

# Asymmetry of recoil protons in neutron $\beta$ decay

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A complete analysis of proton recoil asymmetry in neutron decay in the first order of radiative and recoil corrections is presented. The possible contributions from new physics are calculated in terms of low energy coupling constants, and the sensitivity of the measured asymmetry to models beyond the Standard Model are discussed.

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## I. INTRODUCTION

The free neutron decay, being one of the simplest semileptonic hadron decay processes, is very important in the search for possible manifestations of new physics. The main advantage of neutron decay is the possibility to describe the process with minimal theoretical uncertainties and, as a consequence, the possibility to interpret unambiguously experimental results. The set of experiments for measurements of the neutron lifetime and neutron decay correlations can be used to determine the weak vector coupling constant, to test the universality of the weak interaction, and to search for nonstandard couplings (see, for example, Refs. [1–8], and references therein). The detailed analysis of required experimental accuracy and sensitivity to new physics of different observables for standard setups in neutron decay experiments has been done in the Ref. [9]. However, the recent measurement [10] and the new proposal to measure [11] the integrated asymmetry of recoiled protons in relation to the direction of neutron spin (which is known as a  $C$  angular correlation coefficient [12,13]) raise the question about the sensitivity of this asymmetry to new physics. To be able to estimate the potential sensitivity of the  $C$  asymmetry to new physics and the best accuracy of the measurement of Standard Model parameters (e.g., the ratio of axial-vector and vector coupling constants of weak interaction), one needs to calculate recoil and radiative corrections for the  $C$  asymmetry, as well as all possible contributions from the model beyond the standard one. Moreover, all these calculations must be done in the same framework to keep all possible uncertainties under control.

In this article, we use results of the effective field theory description of neutron  $\beta$  decay [14] as a framework for the calculation of the  $C$  correlation coefficient in the Standard Model (with recoil and radiative corrections). Then we calculate possible corrections from new physics using the most general nonstandard  $\beta$ -decay interactions. This provides a consistent description of the proton recoil asymmetry in terms of low energy coupling constants related to models beyond the Standard one at a level well below that anticipated in the next generation of neutron decay experiments.

## II. PROTON ASYMMETRY IN THE STANDARD MODEL

We have chosen results, based on the effective field theory (EFT) approach, of the description of the polarized neutron decay because this approach provides a general expression for neutron decay distribution function with the accuracy of  $10^{-5}$  in terms of one free parameter—the low energy constant (LEC) (for more details, see Ref. [14]). To calculate the angular correlation coefficient  $C$  with the complete set of recoil and radiative corrections, we use a general expression for the differential neutron decay rate given by Eq. (8) in Ref. [14]. It should be mentioned that in the tree approximation (neglecting recoil corrections and radiative corrections) the EFT results reproduce exactly those of a well-known formula for neutron decay rate [15] in terms of the angular correlations coefficients  $a$ ,  $A$ , and  $B$ :

$$\frac{d\Gamma^3}{dE_e d\Omega_e d\Omega_\nu} = \Phi(E_e) G_F^2 |V_{ud}|^2 (1 + 3\lambda^2) \times \left( 1 + b \frac{m_e}{E_e} + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + A \frac{\vec{\sigma} \cdot \vec{p}_e}{E_e} + B \frac{\vec{\sigma} \cdot \vec{p}_\nu}{E_\nu} \right), \quad (1)$$

Here,  $\vec{\sigma}$  is the neutron spin;  $m_e$  is the electron mass;  $E_e$ ,  $E_\nu$ ,  $\vec{p}_e$ , and  $\vec{p}_\nu$  are the energies and momenta of the electron and antineutrino, respectively; and  $G_F$  is the Fermi constant of the weak interaction (obtained from the  $\mu$ -decay rate). The function  $\Phi(E_e)$  includes normalization constants, phase-space factors, and standard Coulomb corrections. For the Standard Model the angular coefficients depend only on one parameter  $\lambda = -C_A/C_V > 0$ , the ratio of axial-vector to vector nucleon coupling constant [in general,  $C_V = C'_V$  and  $C_A = C'_A$  are low energy coupling constants for the low energy effective Hamiltonian given by Eq. (10)]:

$$a = \frac{1 - \lambda^2}{1 + 3\lambda^2}, \quad A = -2 \frac{\lambda^2 - \lambda}{1 + 3\lambda^2}, \quad B = 2 \frac{\lambda^2 + \lambda}{1 + 3\lambda^2}. \quad (2)$$

(The parameter  $b$  is equal to zero for vector–axial-vector weak interactions.)

