have neglected the difference between perturbed and unperturbed spacings. The spin independent part of the mixing amplitude, $\epsilon_{\beta\gamma}$, is related to $\epsilon'_{\beta\gamma}(J)$ by

$$\epsilon'_{\beta\gamma}(J) = \epsilon_{\beta\gamma}\sqrt{(J-1)J(J+1)(J+2)}, \qquad (2)$$

which reduces to $\epsilon'_{\beta\gamma} = \sqrt{24}\epsilon_{\beta\gamma}$ for $J = 2$. The band mixing parameter $Z_{\beta\gamma}$ is given by

$$Z_{\beta\gamma} = \sqrt{6}\frac{Q_\beta}{Q_\gamma}\epsilon_{\beta\gamma}, \qquad (3)$$

with $Q_\beta = \sqrt{B(E2; 2^+_\beta \to 0^+_1)}$ and, similarly, $Q_\gamma = \sqrt{B(E2; 2^+_\gamma \to 0^+_1)}$. There are several well-deformed nuclei where the above absolute transition strengths have been measured. For example, the ratio of $B(E2; 2^+_\beta \to 0^+_1)/B(E2; 2^+_\gamma \to 0^+_1)$ in ^{156}Gd [12], ^{160}Dy [13], ^{172}Yb [14], and ^{176}Hf [15] is measured as 0.14(1), 0.13(2), 0.18(2), and 0.25(6), respectively. We use an average value of $B(E2; 2^+_\beta \to 0^+_1)/B(E2; 2^+_\gamma \to 0^+_1) = 0.15$, leading to $Z_{\beta\gamma} \sim 0.90\epsilon_{\beta\gamma}$.

Combining the above, $Z_{\beta\gamma}$ can be written as

$$Z_{\beta\gamma} \sim \frac{0.2V_{\beta\gamma}}{\Delta E_{\gamma\beta}}, \qquad (4)$$

which involves the absolute β-γ mixing matrix element and the spacing between the 2^+_γ and 2^+_β levels.

We take the form of $Z_{\beta\gamma}$ given in Eq. (4) for use in the correction factors given in Table I. Values for the $B(E2; 2^+_\gamma \to 0^+_1)/B(E2; 2^+_\gamma \to 4^+_1)$ branching ratio, calculated for several different values of the β-γ mixing matrix element are given in Fig. 4. Figure 4 (top) uses a Z_γ value of 0.035, Fig. 4 (middle) uses a Z_γ value of 0.08, and the bottom panel of Fig. 4 is a combination of the Z_γ values for different energy ranges: $Z_\gamma = 0.035$ in the range $|\Delta E_{\gamma\beta}| >$ 300 keV and $Z_\gamma = 0.08$ in the range $|\Delta E_{\gamma\beta}| <$ 300 keV.

With a Z_γ value of 0.035, the nuclei with a large energy separation between the 2^+_β and 2^+_γ levels are well described by a small $\beta\gamma$ mixing matrix element. For the region of the small $\Delta E_{\gamma\beta}$, however, the predicted branching ratio rises sharply and significantly overpredicts the data. For small values of $\Delta E_{\gamma\beta}$, the data can only be described by significantly larger values of $V_{\beta\gamma}$. Increasing the value of Z_γ to 0.08, as in the middle panel of Fig. 4, results in an overall decrease in the value of the branching ratio. This is clearly not applicable to those nuclei with large values of $\Delta E_{\gamma\beta}$, but does provide a better description of the nuclei with small $\Delta E_{\gamma\beta}$. A rather large $\beta\gamma$ matrix element is also required to reproduce the systematic trend observed in the data. Considering both the systematics observed in Z_γ and the results obtained in the top and middle panels of Fig. 4, we couple two sets of calculations using

