Temperature-dependent particle-number projected moment of inertia

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Expressions of the parallel and perpendicular temperature-dependent particle-number projected nuclear moment of inertia have been established by means of a discrete projection method. They generalize that of the FTBCS method and are well adapted to numerical computation. The effects of particle-number fluctuations have been numerically studied for some even-even actinide nuclei by using the single-particle energies and eigenstates of a deformed Woods-Saxon mean field. It has been shown that the parallel moment of inertia is practically not modified by the use of the projection method. In contrast, the discrepancy between the projected and FTBCS perpendicular moment of inertia values may reach 5%. Moreover, the particle-number fluctuation effects vary not only as a function of the temperature but also as a function of the deformation for a given temperature. This is not the case for the system energy.

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I. INTRODUCTION

The pairing correlations at finite temperature have been the subject of peculiar interest since the early 1960s. The first studies have been performed within the framework of the finite temperature BCS (FTBCS) theory (cf., e.g., Refs. [1–7]). This method has been improved by means of the renormalized RPA at finite temperature [8], the self-consistent thermal RPA [9], and the finite temperature Hartree-Fock-Bogoliubov theory (FTHFB) (cf. Ref. [10] and references therein and Ref. [11]). Another approach uses the path integral formalism [12–22], which allows one to take into account the quantum fluctuations. More recently, the neutron-proton pairing has also been treated at finite temperature [23–27].

However, it is well known that the BCS wave function breaks the particle-number symmetry. This symmetry may be restored at finite temperature by using various methods, such as, for example, a generalization of the Lipkin-Nogami method [28]. Other methods used to eliminate the particle-number fluctuations include particle number projected statistics [29–32] or a combination of the static path approximation (SPA) with either a particle number projection [33] or a parity number projection [34,35].

Moreover, it is well known that the moment of inertia is very sensitive to the pairing correlations and that the BCS approximation is not sufficient for a correct description of this quantity [36,37]. Indeed, the discrepancy between the BCS and experimental values is of the order of 10%–40%. This discrepancy has been significantly reduced, at zero temperature, by performing either a particle-number projection [38–40] or an exact calculation of the moment of inertia [41]. It is worth noticing that the particle-number projection also allows one to correctly describe the back-bending phenomenon [42]. Another approach consists in the inclusion of the neutron-proton pairing effects [43], particularly in nuclei such as $N \simeq Z$ [44–46].

The moment of inertia has also been evaluated at finite temperature within the FTBCS [4,5] and FTHFB [10] methods. However, to our knowledge, the particle-number fluctuations have been eliminated in an approximate way, by either using a combination of the SPA and the RPA [47,48] or by means of a parity-number projection [34].

The purpose of the present work is to explicitly establish an expression for the moment of inertia, at finite temperature, that strictly conserves the number of particles. To this aim, we will generalize the expression established at zero temperature in a previous work [38]. The latter is based on a discrete particle-number projection method.

The paper is organized as follows: The particle-number projection method is recalled in Sec. II. Expressions of the parallel and perpendicular moments of inertia at finite temperature are established in Sec. III. The numerical results dealing with several even-even actinide nuclei, based on the energies and eigenstates of a deformed Woods-Saxon mean field, are presented and discussed in Sec. IV. The main conclusions are summarized in Sec. V.

II. PARTICLE-NUMBER PROJECTION

A. Projected states

In the second quantization formalism, the intrinsic motion of $2P$ paired particles (neutrons or protons) is described by the Hamiltonian

$$H_0 = \sum_{\nu>0} \varepsilon_\nu \, (a_\nu^+ a_\nu + a_{\tilde{\nu}}^+ a_{\tilde{\nu}}) - G \sum_{\nu\mu>0} a_\nu^+ a_{\tilde{\nu}}^+ a_{\tilde{\mu}} a_\mu, \qquad (1)$$

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where the pairing strength G is assumed to be constant and the state $|\widetilde{\nu}\rangle = a_{\widetilde{\nu}}^{+}|0\rangle$ is the time reverse of the state $|\nu\rangle = a_{\nu}^{+}|0\rangle$, of energy ε_ν. In Eq. (1), the time-reversal invariance of H_0 has been taken into account, which implies $\varepsilon_\nu = \varepsilon_{\widetilde{\nu}}$. The ground state is then described by the BCS wave function given by

$$|\psi\rangle = \prod_{\nu>0}(u_\nu + v_\nu a_\nu^{+} a_{\widetilde{\nu}}^{+})|0\rangle, \tag{2}$$

where the parameters u_ν and v_ν represent the occupation and inoccupation amplitudes of the state $|\nu\rangle$. In the quasiparticle representation, defined by using the Bogoliubov-Valatin transformation, the state (2) represents the quasiparticle vacuum whose creation and annihilation operators α_ν^{+} and α_ν are such that $\alpha_\nu|\psi\rangle = 0$, for any ν. However, these BCS wave functions are not eigenstates of the particle-number operator

$$N = \sum_{\nu>0}(a_\nu^{+}\, a_\nu + a_{\widetilde{\nu}}^{+} a_{\widetilde{\nu}}) \tag{3}$$

since only the expectation value of this operator is supposed to be constant and equal to the actual particle number. The quasiparticle states rather describe a superposition of states of nuclei of neighboring masses. They differ by an even number of nucleons and correspond to the same value of the chemical potential λ as well as the gap parameter Δ.