The  $C$  angular coefficient (do not mix with  $C_V$  and  $C_A$ ) has been defined [12] as the angular distribution of the recoil protons in the relation to the direction of the neutron spin, provided all other variables, including proton recoil momentum, are averaged out. (It should be mentioned that another definition of proton asymmetry has been proposed in Ref. [16].) In the tree approximation, it has been calculated

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in Refs. [12] and [13], and numerical corrections to this approximation have been calculated in Ref. [13]. Using this definition, one can calculate the  $C$  coefficient from a general expression for the differential neutron decay rate (Eqs. (18)–(19) in Ref. [14]) with all (in the first order) recoil and radiative corrections. To do this, we use the momentum conservation condition  $\vec{p}_\nu + \vec{p}_e + \vec{p}_p = 0$ , which is multiplied by the neutron spin results in

$$|\vec{p}_\nu| \cos \theta_\nu + |\vec{p}_e| \cos \theta_e + |\vec{p}_p| \cos \theta_p = 0, \quad (3)$$

where  $\vec{p}_p$  is the proton momentum and  $\theta_\nu$ ,  $\theta_e$ , and  $\theta_p$  are angles between neutron spin and directions of antineutrino, electron, and proton momenta, respectively. From Eq. (3) one can see that protons are going to the upper hemisphere ( $\cos \theta_p > 0$ ), if  $|\vec{p}_\nu| \cos \theta_\nu + |\vec{p}_e| \cos \theta_e < 0$ , and to the lower hemisphere ( $\cos \theta_p < 0$ ), if  $|\vec{p}_\nu| \cos \theta_\nu + |\vec{p}_e| \cos \theta_e > 0$ . Therefore, the  $C$  coefficient, being a normalized difference of the neutron decay rate integrated over neutrino and electron angles, must be integrated over the electron energy under these two conditions. The integration over azimuthal angles leads to the  $4\pi^2$  factors. To calculate integrals over  $\theta_\nu$  and  $\theta_e$ , it is convenient to work in  $\cos$  variables:  $\cos \theta_\nu$  and  $\cos \theta_e$ . Thus, these two integrals could be represented in terms of a two-dimensional integral in  $(\cos \theta_\nu, \cos \theta_e)$  space, which must be taken separately over lower and upper parts of the square area in the cosine plane:  $([-1, 1], [-1, 1])$ . The line, dividing the area into two parts, is given by the equation  $|\vec{p}_\nu| \cos \theta_\nu + |\vec{p}_e| \cos \theta_e = 0$ . It should be noted that for both these integrals there are two different regimes of integration:  $|\vec{p}_\nu| > |\vec{p}_e|$  and  $|\vec{p}_\nu| < |\vec{p}_e|$ . For the first case, the integrals should be taken first over  $\cos \theta_\nu$  and then over  $\cos \theta_e$ , and for the second one the integrals should be taken in the opposite order. Applying this procedure for the decay rate given by Eq. (1), one obtains

$$C = \frac{X_1}{2X}(A + B) = \frac{X_1}{2X} \frac{4\lambda}{1 + 3\lambda^2}, \quad (4)$$

where

$$X = 4\sqrt{(E_e^{\max})^2 - m_e^2}(2(E_e^{\max})^4 - 9(E_e^{\max})^2 m_e^2 - 8m_e^4) + 60E_e^{\max} m_e^4 \ln \left( (E_e^{\max} + \sqrt{(E_e^{\max})^2 - m_e^2}) / m_e \right)$$

$$X_1 = 5((E_e^{\max})^5) - 6(E_e^{\max})^3 m_e^2 + 3E_e^{\max} m_e^4 + 2m_e^6 / E_e^{\max} + 12E_e^{\max} m_e^4 \ln (E_e^{\max} / m_e).$$

