PHYSICAL REVIEW C **77**, 054314 (2008)

Quark mean field approach with derivative coupling for nuclear matter

M. Kawabata,[*] S. Akiyama,[†] Y. Futami,[‡] T. Nakasone, and T. Yukino
Department of Physics, Faculty of Science and Technology, Tokyo University of Science, 2641, Noda, Chiba 278-8510, Japan
(Received 20 February 2008; published 22 May 2008)

We propose the quark mean field model including derivative coupling between quarks and scalar mesons in nuclear matter. This model concisely interprets an increasing size of the nucleon as well as a modification of coupling constant in the nuclear environment.

DOI: 10.1103/PhysRevC.77.054314 PACS number(s): 21.65.Mn, 12.39.Ki, 21.30.Fe

I. INTRODUCTION

It is interesting to find a certain relation between quark-gluon degrees of freedom and relativistic meson-nucleon models for nuclear many-body systems. Many studies have been done from this point of view, since Guichon [1] proposed the quark-meson coupling (QMC) model. This model describes nuclear matter as nonoverlapping MIT bags and the quarks inside them couple to scalar σ and vector ω mesons. The QMC model has been extended with reasonable success to various problems of nuclear matter and finite nuclei [2–6]. On the other hand, the quark mean field (QMF) model [7,8] takes the constituent quark model for the nucleon and includes nonlinear self-energy terms in the meson Lagrangian. This model has been applied to study the properties of finite nuclei with success. In particular Tan *et al.* [9] introduced density dependent couplings for the σ and ω mesons with quarks, modeled as functions of the σ mean field for the QMF model, and parametrized the couplings to reproduce the relativistic Brueckner-Hartree-Fock (RBHF) results [10] of nuclear matter. Though this model gave also successful descriptions of finite nuclei, it has many adjustable parameters.

In the present investigation, we introduce a derivative scalar coupling between constituent quarks and scalar mesons for nuclear matter. This idea is essentially an application to quark-meson system of Zimanyi and Moszkouski (ZM) model [11] based on hadronic degrees of freedom. The ZM model differs from the Walecka model [12] only in the form of the coupling of the nucleon to the scalar meson, and can be understood as a model which introduces a effective σ-nucleon coupling constant as a function of the σ meson field, without increasing the number of free parameters. Our model substitutes constituent quarks for nucleons. Therefore, the quark coupling with the σ meson has the σ dependence. Furthermore, the confinement potential has also the σ dependence and is modified in the nuclear medium. We report on several findings resulting from the point of view of the derivative coupling for the QMF models.

In the next section we introduce a quark version of the derivative scalar coupling model for the symmetric nuclear matter and give the necessary detail to understand the origin of the results we obtain. In Sec. III the results are discussed comparing with the QMF models and the ZM model.

[*]kawabata@ph.noda.tus.ac.jp
[†]akiyama@ph.noda.tus.ac.jp
[‡]futami@ph.noda.tus.ac.jp

II. QUARK-MESON DERIVATIVE COUPLING MODEL FOR NUCLEAR MATTER

As the first step, we consider a quark version of the derivative coupling model (ZM model) for nuclear matter. In our case, the dynamical degrees of freedom are quark fields (q) with the constituent quark mass m_q, scalar meson fields (σ), and vector meson fields (ω). On the analogy of Ref. [11], we assume the following effective Lagrangian density for quark-meson many body systems:

$$\mathscr{L}^q = \bar{q}\left[\tilde{m}^{-1} i\gamma^\mu\partial_\mu - m_q - \chi_c - \tilde{m}^{-1} g^q_\omega \gamma^\mu\omega_\mu\right]q - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2_\omega\omega_\mu\omega^\mu + \tfrac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m^2_\sigma\sigma^2\right), \quad (1)$$

where

$$\tilde{m}^{-1} = 1 + g^q_\sigma\sigma/m_q, \quad (2)$$

g^q_σ and g^q_ω denote the quark-meson couplings, $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, and χ_c expresses the confinement potential given by the gluon dynamics. We take into account the confinement in terms of the harmonic oscillator potential $\chi_c = kr^2/2$ with $k = 1000$ MeV/fm^2 [8].