It clearly appears from Eq. (2) that the quasiparticle states describe a superposition of states with $0, 2, 4, \ldots, 2\Omega$ particles, with Ω being the total degeneracy of pairs of the system. It has been shown that the sequence of states that correspond to $2P$ paired particles [49–54],

$$|\psi_n\rangle = C_n \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_{j>0}(u_j + z_k v_j a_j^{+} a_{\widetilde{j}}^{+})|0\rangle + cc \right\}, \tag{4}$$

where C_n is a normalization factor and

$$\xi_k = \begin{cases} \frac{1}{2} & \text{if} \quad k = 0 \text{ or } k = n+1, \\ 1 & \text{if} \quad 1 \leqslant k \leqslant n, \end{cases} \qquad z_k = \exp\left(i\frac{k\pi}{n+1}\right),$$

with n being a nonzero integer and cc the complex conjugate with respect to z_k, converges toward the projected state.

As has been shown in Refs. [49–51], as soon as the relation $2(n+1) > \max(P, \Omega - P)$ is satisfied, the projected state defined by Eq. (4) coincides with the P pairs component. In practice, the convergence is reached when $n \simeq 3$ or 4.

The particle-number fluctuations may also be easily eliminated in the states that correspond to any quasiparticle number. In particular, the projected one- and two-quasiparticle states read

$$|(\nu)_n\rangle = C_n^{\nu} \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-P} a_\nu^{+} \prod_{\substack{j>0 \\ j\neq\nu}}(u_j + z_k v_j a_j^{+} a_{\widetilde{j}}^{+})\,|0\rangle + cc \right\}, \tag{5}$$

$$|(\nu\mu)_n\rangle = C_n^{\nu\mu} \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-(P-1)} a_\nu^{+} a_{\widetilde{\mu}}^{+} \times \prod_{\substack{j>0 \\ j\neq(\nu\mu)}}(u_j + z_k v_j a_j^{+} a_{\widetilde{j}}^{+})\,|0\rangle + cc \right\}, \tag{6}$$

where C_n^{ν} and $C_n^{\nu\mu}$ are normalization factors.

B. Projected state energies

The normalization factors and energies of the states (4) to (6) may be easily evaluated using the properties [50]

$$\langle\psi_n|\,\mathcal{O}\,|\psi_n\rangle = 2\,(n+1)\,C_n\,\langle\psi_n|\,\mathcal{O}\,|\psi\rangle \tag{7}$$

and

$$\begin{aligned} &\langle(\nu_1 \cdots \nu_l)_n|\mathcal{O}|(\nu_1 \cdots \nu_l)_n\rangle \\ &\quad = 2(n+1)C_n^{\nu_1\cdots\nu_l}\langle(\nu_1 \cdots \nu_l)_n|\mathcal{O}\alpha_{\nu_1}^{+} \cdots \alpha_{\nu_l}^{+}|\psi\rangle, \\ &l = 1, 2, \ldots, \end{aligned} \tag{8}$$

which are valid for any operator $\mathcal{O}$ that conserves the particle number. One then has

$$2(n+1)C_n^2 \times \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_{j>0} \left(u_j^2 + z_k v_j^2\right) + cc \right\} = 1,$$

$$2(n+1)\left(C_n^{\nu}\right)^2 \times \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_{j\neq\nu} \left(u_j^2 + z_k v_j^2\right) + cc \right\} = 1,$$

$$2(n+1)\left(C_n^{\nu\mu}\right)^2 \times \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-(P-1)} \prod_{j\neq(\nu,\mu)} \left(u_j^2 + z_k v_j^2\right) + cc \right\} = 1.$$

Extracting the real part gives

$$4(n+1)C_n^2 \sum_{k=0}^{n+1} \xi_k R_k \cos\psi_k = 1, \tag{9}$$

$$4(n+1)\left(C_n^{\nu}\right)^2 \sum_{k=0}^{n+1} \xi_k R_k^{\nu} \cos\left(\psi_k^{\nu}\right) = 1, \tag{10}$$

$$4(n+1)\left(C_n^{\nu\mu}\right)^2 \sum_{k=0}^{n+1} \xi_k R_k^{\nu\mu} \cos\left(\psi_k^{\nu\mu}\right) = 1, \tag{11}$$

with the notation

$$x_k = \frac{k\pi}{2(n+1)} \qquad \tan\varphi_{\nu k} = \left(v_\nu^2 - u_\nu^2\right)\tan x_k,$$

$$|\varphi_{\nu k}| \leqslant \frac{\pi}{2}, \qquad \rho_{\nu k} = \sqrt{1 - 4u_\nu^2 v_\nu^2 \sin^2 x_k},$$

$$R_k = \prod_j \rho_{jk}, \qquad R_k^{\nu_1,\ldots,\nu_l} = \prod_{j\neq\nu_1,\ldots,\nu_l} \rho_{jk},$$

$$\psi_k = \sum_j \varphi_{jk} + (\Omega - 2P)\, x_k,$$
$$\psi_k^{\nu_1,\ldots,\nu_l} = \sum_{j\neq \nu_1,\ldots,\nu_l} \varphi_{jk} + (\Omega - 2P)\, x_k.$$