Equation (4) exactly reproduces the results of the calculations of Refs. [12] and [13] [the coefficient in Eq. (4) has a different sign because we define the positive direction of recoil protons as the direction of the neutron spin polarization]. To obtain a general expression with radiative and recoil corrections, one must apply the same procedure for the general neutron decay rate given by Eqs. (8)–(19) in Ref. [14]. These calculations are rather cumbersome but can be done exactly, without any approximation. Then, one can represent all corrections to the  $C$  coefficient in Eq. (4) as a sum of three terms

$$\Delta C = \Delta C_\alpha + \Delta C_\delta + \Delta C_{\text{rec}}, \quad (5)$$

where  $\Delta C_\alpha$  contains Coulomb and radiative corrections, which do not depend on the nucleon structure (they are also known as the “outer” corrections),  $\Delta C_\delta$  is the part of radiative

corrections that is dependent on the nucleon structure (or the “inner” corrections), and  $\Delta C_{\text{rec}}$  represents recoil corrections. For recoil corrections we have

$$\begin{aligned} \Delta C_{\text{rec}} = & \frac{5}{12m_n X(1 + 3\lambda^2)} \left\{ 9\lambda\mu_V [(E_e^{\max})^2 - m_e^2](E_e^{\max})^4 \right. \\ & - 4(E_e^{\max})^2 m_e^2 + 3m_e^4 + 4m_e^4 \ln (E_e^{\max} / m_e) \\ & + \lambda [31(E_e^{\max})^6] - 117(E_e^{\max})^4 m_e^2 + 279(E_e^{\max})^2 m_e^4 \\ & - 211m_e^6 + 18m_e^8 / (E_e^{\max})^2 - 12m_e^4 (9(E_e^{\max})^2 \\ & + 11m_e^2) \ln (E_e^{\max} / m_e) + 3\mu_V [-(E_e^{\max})^4 m_e^2 \\ & - 9(E_e^{\max})^2 m_e^4 + 9m_e^6 + m_e^8 / (E_e^{\max})^2 \\ & - 12m_e^4 ((E_e^{\max})^2 + m_e^2) \ln (E_e^{\max} / m_e) \\ & + 3\lambda^2 [-2(E_e^{\max})^6] + 17(E_e^{\max})^4 m_e^2 + 9(E_e^{\max})^2 m_e^4 \\ & - 25m_e^6 + m_e^8 / (E_e^{\max})^2 - 12m_e^4 (5(E_e^{\max})^2 + m_e^2) \\ & \left. \times \ln (E_e^{\max} / m_e) \right] - \frac{6X_1\lambda}{5X(1 + 3\lambda^2)} \\ & \times E_e^{\max} \sqrt{(E_e^{\max})^2 - m_e^2} [\lambda^2 (52(E_e^{\max})^4 \\ & - 124(E_e^{\max})^2 m_e^2 + 507m_e^4) + (12(E_e^{\max})^4 \\ & - 4(E_e^{\max})^2 m_e^2 + 277m_e^4)] \left. \right\}, \quad (6) \end{aligned}$$

and for the strong interaction dependent part of the radiative corrections we have

$$\Delta C_\delta = -\frac{(4\lambda X_1 - X)}{2X(1 + 3\lambda^2)} \frac{\alpha}{2\pi} e_V^R, \quad (7)$$

where  $e_V^R$  is the LEC of the EFT [14]. The expression for  $\Delta C_\alpha$  is very long and too complicated to be presented here. However, one observes that all coefficients in the expressions for  $\Delta C$  (including  $\Delta C_\alpha$ ) depend only on the mass of electron and the maximal electron energy. Therefore, one can rewrite these expressions in a simple form (and without a loss of accuracy) by replacing the mass of electron and the maximal electron energy with their values:  $m_e = 0.511099$  MeV and  $E_e^{\max} = 1.293332$  MeV. Then all dependencies on these parameters collapse to numerical coefficients in the front of neutron decay variables and the complete set of corrections  $\Delta C$  can be written as