This model Lagrangian includes a derivative coupling between quarks and scalar mesons. We proceed to rescale the field q as follows: $q \to \tilde{m}^{1/2}q$. Considering that $\tilde{m}$ does not depend on space-time coordinates, the rescaled Lagrangian density is expressed by

$$\mathscr{L}^q_R = \bar{q}\left[i\gamma^\mu\partial_\mu - \left(m_q - g^{*q}_\sigma(\sigma)\sigma\right) - \chi^*_c(\sigma) - g^q_\omega\gamma^\mu\omega_\mu\right]q - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2_\omega\omega_\mu\omega^\mu + \tfrac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m^2_\sigma\sigma^2\right), \quad (3)$$

where $g^{q*}_\sigma(\sigma) = \tilde{m}g^q_\sigma$ and $\chi^*_c(\sigma) = \tilde{m}\chi_c$. Then, the quark field $q(t, \boldsymbol{r})$ in a nucleon satisfies the equation of motion

$$\left[i\gamma^\mu\partial_\mu - \left(m_q - g^{*q}_\sigma(\sigma)\sigma\right) - \chi^*_c(\sigma) - g^q_\omega\gamma^0\omega_0\right]q = 0. \quad (4)$$

This model introduces the σ dependent scalar coupling $g^{q*}_\sigma(\sigma)$ and confinement potential $\chi^*_c(\sigma)$ in nuclear medium, however, g^q_ω is not modified. When the meson fields are replaced by the constant classical fields, we can rewrite the Dirac equation as

$$[-i\boldsymbol{\alpha}\nabla + \beta(m_q - \delta m_q) + \beta\chi^*_c(\sigma)]q(\boldsymbol{r}) = \varepsilon^* q(\boldsymbol{r}). \quad (5)$$

The quark mass correction δm_q and energy ε^* are defined as

$$\delta m_q = g_\sigma^{q*}(\sigma)\sigma, \tag{6}$$
$$\varepsilon^* = \varepsilon - g_\omega^q \omega_0, \tag{7}$$

where ε is the energy of the quark under the influence of meson mean fields. The time component ω_0 of the vector field introduces a shift in the energy. We take the constituent quark mass to be one third of the nucleon mass: $m_q = M_N/3 =$ 313 MeV. The quark mass correction δm_q is related to the effective nucleon mass $M_N^*(\sigma)$. The $M_N^*(\sigma)$ is expressed as

$$M_N^*(\sigma) = \sqrt{(3\varepsilon^*(\sigma) + E_{\text{spin}})^2 - \langle \boldsymbol{p}_{\text{c.m.}}^2 \rangle_N}. \tag{8}$$

Here we follow Ref. [7] to take into account the spin correlations E_{spin} for $M_N^*(\sigma)$ and remove the spurious center of mass motion $\langle \boldsymbol{p}_{\text{c.m.}}^2 \rangle_N$. Since the three constituent quarks are independent, we have

$$\langle \boldsymbol{p}_{\text{c.m.}}^2 \rangle_N = \sum_{i=1}^{3} \langle \boldsymbol{p}_i^2 \rangle_N, \tag{9}$$

where $\langle \boldsymbol{p}_i^2 \rangle_N$ is the expectation value of the momentum squared of the ith quark in a nucleon.

We now move to the second step, in which the change of the properties of the nuclear many-body system will be solved using the change of the nucleon properties obtained in the first step. To perform the nuclear matter calculation, we use the relativistic mean field (RMF) approximation containing nucleon (ψ), neutral scalar (σ), and vector (ω) mesons. As we treat the symmetric nuclear matter, the Lagrangian density is

$$\mathscr{L}^N = \bar{\psi}[i\gamma^\mu \partial_\mu - M_N^*(\sigma) - g_\omega \gamma^\mu \omega_\mu]\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2 \omega_\mu \omega^\mu + \tfrac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right), \tag{10}$$

where g_ω is the ω-nucleon coupling constant. For uniform nuclear matter in the mean-field approximation, meson fields are just constants. The Euler-Lagrange equations for the fields ψ, σ, and ω lead to the following equations of motion:

$$[i\gamma^\mu \partial_\mu - M_N^*(\sigma) - g_\omega \omega_0 \gamma^0]\psi = 0, \tag{11}$$
$$m_\sigma^2 \sigma = -\frac{\partial M_N^*}{\partial \sigma}\rho_s, \tag{12}$$
$$m_\omega^2 \omega_0 = g_\omega \rho_B. \tag{13}$$

Here the scalar density $\rho_s = \langle \bar{\psi}\psi \rangle$ and the baryon density $\rho_B = \langle \psi^\dagger \psi \rangle$. The bracket $\langle\ \rangle$ means the expectation value of the operator for the nuclear ground state. The results in the first step are put into M_N^*. Equation (11) describes the motion of a nucleon with the mass $M_N^*(\sigma)$ instead of the bare mass M_N.