In the same way, the energies may be written

$$\begin{aligned} E_n^0 &= \langle \psi_n | H_0 | \psi_n \rangle \\ &= E_0 + 2(n+1) C_n^2 G \sum_{\gamma,\eta} u_\gamma^3 u_\eta v_\eta^3 v_\gamma \\ &\quad \times \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-P} (z_k - 1)^2 \prod_{j\neq(\gamma,\eta)} \left(u_j^2 + z_k v_j^2\right) + cc \right\}, \end{aligned} \tag{12}$$

$$\begin{aligned} E_n^\nu &= \langle (\nu)_n |\, H_0 \,| (\nu)_n \rangle \\ &= E_0 + E_\nu + 2\,(n+1)\left(C_n^\nu\right)^2 G \sum_{\gamma,\eta\neq(\nu)} u_\gamma^3 u_\eta v_\eta^3 v_\gamma \\ &\quad \times \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-P} (z_k - 1)^2 \prod_{j\neq(\nu,\gamma,\eta)} \left(u_j^2 + z_k v_j^2\right) + cc \right\}, \end{aligned} \tag{13}$$

$$\begin{aligned} E_n^{\nu\mu} &= \langle (\nu\mu)_n | H_0 | (\nu\mu)_n \rangle \\ &= E_0 + E_\nu + E_\mu + 2(n+1)\left(C_n^{\nu\mu}\right)^2 G \\ &\quad \times \sum_{\gamma,\eta\neq(\nu,\mu)} u_\gamma^3 u_\eta v_\eta^3 v_\gamma \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-(P-1)} (z_k - 1)^2 \right. \\ &\quad \left. \times \prod_{j\neq(\nu,\mu,\gamma,\eta)} \left(u_j^2 + z_k v_j^2\right) + cc \right\}, \end{aligned} \tag{14}$$

where E_0 and E_ν, respectively, represent the BCS and quasiparticle energies given by

$$E_0 = 2 \sum_{\nu>0} \left(\varepsilon_\nu - \lambda - \frac{G}{2} v_\nu^2\right) v_\nu^2 - \frac{\Delta^2}{G},$$
$$E_\nu = \sqrt{\left(\varepsilon_\nu - \lambda - G v_\nu^2\right)^2 + \Delta^2},$$

with

$$\Delta = G \sum_{\nu>0} u_\nu v_\nu.$$

After extraction of the real part, the energies (12) to (14) become

$$\begin{aligned} E_n^0 &= E_0 - 16(n+1) C_n^2 G \sum_{\gamma,\eta} u_\gamma^3 u_\eta v_\eta^3 v_\gamma \\ &\quad \times \sum_{k=0}^{n+1} \xi_k R_k^{\gamma\eta} \sin^2 x_k \cos \psi_k^{\gamma\eta}, \end{aligned} \tag{15}$$

$$\begin{aligned} E_n^\nu &= E_0 + E_\nu - 16(n+1)\left(C_n^\nu\right)^2 G \sum_{\gamma,\eta\neq(\nu)} u_\gamma^3 u_\eta v_\eta^3 v_\gamma \\ &\quad \times \left\{ \sum_{k=0}^{n+1} \xi_k R_k^{\nu\gamma\eta} \sin^2 x_k \cos \psi_k^{\nu\gamma\eta} \right\}, \end{aligned} \tag{16}$$

$$\begin{aligned} E_n^{\nu\mu} &= E_0 + E_\nu + E_\mu \\ &\quad - 16(n+1)\left(C_n^{\nu\mu}\right)^2 G \sum_{\gamma,\eta\neq(\nu,\mu)} u_\gamma^3 u_\eta v_\eta^3 v_\gamma \\ &\quad \times \left\{ \sum_{k=0}^{n+1} \xi_k R_k^{\nu\mu\gamma\eta} \sin^2 x_k \cos\left(\psi_k^{\nu\mu\gamma\eta} + 2x_k\right) \right\}. \end{aligned} \tag{17}$$

III. MOMENT OF INERTIA

Let us start with the grand-partition function

$$Z = \mathrm{Tr}\, e^{-\beta H}, \tag{18}$$

where β is the inverse of the temperature T and H is the Hamiltonian of the system. When the latter is cranked around the Oi ($i = x, z$) axis (with Oz being the symmetry axis) of a rotating frame, H is given by

$$H = H_0 - \lambda N - \hbar\omega J_i, \tag{19}$$

where ω is the rotation frequency and J_i is the i projection of the angular momentum. The usual Inglis expression [55] of the energy may be easily generalized to include the temperature effects as well as the pairing correlations by using the previously defined projected wave functions. This energy expanded to the second order in ω by means of the thermodynamical perturbation theory is given by

$$E = -\frac{\partial LnZ}{\partial\beta} \simeq E_0 - \hbar^2\omega^2 \int_0^\beta \langle J_i(\beta)\, J_i(\gamma)\rangle\, d\gamma, \tag{20}$$

where

$$J_i(\alpha) = e^{\alpha(H_0-\lambda N)} J_i e^{-\alpha(H_0-\lambda N)}, \quad \alpha = \beta, \gamma. \tag{21}$$