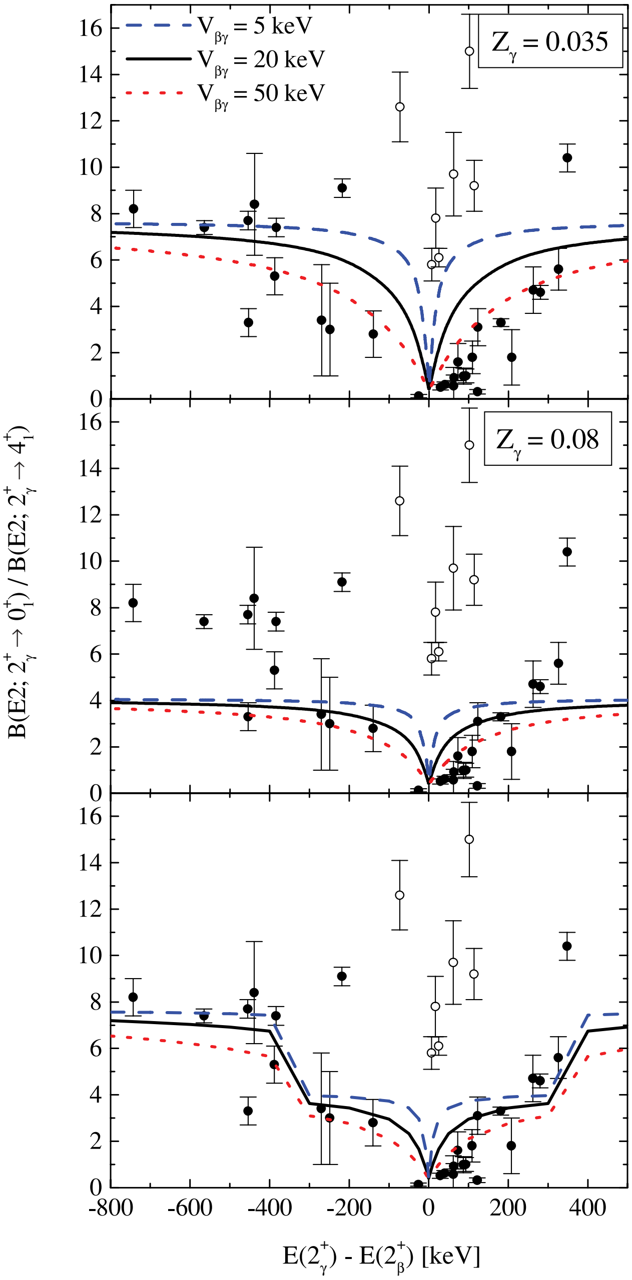

FIG. 4. (Color online) Branching ratio, $B(E2; 2^+_\gamma \to 0^+_1)/B(E2; 2^+_\gamma \to 4^+_1)$, plotted as a function of the energy difference between the 2^+_γ and 2^+_β levels. The data are the same as in Fig. 1. The lines represent calculations using the correction factors given in Table I for different strengths for the matrix element between the 2^+_γ and 2^+_β levels. In the top (middle) panel a Z_γ value of 0.035 (0.08) is used in the calculation. The bottom panel is a combination of the top and middle panels with $Z_\gamma = 0.035$ used in the range $|\Delta E_{\gamma\beta}| > 300$ keV and $Z_\gamma = 0.08$ used in the range $|\Delta E_{\gamma\beta}| < 300$ keV.

$Z_\gamma = 0.035$ in the range $|\Delta E_{\gamma\beta}| > 300$ keV and $Z_\gamma = 0.08$ in the range $|\Delta E_{\gamma\beta}| < 300$ keV, in the bottom panel of Fig. 4.

In Fig. 5, we apply the same analysis to the branching ratio $B(E2; 2^+_\gamma \to 2^+_1)/B(E2; 2^+_\gamma \to 0^+_1)$, varying Z_γ in the energy ranges described above. The data for large $\Delta E_{\gamma\beta}$ are again well described by $Z_\gamma = 0.035$ and a small $V_{\beta\gamma}$ term.