$$\begin{aligned} \Delta C = & \frac{1}{(1 + 3\lambda^2)} \left[ \frac{\alpha}{2\pi} (23.19375\lambda + 4.45619\lambda^2) \right. \\ & + \frac{\alpha}{2\pi} e_V^R (0.2748 - 1.0993\lambda) \\ & + \frac{1}{m_n} (2.25672\lambda - 0.265737\lambda^2 \\ & - 0.0113986\mu_V - 0.583714\lambda\mu_V) \\ & \left. - \frac{1}{m_n} \frac{\lambda}{(1 + 3\lambda^2)} (3.1326 + 7.775\lambda^2) \right], \quad (8) \end{aligned}$$

where neutron mass  $m_n$  is in MeV. The first term [the first line of Eq. (9)] is  $\Delta C_\alpha$ , the second term is  $\Delta C_\delta$ , and last three lines are recoil corrections. Now, using  $m_n = 939.57$  MeV,  $\mu_V = 3.7$ ,  $\alpha = 1/137.036$ , and  $\lambda = 1.2695$ , one obtains

$$\Delta C = 0.0065 - 0.00022e_V^R. \quad (9)$$

Thus, all radiative and recoil corrections are expressed in terms of only one unknown parameter (the EFT low energy constant), which is supposed to be obtained from another independent experiment, if possible, or should be calculated from basic principles (for example, in lattice QCD). In the framework of the EFT, it could be estimated as  $e_V^R \simeq 20$  (see Ref. [14] for details). Discussions of another way of estimating  $e_V^R$  and its accuracy are given in the last section.

### III. NEUTRON $\beta$ DECAY BEYOND THE STANDARD MODEL

Now, when we understand all contributions to the  $C$  angular correlation from the Standard Model, we can consider how possible contributions from new physics can change the value of the  $C$  asymmetry. To calculate the possible contributions to the  $C$  coefficient from the models beyond the Standard Model, one can use the most general form of the Hamiltonian for the description of neutron  $\beta$  decay in terms of low energy coupling constants  $C_i$  (do not confuse with  $C$  angular correlation coefficient) by [15,17]

$$\begin{aligned}
H_{\text{int}} = & (\hat{\psi}_p \psi_n)(C_S \hat{\psi}_e \psi_\nu + C'_S \hat{\psi}_e \gamma_5 \psi_\nu) \\
& + (\hat{\psi}_p \gamma_\mu \psi_n)(C_V \hat{\psi}_e \gamma_\mu \psi_\nu + C'_V \hat{\psi}_e \gamma_\mu \gamma_5 \psi_\nu) \\
& + \frac{1}{2}(\hat{\psi}_p \sigma_{\lambda\mu} \psi_n)(C_T \hat{\psi}_e \sigma_{\lambda\mu} \psi_\nu + C'_T \hat{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_\nu) \\
& - (\hat{\psi}_p \gamma_\mu \gamma_5 \psi_n)(C_A \hat{\psi}_e \gamma_\mu \gamma_5 \psi_\nu + C'_A \hat{\psi}_e \gamma_\mu \psi_\nu) \\
& + (\hat{\psi}_p \gamma_5 \psi_n)(C_P \hat{\psi}_e \gamma_5 \psi_\nu + C'_P \hat{\psi}_e \psi_\nu) \quad (10) \\
& + \text{Hermitian conjugate,}
\end{aligned}$$

where the index  $i = V, A, S, T,$  and  $P$  corresponds to vector, axial-vector, scalar, tensor, and pseudoscalar nucleon interactions. In this presentation, the constants  $C_i$  can be considered as effective constants of nucleon interactions with defined Lorentz structure, assuming that all high energy degrees of freedom (for the Standard Model and any given extension of the Standard Model) are integrated out. Because we are interested in the  $C$  angular correlation coefficient, which is the time reversal conserving one, all constants  $C_i$  can be chosen to be real. Also, for the sake of simplicity, we redefine all coupling constants  $C_i$  and  $C'_i$  in Eq. (10) by normalizing them by the vector coupling constant of the Standard Model [ $C_i \rightarrow C_i/(G_F \cos \theta_C)$ ].