This expression allows one to derive a good approximation of the moment of inertia of the system, which is then defined by

$$\Im_i = 2\hbar^2 \int_0^\beta \langle J_i(\beta) J_i(\gamma)\rangle d\gamma. \tag{22}$$

The thermal average in Eq. (22) must be evaluated by using the grand-canonical ensemble associated to the Hamiltonian $H_0 - \lambda N$, that is,

$$\langle J_i(\beta)\, J_i(\gamma)\rangle = \frac{\mathrm{Tr}\, J_i(\beta)\, J_i(\gamma)\, e^{-\beta(H_0-\lambda N)}}{\mathrm{Tr}\, e^{-\beta(H_0-\lambda N)}}. \tag{23}$$

If one neglects the interaction between the quasiparticles, this thermal average can easily be made explicit using the representation basis defined in Sec. II. Indeed, in the latter, H_0 may be written in the diagonal form

$$H_0 = E_n^0 + \sum_{\nu>0} E_n^\nu \left(P_n^\nu + P_n^{\widetilde{\nu}}\right), \tag{24}$$

where P_n^ν (respectively $P_n^{\widetilde{\nu}}$) is the projector on the state $|(\nu)_n\rangle$ (respectively $|(\widetilde{\nu})_n\rangle$) given by Eq. (5) and of which the occupation number is given by

$$f_n^\nu = \left\langle P_n^\nu \right\rangle = \left\langle P_n^{\widetilde{\nu}} \right\rangle = \frac{1}{1 + e^{\beta E_n^\nu}}. \tag{25}$$

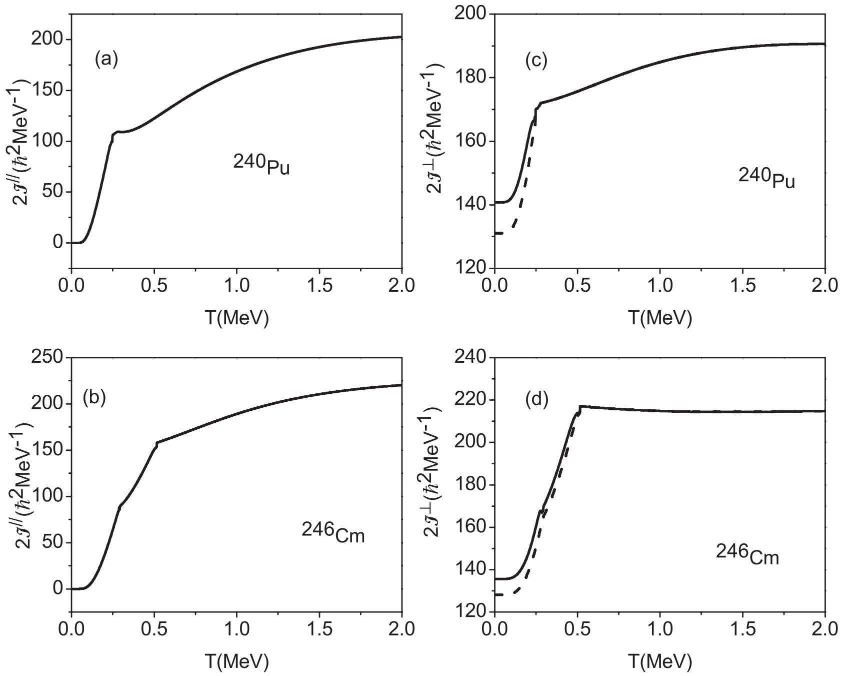

FIG. 1. Variation of the parallel [(a) and (b)] and perpendicular [(c) and (d)] moments of inertia, evaluated by using the FTBCS (dashed lines) and projection (solid lines) methods, for the ground state of the nuclei ^{240}Pu and ^{246}Cm as a function of the temperature.

This occupation number is analogous to that of the quasiparticles but the energies E_ν are replaced by the corresponding projected ones, E_n^ν.