The total energy density of nuclear matter can be obtained by filling all nucleons up to the Fermi level k_F:

$$E = \frac{4}{(2\pi)^3}\int^{k_F} d^3k \sqrt{k^2 + M_N^*(\sigma)^2} + \frac{1}{2}m_\sigma^2 \sigma^2 + \frac{1}{2}m_\omega^2 \omega_0^2. \tag{14}$$

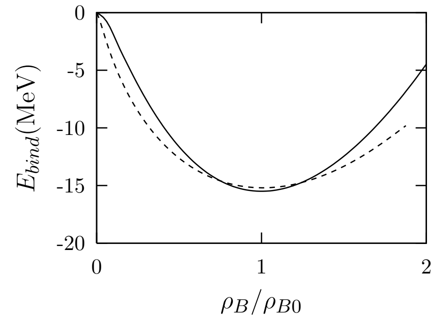

FIG. 1. The binding energy per nucleon E_{bind} as a function of the baryon density. The results in the present model are shown by the solid line, while those in the QMF model [9] are shown by the dashed line, for comparison.

The binding energy per nucleon in nuclear matter is then given by

$$E_{\text{bind}} = \frac{E}{\rho_B} - M_N. \tag{15}$$

III. RESULTS AND DISCUSSIONS

First of all, we take the meson masses as $m_\sigma = 550$ MeV, and $m_\omega = 783$ MeV. Then there are two parameters (g_σ^q, g_ω^q) which need to be determined. These values can be chosen to fit the nuclear binding energy $E_{\text{bind}} = -15.5$ MeV at the saturation density $\rho_{B0} = 0.16$ fm^{-3}. This gives the values $g_\sigma^q = 2.01$ and $g_\omega^q = 2.64$.

Figure 1 presents how the binding energy per nucleon varies with the baryon density ρ_B in the models. The present model gives a slightly stiff equation of state compared to that of the QMF model [9].

We show in Fig. 2 the density dependence of the effective nucleon mass (8). The present value of the effective mass at ρ_{B0} is $M_N^* = 754$ MeV ($= 0.8\, M_N$), which is somewhat smaller than that in the ZM model [11], but significantly larger than that in the QMF model [8].

Since the relation between the effective nucleon mass and the quark mass correction δm_q has been studied in Refs. [6,7], we also examine it in the present model.

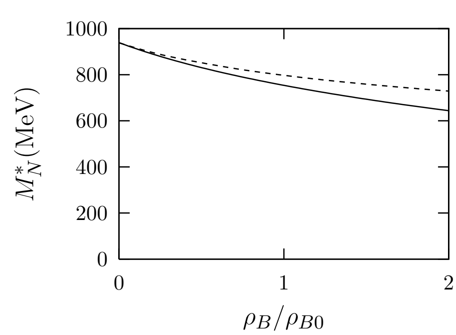

FIG. 2. Nucleon effective mass in nuclear matter as a function of baryon density for the models. The results from the present model are shown by the solid line. The dashed line corresponds to the ZM model.

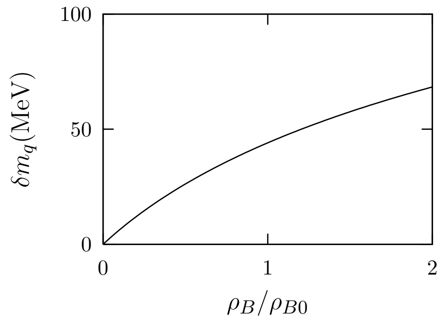

FIG. 3. The quark mass correction δm_q as a function of baryon density.

We have plotted the density dependence of δm_q in Fig. 3. The value of δm_q is suppressed below 70 MeV for $\rho_B \lesssim 2\rho_{B0}$ and is 45 MeV at ρ_{B0}.

In Fig. 4 the effective nucleon mass M_N^* is plotted as a function of δm_q. There is a linear relation approximately in the region $\delta m_q \lesssim 70$ MeV.