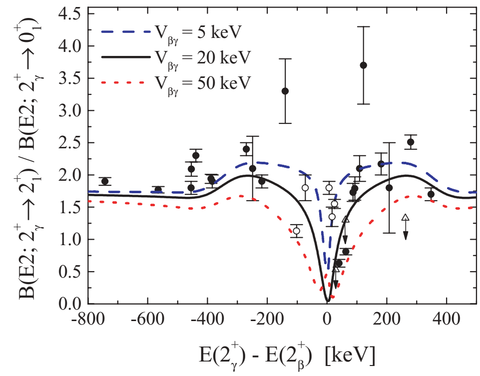

FIG. 5. (Color online) Branching ratio, $B(E2; 2^+_\gamma \to 2^+_1)/B(E2; 2^+_\gamma \to 0^+_1)$, plotted as a function of the energy difference between the 2^+_γ and 2^+_β levels. The data are the same as in Fig. 2. The lines represent calculations using the correction factors given in Table I and assuming different strengths for the matrix element between the 2^+_γ and 2^+_β levels. The Z_γ values used in the calculations are $Z_\gamma = 0.035$ in the range $|\Delta E_{\gamma\beta}| < 300$ keV and $Z_\gamma = 0.08$ in the range $|\Delta E_{\gamma\beta}| < 300$ keV.

Excluding the SU(3)-like nuclei, the data for nuclei with small $\Delta E_{\gamma\beta}$ are also again well described by the larger Z_γ term and a large $\beta\gamma$ mixing matrix element. It is clear, however, that the data are lacking in the region for $\Delta E_{\gamma\beta}$ values between -300 and 0 keV, that would firmly distinguish which interaction strength is most appropriate. In addition, this branching is not very sensitive to the mixing matrix element. For example, for $\Delta E_{\gamma\beta} = 200$, $V_{\beta\gamma} = 5$ keV predicts a branching ratio of 2.2 whereas $V_{\beta\gamma} = 20$ keV predicts a branching ratio of 1.9. For a number of points, these predictions lie within the range of the experimental error. There are several data points, however, that still support a strong interaction strength between the 2^+_γ and 2^+_β levels.

In formulating the above analysis, we assumed that the deviations observed in the branching ratios of the $2^+_\gamma \to$ ground transitions stem entirely from mixing with the ground band and the β band. While the correlation between the variation in the Alaga ratios and the energy difference between the 2^+_γ and 2^+_β levels suggests that the β band in fact is responsible for the observed deviations, we cannot rule out mixing from higher lying states using an analysis of only $E2$ ratios.

The amount of $K = 0$ mixing into the γ band can be further investigated using the ratio of $E0$ and $E2$ reduced transition probabilities in the $2^+_\gamma \to 2^+_1$ transition. The $X(E0/E2)$ ratio can be determined experimentally from

$$X(E0/E2; 2^+_\gamma \to 2^+_1) = 2.54 \times 10^9 A^{4/3} E^5_\gamma q^2(E0/E2; 2^+_\gamma \to 2^+_1) \frac{\alpha_K(E2)}{\Omega_K(Z,k)}, \quad (5)$$

where E_γ is in units of MeV and $\Omega_K(Z,k)$ is an electronic factor available from the tables given in Ref. [16]. The $E0/E2$ mixing ratio is calculated from the experimental and

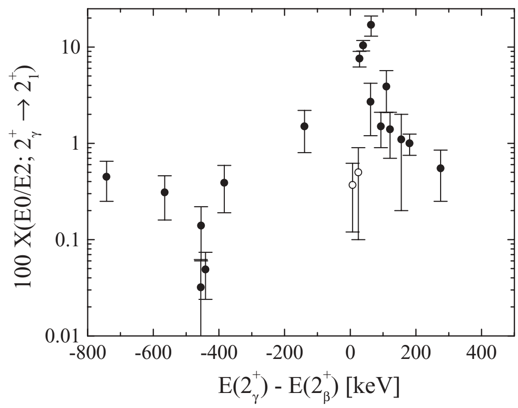

FIG. 6. $X(E0/E2; 2^+_\gamma \to 2^+_1)$ ratio plotted as a function of the energy difference between the 2^+_γ and 2^+_β levels. Data are from Ref. [7], except for a select number of W and Os isotopes that are taken from Refs. [17] and [18].

theoretical K conversion as

$$q^2(E0/E2) \sim \frac{\alpha(\exp)}{\alpha(E2)} - 1 \tag{6}$$

in the simplest case where the $M1$ component of the transition is negligible. Because $E0$ transitions follow the $\Delta K = 0$ selection rule, $X(E0/E2; 2^+_\gamma \to 2^+_1) > 0$ would indicate the existence of a $K = 0$ component in the 2^+_γ band level.