To see explicitly the influence of a nonstandard interaction on the  $C$  angular coefficient, we follow the procedure described in Ref. [9]. First, we rewrite the coupling constants  $C_i$  as a sum of a contribution from the standard model  $C_i^{\text{SM}}$  and a possible contribution from new physics  $\delta C_i$ :

$$\begin{aligned}
C_V &= C_V^{\text{SM}} + \delta C_V \\
C'_V &= C_V^{\text{SM}} + \delta C'_V \\
C_A &= C_A^{\text{SM}} + \delta C_A \\
C'_A &= C_A^{\text{SM}} + \delta C'_A \\
C_S &= \delta C_S \\
C'_S &= \delta C'_S \\
C_T &= \delta C_T \\
C'_T &= \delta C'_T.
\end{aligned} \quad (11)$$

The pseudoscalar coupling constants are neglected here, because we treat [15] nucleons nonrelativistically. Then, we apply the above-described procedure to the calculation of the  $C$  angular correlation coefficient from the Hamiltonian (10) using Eq. (5) of Ref. [9]. [It should be noted that in the case of all  $\delta C_i$  being equal to zero, the results is Eq. (4).] The obtained corrections to the  $C$  correlation coefficient due to contributions from nonstandard modes can be written as

$$\begin{aligned}
\delta C_{\text{NewPhys}} = & \frac{X_1 L_1}{2X(1+3\lambda^2)} + \frac{X_3 L_3}{2X(1+3\lambda^2)} - \frac{X_1 2\lambda}{X(1+3\lambda^2)} \\
& \times \left[ \frac{L_0}{(1+3\lambda^2)} + \frac{X_2 L_2}{X(1+3\lambda^2)} \right], \quad (12)
\end{aligned}$$

where

$$\begin{aligned}
X_2 &= 10m_e(E_e^{\text{max}}) \sqrt{(E_e^{\text{max}})^2 - m_e^2} (2(E_e^{\text{max}})^2 + 13m_e^2) \\
& - 30m_e(4(E_e^{\text{max}})^2 m_e^2 + m_e^4) \\
& \times \ln((E_e^{\text{max}} + \sqrt{(E_e^{\text{max}})^2 - m_e^2})/m_e) \\
X_3 &= 5m_e(3(E_e^{\text{max}})^4 + 12(E_e^{\text{max}})^2 m_e^2 - 15m_e^4 \\
& - 12(2(E_e^{\text{max}})^2 m_e^2 + m_e^4) \ln(E_e^{\text{max}}/m_e)).
\end{aligned}$$

The coefficients  $L_i$  depend only on new physics contributions:

$$\begin{aligned}
L_0 &= (\delta C_V + \delta C'_V) + (\delta C_V^2 + \delta C_V'^2 + \delta C_S^2 + \delta C_S'^2)/2 \\
& + 3[\lambda(\delta C_A + \delta C'_A) + (\delta C_A^2 + \delta C_A'^2 \\
& + \delta C_T^2 + \delta C_T'^2)/2], \quad (13)
\end{aligned}$$

$$\begin{aligned}
L_1 &= -2(\delta C_A + \delta C'_A) + 3\delta C_T \delta C'_T - 2(\delta C_V \delta C'_A \\
& + \delta C'_V \delta C_A) + 2\lambda(\delta C_V + \delta C'_V), \quad (14)
\end{aligned}$$

$$\begin{aligned}
L_2 &= \sqrt{1 - \alpha^2}[(\delta C_S + \delta C'_S) + \delta C_S \delta C_V + \delta C'_S \delta C'_V \\
& + 3(\lambda(\delta C_T + \delta C'_T) + \delta C_T \delta C_A + \delta C'_T \delta C'_A)],
\end{aligned}$$

$$\begin{aligned}
L_3 &= \sqrt{1 - \alpha^2}[-2\lambda(\delta C_T + \delta C'_T) - \lambda(\delta C_S + \delta C'_S) \\
& + (\delta C_T + C'_T) + 2\delta C_T \delta C'_A + 2\delta C_A \delta C'_T + \delta C_S \delta C'_A \\
& + \delta C_A \delta C'_S + \delta C_V \delta C'_T + \delta C_T \delta C'_V]. \quad (15)
\end{aligned}$$