To investigate the situation in more detail we expand M_N^* about the point $\delta m_q = 0$ according to

$$M_N^* = M_N + \left.\frac{\partial M_N^*}{\partial(\delta m_q)}\right|_{\delta m_q=0} \delta m_q + \cdots. \tag{16}$$

For $\delta m_q \ll M_N$ we can neglect the remaining terms in the expansion of Eq. (16). From Figs. 3 and 4, $\delta m_q \ll M_N$ is satisfied for $\rho_B \leqslant 2\rho_{B0}$, and $\partial M_N^*/\partial(\delta m_q)$ is approximately constant. Therefore, M_N^* can be written as

$$M_N^* = M_N - 4.2\delta m_q = M_N - 1.4 \times 3\delta m_q. \tag{17}$$

The effective nucleon mass M_N^* may be also written by the following equation:

$$M_N^* = M_N - g_\sigma^*(\sigma)\sigma, \tag{18}$$

with a σ-nucleon coupling $g_\sigma^*(\sigma)$ which depends explicitly on the σ-field. Comparing Eq. (17) with Eq. (18) and using Eq. (6), a modification of the σ-nucleon coupling in the nuclear environment is analytically expressed by the effective σ-quark coupling $g_\sigma^{q*}(\sigma)$:

$$\begin{aligned} g_\sigma^*(\sigma) &\simeq - \left.\frac{\partial M_N^*}{\partial(\delta m_q)}\right|_{\delta m_q=0} g_\sigma^{q*}(\sigma) \\ &= 1.4 \times 3 g_\sigma^{q*}(\sigma). \end{aligned} \tag{19}$$

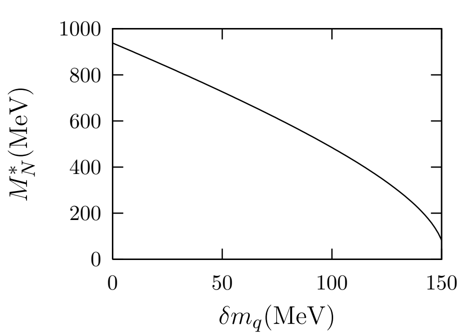

FIG. 4. The effective nucleon mass M_N^* as a function of the quark mass correction δm_q.

TABLE I. The results for U_s, U_v, $U_v - U_s$ and K at the saturation density ρ_{B0}.

Model	U_s (MeV)	U_v (MeV)	$U_v - U_s$ (MeV)	K (MeV)
This work	−186	126	311	245
QMF [9]	−350	270	620	159
ZM [11,14]	−141	83	223	225

Here $g_\sigma^{q*}(\sigma)$ is monotone decreasing function like that of Ref. [9], but the form of the function is different in both models.

The second term in the right-hand side of Eq. (17) is defined as the scalar potential (U_s), which shifts the nucleon mass from M_N to M_N^*. For comparing with the ZM model [11,14] we rewrite U_s as follows:

$$U_s = -1.4 \times \tilde{m} g_\sigma \sigma, \tag{20}$$

where $g_\sigma = 3g_\sigma^q$. If in Eq. (2), we replace m_q and g_σ^q as $M_N = 3m_q$ and $g_\sigma = 3g_\sigma^q$ respectively, we see that $\tilde{m}$ is identical with the corresponding quantity of the original ZM model. Equation (20) is, however, different from the scalar potential of ZM model by factor 1.4. The difference comes from the quark structure of the nucleons in nuclear medium.

The vector potential (U_v) is given by $U_v = g_\omega \omega_0$. In regard to g_ω, we take $g_\omega = 3g_\omega^q$ [13], where g_ω has no σ dependence differing from Ref. [9].

Figure 5 illustrates U_s and U_v as functions of ρ_B. Table I shows the results of the models for $U_s, U_v, U_v - U_s$ and incompressibility K at the saturation density of the nuclear matter. For finite nuclei, $U_v - U_s$ roughly gives us the strength of the spin-orbit splitting. There exist appreciable differences between the QMF results [9] and those in the present model. In Refs. [8,9], U_s and U_v are parametrized to reproduce RBHF results. If one accepts the value of the RBHF as reasonable, $U_v - U_s$ of the present model is too small: it is almost half of the QMF model [9], but is larger than that found in the original ZM model. On the other hand, a value of K is consistent with the accepted empirical value (210 ± 30 MeV). It had been considered a defect of the original ZM model that the spin-orbit interaction was too small to explain the observed spin-orbit splitting for finite nuclei. In order to remove this defect, Biro and Zimanyi [15] introduced an additional tensor