Taking into account the fact that the projected states $|\psi_n\rangle$, $|(\nu)_n\rangle$, and $|(\nu\mu)_n\rangle$ are orthononormalized and hence that the corresponding projectors P_n, P_n^ν, and $P_n^{\nu\mu}$ are orthogonal, one may write

$$J_i(\alpha) = \sum_{\nu\mu} \left\{ \frac{1}{2}\left[e^{-\alpha\left(E_n^\nu + E_n^\mu\right)} P_n J_i P_n^{\nu\mu} + e^{\alpha\left(E_n^\nu + E_n^\mu\right)} P_n^{\nu\mu} J_i P_n \right] + e^{\alpha\left(E_n^\nu - E_n^\mu\right)} P_n^\nu J_i P_n^\mu \right\}. \qquad (26)$$

The thermal average in Eq. (22) becomes

$$\langle J_i(\beta) J_i(\gamma)\rangle = \sum_{\nu\mu} \left\{ \frac{1}{2}\left[e^{(\gamma-\beta)(E_n^\nu + E_n^\mu)} + e^{(\beta-\gamma)(E_n^\nu + E_n^\mu)} \right] \times \left(1 - f_n^\nu\right)\left(1 - f_n^\mu\right) |\langle \psi_n | J_i | (\nu\mu)_n\rangle|^2 + e^{(\beta-\gamma)(E_n^\nu - E_n^\mu)} f_n^\nu \left(1 - f_n^\mu\right) |\langle (\nu)_n | J_i | (\mu)_n\rangle|^2 \right\}. \qquad (27)$$

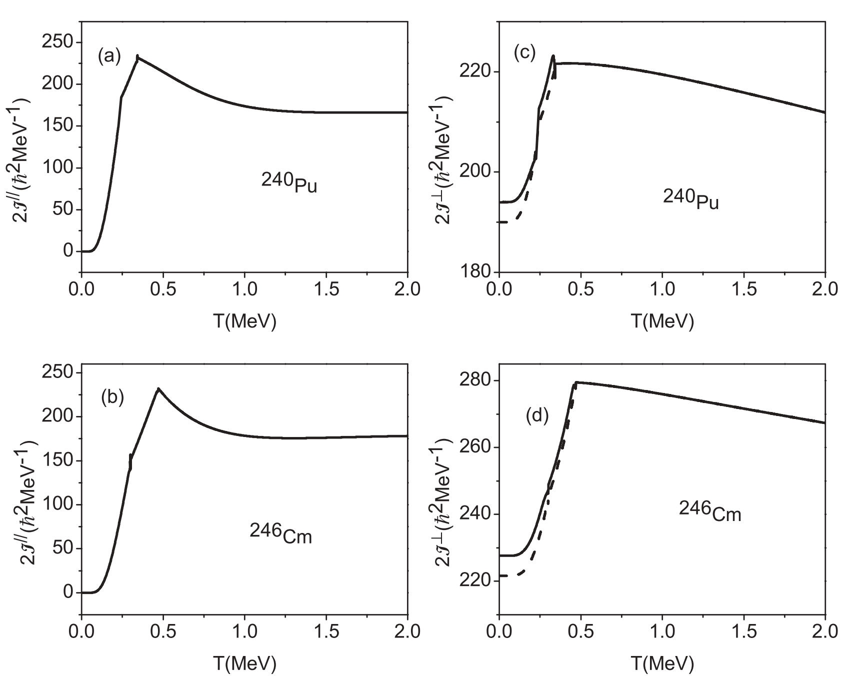

FIG. 2. Same as Fig. 1 for the isomeric state.