In the above expressions, we have neglected radiative corrections and recoil effects for the new physics contributions, but kept Coulomb corrections because they can be important for a low energy part of the electron spectrum. These expressions without Coulomb corrections correspond to results in Ref. [12] obtained for the case of  $C_S = -C'_S, C_T = -C'_T, C_V = -C'_V,$  and  $C_A = -C'_A$ . Contributions from nonstandard  $\beta$ -decay interactions have been considered also in Ref. [13] (in the VAST model family parametrization [13]) and they have the same order of magnitude as contributions in Eq. (12). However, because the results presented in Ref. [13] are numeric ones, it is difficult to make the exact comparison in a general case.

From Eq. (12), one can see that, as in the case of radiative and recoil corrections, all coefficients in the expression are functions only of electron mass and maximum electron energy. Therefore, we simplify the general expressions for the contributions from new physics, by substituting numerical values for all known parameters (electron mass, electron maximal energy, as well as for  $\alpha = 1/137.036$  and  $\lambda = 1.2695$ ) and keep only

TABLE I. Possible manifestations of new physics.

Model	Exotic fermion	Leptoquark	Contact interactions	SUSY	Higgs
$\bar{a}_{LL}$		0.2–0.03			
$\bar{a}_{LR}$	0.01	0.01			
$\bar{A}_{LL} + \bar{A}_{LR}$			0.01	$7.5 \cdot 10^{-4}$	$3 \cdot 10^{-6}$
$-\bar{A}_{LL} + \bar{A}_{LR}$		$3 \cdot 10^{-6}$			

first order contributions from nonstandard interactions. Then, Eq. (12) transforms into

$$\begin{aligned} \delta C_{\text{NewPhys}} &= 0.05657(\delta C_V + \delta C'_V) + 0.04456(\delta C_A + \delta C'_A) \\ &\quad - 0.06234(\delta C_S + \delta C'_S) + 0.02132(\delta C_T + \delta C'_T). \end{aligned} \quad (16)$$

Instead of the presentation of these corrections in terms of low energy coupling constants related to the Lorentz structure of weak interactions, we can rewrite them in terms of quark and lepton current constants  $\bar{a}_{jl}$  and  $\bar{A}_{jl}$ , defined in Ref. [7]. Using the transformation rules [9]

$$\begin{aligned} \delta C_V + \delta C'_V &= 2(\bar{a}_{LL} + \bar{a}_{LR}), \\ \delta C_A + \delta C'_A &= 2\lambda(\bar{a}_{LL} - \bar{a}_{LR}), \\ \delta C_S + \delta C'_S &= 2g_S(\bar{A}_{LL} + \bar{A}_{LR}), \\ \delta C_T + \delta C'_T &= 4g_T\bar{\alpha}_{LL}, \end{aligned} \quad (17)$$

and assuming [7]  $g_S = 1$  and  $g_T = 1$ , we obtain the expression for corrections from new physics as

$$\begin{aligned} \delta C_{\text{NewPhys}} &= 0.11314(\bar{a}_{LL} + \bar{a}_{LR}) + 0.11314(\bar{a}_{LL} - \bar{a}_{LR}) \\ &\quad - 0.12468(\bar{A}_{LL} + \bar{A}_{LR}) + 0.08528\bar{\alpha}_{LL}. \end{aligned} \quad (18)$$

The parameters  $\bar{a}_{jl}$ ,  $\bar{\alpha}_{jl}$ , and  $\bar{A}_{jl}$  describe contributions to the low energy Hamiltonian from current-current interactions in terms of  $j$  type of leptonic current and  $i$  type of quark current. For example,  $\bar{a}_{LR}$  is the contribution to the Hamiltonian from left-handed leptonic current and right-handed quark current normalized by the size of the Standard Model (left-left current) interactions.  $g_S$  and  $g_T$  are form factors at zero-momentum transfer in the nucleon matrix element of scalar and tensor currents. For more details, see Ref. [7].