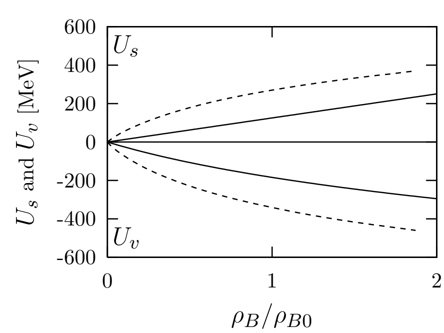

FIG. 5. The scalar and vector potentials of nuclear matter, U_s and U_v, as functions of the baryon density ρ_B. The lines are labeled as in Fig. 1.

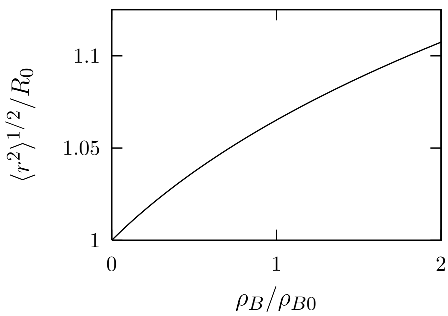

FIG. 6. The density dependence of the nucleon radius, in units of the free nucleon radius $R_0 = 0.6$ fm.

coupling term in the ZM Lagrangian. The similar idea might be applied to the present model.

We also plot the ratio of the nucleon rms radius in nuclear matter to that in free space as a function of density in Fig. 6. The $6 \sim 7\%$ increase in the nucleon size at ρ_{B0} is comparable to those suggested in Refs. [7,8]. This is due to the change of the position probability density of quarks in nuclear medium. The change comes from the modification of k ($kr^2/2 \rightarrow (\tilde{m}k)r^2/2$) in Eq. (3).

In summary, we have investigated the QMF model with the derivative scalar coupling of σ-quark meson. As a result, a σ dependent coupling constant $g_\sigma^{q\,*}(\sigma)$ is introduced and the confinement potential is modified in the nuclear medium. The ω-quark coupling constant does not have a σ dependence. This matter is one of the present pending question. Regardless of only two free parameters, the present model gives several important findings related to the quark-meson coupling in nuclear matter.

ACKNOWLEDGMENT

We are grateful to K. Saito for fruitful discussions and helpful suggestions.

[1] P. A. M. Guichon, Phys. Lett. **B200**, 235 (1988).
[2] K. Saito and A. W. Thomas, Nucl. Phys. **A574**, 659 (1994).
[3] P. A. M. Guchon, K. Saito, E. Rodionov, and A. W. Thomas, Nucl. Phys. **A601**, 349 (1996).
[4] Xuemin Jin and B. K. Jennings, Phys. Rev. C **54**, 1427 (1996).
[5] H. Muller and B. K. Jennings, Nucl. Phys. **A640**, 55 (1998).
[6] P. Wang, R. K. Su, H. Q. Song, and L. L. Zhang, Nucl. Phys. **A653**, 166 (1999).
[7] H. Toki, U. Meyer, A. Faessler, and R. Brockmann, Phys. Rev. C **58**, 3749 (1998).
[8] H. Shen and H. Toki, Phys. Rev. C **61**. 045205 (2000).
[9] Y. H. Tan, H. Shen, and P. Z. Ning, Phys. Rev. C **63**, 055203 (2001).
[10] R. Brockmann and R. Machleidt, Phys. Rev. C **42**, 1965 (1990).
[11] J. Zimanyi and S. A. Moszkowski, Phys. Rev. C **42**, 1416 (1990).
[12] J. D. Walecka, Ann. Phys. (NY) **83**, 491 (1974); B. D. Serot and J. D. Walecka, *Advances in Nuclear Physics* (Plenum, New York, 1986), Vol. 16.
[13] S. Fleck, W. Bentz, K. Shimizu, and K. Yazaki, Nucl. Phys. **A510**, 731 (1990).
[14] A. Delfino, C. T. Coelho, and M. Malheiro, Phys. Rev. C **51**, 2188 (1995).
[15] T. S. Biro and J. Zimanyi, Phys. Lett. **B391**, 1 (1997).