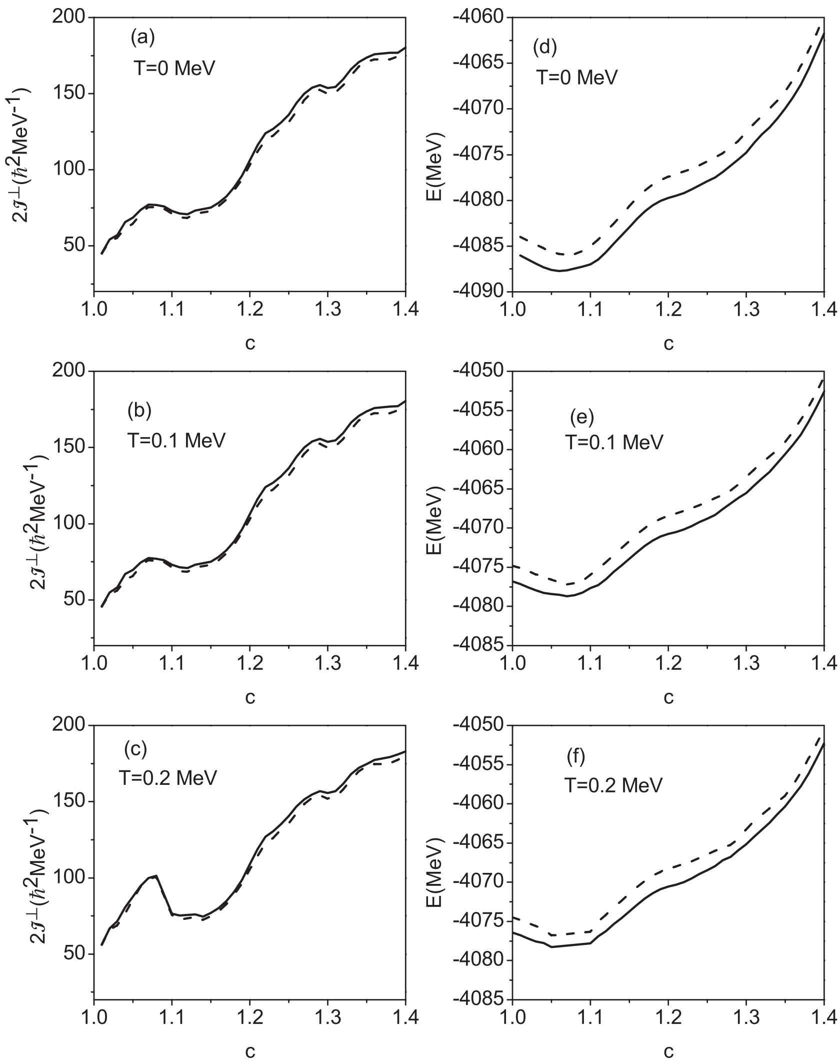

FIG. 3. Variation of the perpendicular moment of inertia [(a)–(c)] and total system energy [(d)–(f)] of the nucleus ^{226}Ra, evaluated by using the FTBCS (dashed lines) and projection (solid lines) methods as a function of the elongation parameter c, for a fixed value of the neck parameter h ($h = 0$), for various values of the temperature.

After integration relative to γ, one obtains, for the moment of inertia,

$$\begin{aligned}\Im_n^i &= \frac{1}{2}\hbar^2 \sum_{\nu\mu} \frac{|\langle \psi_n | J_i | (\nu\mu)_n \rangle|^2}{E_n^\nu + E_n^\mu}\left(\tanh\frac{1}{2}\beta E_n^\nu + \tanh\frac{1}{2}\beta E_n^\mu\right)\\ &\quad + \frac{1}{2}\hbar^2 \sum_{\nu\mu} \frac{|\langle (\nu)_n | J_i | (\mu)_n \rangle|^2}{E_n^\nu - E_n^\mu}\left(\tanh\frac{1}{2}\beta E_n^\nu - \tanh\frac{1}{2}\beta E_n^\mu\right). \end{aligned} \tag{28}$$

This expression is similar to that obtained by using the FTBCS method. By replacing the matrix elements of $J_i (i = x, z)$ using their expectation values over the projected states in Eq. (28), one obtains the perpendicular and parallel moments of inertia relative to the Ox and Oz axes, respectively:

$$\begin{aligned}\Im_n^\perp &= \frac{1}{2}\hbar^2 \sum_{\nu\mu} |\langle \nu | J_x | \mu \rangle|^2 \left\{ \left(\frac{C_n}{C_n^{\nu\mu}}\right)^2 \frac{(u_\nu v_\mu - u_\mu v_\nu)^2}{E_n^\nu + E_n^\mu} \right.\\ &\quad \times \left(\tanh\frac{1}{2}\beta E_n^\nu + \tanh\frac{1}{2}\beta E_n^\mu\right)\\ &\quad + \left(\frac{C_n^\nu}{C_n^\mu}\right)^2 \frac{(u_\nu u_\mu + v_\nu v_\mu)^2}{E_n^\nu - E_n^\mu}\\ &\quad \left. \times \left(\tanh\frac{1}{2}\beta E_n^\nu - \tanh\frac{1}{2}\beta E_n^\mu\right)\right\} \end{aligned} \tag{29}$$

and

$$\Im_n^\parallel = \frac{1}{2}\hbar^2 \beta \sum_\nu \frac{|\langle \nu | J_z | \nu \rangle|^2}{\cosh^2 \frac{1}{2}\beta E_n^\nu}, \tag{30}$$

where $\langle \nu | J_i | \mu \rangle$ means the matrix element of J_i using the single-particle basis.

It is worth noticing that Eqs. (29) and (30) of the perpendicular and parallel moments of inertia differ from their homologous counterparts obtained by using the FTBCS formalism [5,7] only by the projected state energies and the appearance of the multiplying factors $(C_n/C_n^{\nu\mu})^2$ and $(C_n^\nu/C_n^\mu)^2$ in $\Im_n^\perp$.

However, in the previous expressions, the chemical λ and the gap parameter Δ, as well as the u_ν and v_ν parameters, which are temperature dependent, are obtained by solving the