The expected values of these parameters vary over a wide range from 0.07 to  $10^{-6}$  (see Table I and Ref. [7] for the comprehensive analysis and for discussions of significance of each of these parameters for models beyond the Standard one).

#### IV. CONCLUSIONS

Taking into account the results of Eqs. (4), (5), (12), and (18), one can write the complete expression for the  $C$  angular coefficient ( $C_{\text{total}}$ ) as a sum of the tree-level approximation  $C$ , radiative and recoil corrections in the Standard Model  $\Delta C$ , and possible contributions from new physics  $\delta C_{\text{NewPhys}}$ :

$$C_{\text{total}} = C + \Delta C + \delta C_{\text{NewPhys}}. \quad (19)$$

It should be noted that this equation is the exact expression of the  $C$  angular correlation coefficient in the first order of recoil corrections, radiative corrections, and low energy contributions from new physics. Therefore, it could be considered as the complete expression up to the level of accuracy of  $10^{-5}$ , provided the EFT LEC is given. Otherwise, it could be considered as a parametrization in terms of one free parameter—LEC with the same accuracy of  $10^{-5}$ . Would the parameter  $e_V^R$  be determined from another independent experiment (for example, from the precise measurement of neutrino-deuteron cross-sections) or calculated using lattice QCD approach, Eq. (19) could be used to test the Standard Model up to the level of accuracy of about  $10^{-5}$ , by comparing a theoretical prediction with experimental results. Unfortunately, neutrino experiments and QCD calculations with the required accuracy are rather difficult problems and we cannot rely on them at the present time.

To understand the desirable level of accuracy in a search for new physics, one can use first a conservative approach: the estimate for the LEC as  $e_V^R \simeq 20$  given in Ref. [14]. Then, the level of theoretical uncertainties due to strong interactions, according to Eq. (9), is about 0.0044, which is comparable to the claimed experimental accuracy 0.0026 of the recent experiment [10]. However, as it was mentioned in Ref. [14], by comparing the results of the EFT approach and the calculations of radiative corrections for total neutron decay rate [18–20], one can find the correspondence between these two calculations, which results [14] in the following equation

$$\begin{aligned} e_V^R &= -\frac{5}{4} - 4 \ln\left(\frac{m_W}{m_Z}\right) + 3 \ln\left(\frac{m_W}{m_N}\right) + \ln\left(\frac{m_W}{m_A}\right) \\ &\quad + 2C_{\text{Born}} + A_g. \end{aligned} \quad (20)$$

Here  $m_W$ ,  $m_Z$  are the masses of the  $W$ ,  $Z$  bosons and  $m_A$  is the axial mass scale, which are rather well known. The source of theoretical uncertainties is related to two last terms  $C_{\text{Born}}$  and  $A_g$  (see, for details, Refs. [18–21]). Changing from the EFT “ideology” with one unknown LEC to direct calculations using strong interaction models, we lost the attractive feature of the model independent EFT approach and have to deal with dependencies on strong interaction models applied for the description of the internal structure of nucleons. On the other hand, in the given framework [18–20], which is actually a very well recognized standard approach to general analysis of weak interactions, we can reduce uncertainties in the estimation of LEC to the uncertainties of calculations of  $C_{\text{Born}}$  and  $A_g$  terms. Then, using the results of recent calculations of the terms [21]  $C_{\text{Born}} \simeq 0.829$  and  $A_g \simeq -0.34$  with the claimed level of uncertainty of 10%, one can reduce the level of uncertainty of

the obtained theoretical description of the  $C$  angular coefficient to the level of about  $10^{-5}$ , i.e., to the level of validity of the description of neutron decay in Ref. [14].

Accepting these estimates, one can see from Eqs. (16) and (18) that precise measurements of the  $C$  angular correlation can provide limits for nonstandard interactions in terms of  $\delta C_i$  coupling constants up to the level of about  $(2 - 5) \cdot 10^{-4}$ , or, in terms of parameters related to nonstandard currents, up to the level of about  $10^{-4}$ . However, to be able to constrain

new physics parameters at this level, the currently achieved experimental accuracy [10] must be improved by two orders of magnitude.

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