The experimental $X(E0/E2; 2^+_\gamma \to 2^+_1)$ values are given in Fig. 6, plotted as a function of the energy separation between the 2^+_β and 2^+_γ levels. The errors associated with the $X(E0/E2)$ values are often large, on the order of 50%. Nevertheless, there is often more than an order of magnitude difference between the experimental values when $\Delta E_{\gamma\beta}$ is large (small values of $X(E0/E2)$) compared to when $\Delta E_{\gamma\beta}$ is small. Indeed, for $|\Delta E_{\gamma\beta}| < 300$ keV, there is a significant contribution of $E0$ strength in the $X(E2/E0)$ ratio. This re-affirms the argument from the $E2$ transitions that large β-γ matrix elements are needed to reproduce the $E2$ branching ratios in these nuclei. The only exception are (open symbols in Fig. 6) nuclei close in structure to SU(3) where the mixing is minimal and $X(E0/E2)$ is small as well.

In conclusion, we applied a simple bandmixing formalism incorporating mixing between the ground, β, and γ bands to the $E2$ branching ratio from the 2^+_γ state in well-deformed rare-earth nuclei. It is found that in those nuclei where the 2^+_γ and 2^+_β levels are well separated in energy, γ-ground mixing and a small mixing matrix element between the β and γ bands reasonably reproduce the experimental branching ratios. However, there seems to be two distinct classes of deformed nuclei with nearly degenerate β and γ bands: those near the SU(3) limit where these two intrinsic states belong to the same representation and have virtually no mixing, and a second class with nearly maximal mixing and anomalously large mixing matrix elements. Evidence that this large mixing matrix element indeed arises from mixing with the $K = 0$, β excitation is supported by large $X(E0/E2; 2^+_\gamma \to 2^+_1)$ ratios in nuclei where the energies of the 2^+_γ and 2^+_β levels are similar. The $X(E0/E2)$ values are a highly sensitive signature of such mixing.

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[1] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II.

[2] G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. **29**, 9 (1955).

[3] P. O. Lipas, Nucl. Phys. **39**, 468 (1962).

[4] B. R. Mottelson, J. Phys. Soc. Jpn. Suppl. **24**, 87 (1968).

[5] L. L. Riedinger, N. R. Johnson, and J. H. Hamilton, Phys. Rev. **179**, 1214 (1969).

[6] J. H. Hamilton, F. E. Coffman, A. V. Ramayya, and K. Baker, Phys. Rev. C **3**, 960 (1971).

[7] http://www.nndc.bnl.gov/ensdf.

[8] A. Arima and F. Iachello, Ann. Phys. (NY) **99**, 253 (1976); **111**, 201 (1978); **123**, 468 (1979).

[9] A. Arima and F. Iachello, in *Advances in Nuclear Physics*, edited by J. Negele and E. Vogt (Plenum, New York, 1984), Vol. **13**, p. 139.

[10] R. F. Casten, P. von Brentano, and A. M. I. Haque, Phys. Rev. C **31**, 1991(R) (1985).

[11] R. F. Casten, D. D. Warner, and A. Aprahamian, Phys. Rev. C **28**, 894 (1983).

[12] C. W. Reich, Nucl. Data Sheets **99**, 753 (2003).

[13] C. W. Reich, Nucl. Data Sheets **105**, 557 (2005).

[14] Balraj Singh, Nucl. Data Sheets **75**, 199 (1995).

[15] M. S. Basunia, Nucl. Data Sheets **107**, 791 (2006).

[16] D. A. Bell, C. E. Aveledo, M. G. Davidson, and J. P. Davidson, Can. J. Phys. **48**, 2542 (1970).

[17] T. Kibedi, G. D. Dracoulis, A. P. Byrne, and P. M. Davidson, Nucl. Phys. **A688**, 669 (2001).

[18] T. Kibedi, G. D. Dracoulis, A. P. Byrne, P. M. Davidson, and S. Kuyucak, Nucl. Phys. **A567**, 183 (1994).