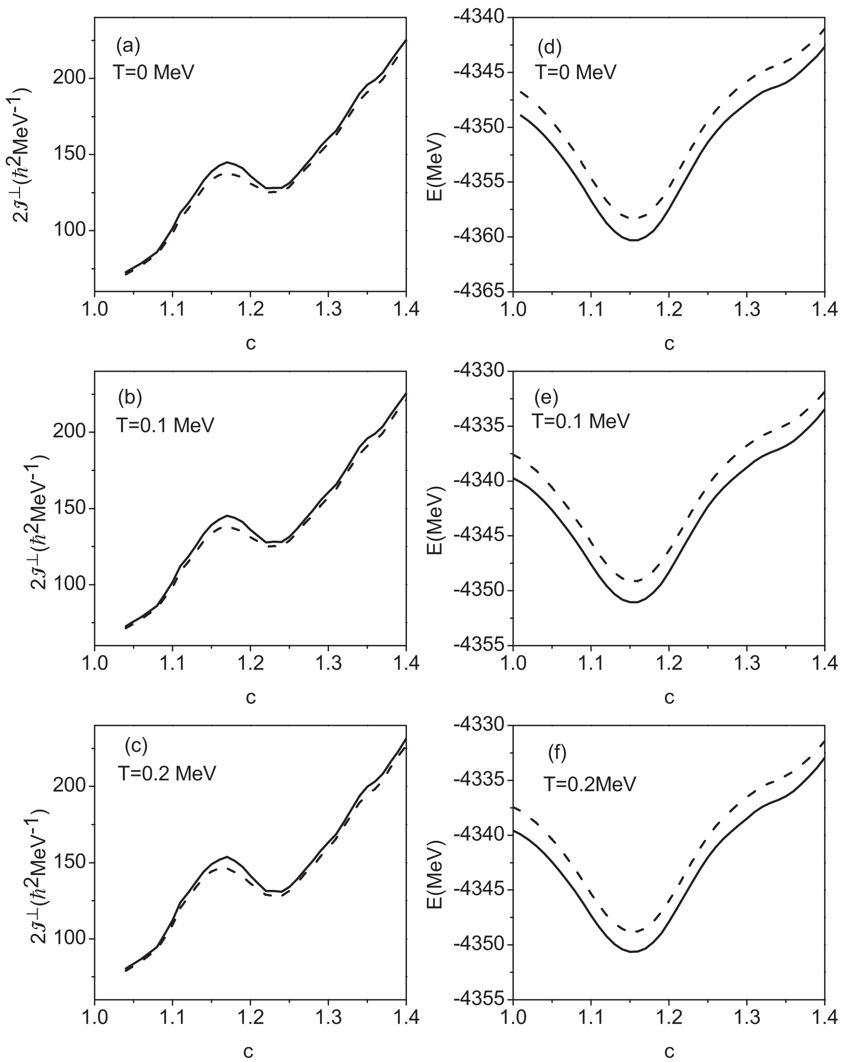

FIG. 4. Same as Fig. 3 for the nucleus ^{248}Cf.

usual FTBCS gap equations, that is [2],

$$\frac{2}{G} = \sum_{\nu>0} \frac{\tanh \frac{1}{2}\beta E_\nu}{E_\nu},$$

$$N = \sum_{\nu>0} \left[1 - \frac{\varepsilon_\nu - \lambda - G v_\nu^2}{E_\nu} \tanh \frac{1}{2}\beta E_\nu \right].$$

It is worth noticing that the single-particle energies should be in principle temperature dependent. However, as has been shown in several numerical calculations, by using realistic potentials, this dependence is small and the temperature effects on the single-particle energies may be neglected [56].

IV. NUMERICAL RESULTS AND DISCUSSION

The previously described formalism has been applied to some even-even actinide nuclei (i.e., ^{226}Ra, ^{230}Th, ^{236}U, ^{240}Pu, ^{246}Cm, ^{248}Cf, ^{254}Fm, and ^{256}No). The single-particle energies and states are those of a Woods-Saxon deformed mean field with the parameters described in Ref. [57]. The nuclear deformation is described by using the elongation parameter c and the neck parameter h [4,58,59]. The pairing strengths are given by

$$A.G_n = 19.2 - 7.4\tfrac{N-Z}{A},$$
$$A.G_p = 19.2 + 7.4\tfrac{N-Z}{A}.$$

As a first step, we have studied the convergence of the projected perpendicular moment of inertia as a function of the extraction degree of the false components n. The values that correspond to the ground and isomeric states of the nuclei ^{230}Th and ^{248}Cf, chosen as an example, are given in Table I for several values of the temperature. The usual FTBCS values are also given in the same table. One then observes a rapid convergence for any value of T. Indeed, the convergence is reached when $n \geqslant 4$ in each case. This is because the false components in the BCS wave functions mainly correspond to a pair of particle numbers close to that of the studied nucleus [38]. In all that follows, we will use the value $n = 5$.

We have then studied the variations of the perpendicular and parallel moments of inertia evaluated using Eqs. (29) and (30) as a function of the temperature ($0 \leqslant T \leqslant 2$ MeV). These variation are given in Fig. 1 for the ground state and in Fig. 2 for the isomeric state of the nuclei ^{240}Pu and ^{246}Cm

TABLE I. Variation of the FTBCS and projected perpendicular moment of inertia [$2\Im^{\perp}(\hbar^2\ \mathrm{MeV}^{-1})$] as a function of the extraction degree of the false components n for the ground and isomeric states of the ^{230}Th and ^{248}Cf nuclei.

	^{230}Th ground state	^{230}Th isomeric state	^{248}Cf ground state	^{248}Cf isomeric state
		$T = 0$ MeV		
FTBCS	98.742	163.262	136.115	223.078
$n = 0$	99.147	166.893	136.808	224.310
$n = 1$	101.536	168.918	141.721	227.445
$n = 2$	101.786	169.348	142.251	228.465
$n = 3$	101.789	169.356	142.260	228.509
$n = 4$	101.789	169.357	142.260	228.510
$n = 5$	101.789	169.357	142.260	228.510
		$T = 0.1$ MeV		
FTBCS	99.261	164.899	136.564	223.522
$n = 0$	99.668	168.802	137.258	224.809
$n = 1$	102.010	170.670	142.166	227.924
$n = 2$	102.257	171.098	142.694	228.943
$n = 3$	102.260	171.107	142.703	228.987
$n = 4$	102.260	171.107	142.703	228.987
$n = 5$	102.260	171.107	142.703	228.987
		$T = 0.2$ MeV		
FTBCS	109.931	176.441	146.830	231.007
$n = 0$	109.537	173.270	147.606	233.301
$n = 1$	111.051	175.625	152.299	235.904
$n = 2$	111.220	175.631	152.785	236.881
$n = 3$	111.221	175.631	152.794	236.921
$n = 4$	111.221	175.631	152.794	236.922
$n = 5$	111.221	175.631	152.794	236.922

chosen as an example. The values deduced from the usual FTBCS method are also given in these figures.

It can be then seen that the parallel moment of inertia is practically unmodified by the use of the particle-number projection method whereas the perpendicular moment of inertia is significantly modified (i.e., it is systematically increased). The fact that the variations induced by the particle-number fluctuations are more important in $\Im^{\perp}$ than in $\Im^{\parallel}$ was foreseeable. Indeed, in $\Im^{\parallel}$ only the quasiparticle energies are affected by the projection, whereas in $\Im^{\perp}$ the projection introduces multiplying factors besides the modifications in the quasiparticle energies.

With regard to the perpendicular moment of inertia, one notes that the effect of particle-number fluctuations is maximal for $T = 0$ and then decreases until it vanishes above the critical temperature. Indeed, in the latter region, the pairing correlations vanish and the FTBCS and projected moments of inertia join since we used a projection-after-variation method. It is worth noticing that Alhassid *et al.* [34] obtained similar curves for the iron isotopes using a parity-number projection method.

The fact that the discrepancy between the FTBCS and projected values of $\Im^{\perp}$ is maximal at zero temperature is also illustrated in Table II, where we report the average relative discrepancies [i.e., $(\Im^{\perp}_n - \Im^{\perp}_{\mathrm{FTBCS}})/\Im^{\perp}_{\mathrm{FTBCS}}$] for the ground and isomeric states of the considered nuclei. The effects of the particle-number fluctuations vary not only as a function of the temperature but also as a function of the deformation. Indeed, at zero temperature, for example, the average relative discrepancy is 4.74% for the ground state and 2.74% for the isomeric one. We have then studied the variations of $\Im^{\perp}_n$ and $\Im^{\perp}_{\mathrm{FTBCS}}$ as a function of the deformation, for various temperatures. These variations, as a function of the elongation parameter c and for a given neck parameter h ($h = 0$), are given in Figs. 3 and 4 for the ^{226}Ra and ^{248}Cf nuclei, chosen as an example. It then clearly appears that the effect of the particle-number projection on $\Im^{\perp}$ is not constant with respect to the deformation. This is particularly the case for the ^{248}Cf nucleus, for which the relative discrepancies between the projected and FTBCS values vary from less than 2% to more than 5%. Therefore, the perpendicular moment of inertia behavior differs from that of the energy. Indeed, it has been already shown (cf. Refs. [53,60,61]) that, at zero temperature, the discrepancy between the BCS and projected energies is quasiconstant with regard to the deformation. This result remains valid at finite temperature (the difference between the FTBCS and projected values being of the order of 2 MeV) as can be shown in Figs. 3 and 4, where we have also reported the variations of the FTBCS and projected energies as a function of the deformation.

TABLE II. Average relative discrepancies between the FTBCS and projected perpendicular moments of inertia for the ground state (second column) and the isomeric state (third column) for various temperatures.

T (MeV)	Ground state	Isomeric state
0	4.74%	2.74%
0.1	4.72%	2.73%
0.2	4.15%	2.04%
0.3	1.77%	1.10%
0.4	1.49%	0.76%
0.5	0.38%	0.18%

V. CONCLUSION

We have studied the effect of the particle-number fluctuations inherent to the BCS theory on the moment of inertia of some even-even actinide nuclei at finite temperature. We have established an *explicit* expression of the parallel and perpendicular moments of inertia based on a discrete particle-number projection method. The obtained expressions are simple and generalize that of the FTBCS method. They are well adapted to numerical computation.

A numerical study as a function of the temperature, for the ground and isomeric states of the considered nuclei, based on the energies and eigenstates of a Woods-Saxon mean field, has shown that the parallel moment of inertia is practically unaffected by the particle-number fluctuations. For the perpendicular moment of inertia, the projection effect, which corresponds to a systematic increase (of the order of 5% on average for the ground state and 3% for the isomeric one), is maximal at zero temperature and then decreases, until it vanishes above the critical temperature since we used a projection-after-variation method. It has also been shown that the effects of particle-number fluctuations vary as a function of the deformation for a given temperature. This is not the case for the system energy.

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