Sea-quark effects in the pion charge form factor

Q. B. Li* and D. O. Riska†
Helsinki Institute of Physics, POB 64, 00014 University of Helsinki, Finland
(Received 23 August 2007; published 23 April 2008)

It is shown that the data on the pion charge form factor admit the possibility for a substantial sea-quark component in the pion wave function. If the charge form factor is calculated with instant form kinematics in a constituent quark model that is extended to include explicit $(q\bar{q})^2$ components in the pion wave function, that component will give the dominant contribution to the calculated π^+ charge form factor at large values of momentum transfer. The present experimental values Q^2 can be described fairly well with $(q\bar{q})^2$ component admixtures of up to 50%. The sensitivity of the calculated π^+ charge form factor to whether one of the quarks or one of the antiquarks is taken to be in the P state is small.

DOI: 10.1103/PhysRevC.77.045207 PACS number(s): 14.40.Aq, 12.39.Ki, 13.40.Gp, 24.85.+p

I. INTRODUCTION

The small mass of the pion, in comparison to that of the vector mesons, suggests that the pion is a collective state of $(q\bar{q})^n$ configurations, with many values of n. Therefore, it is something of a riddle that it is possible to describe the empirical charge form factors of the charged pions satisfactorily with phenomenological wave functions under the assumption that they are pure quark-antiquark states [1]. It is then a natural question to ask whether the empirical pion charge form factors can exclude the presence of the expected multiquark configurations in the pion.

The negative parity of the pion requires that in the simplest sea-quark configuration, $(q\bar{q})^2$, at most three of the constituents can be in the ground state and that either one of the quarks or one of the antiquarks is raised to the P state (or a higher odd-L state). Because this is energetically unfavorable, it suggests that the probability of that configuration may be small in comparison to that of the $q\bar{q}$ component. The situation is analogous to that in baryons, for which positive parity requires that in a $qqqq\bar{q}$ admixture either one of the quarks or the antiquark has to be in the P state [2].

Here the charge form factor of the (charged) pion is calculated in an extension of the constituent quark model to include admixtures of the simplest sea-quark configurations $(q\bar{q})^2$ in instant form kinematics. The calculation is made for both the case where one of the quarks is in the P state and for the case where one of the antiquarks is in the P state. It is found in both cases that inclusion of the sea-quark configuration allows for a good description of the empirical form factor, even if it represents as much as half of the wave function. The main point is, however, that as soon as there is a nonvanishing probability for the sea-quark component in the wave function, that component will lead to the dominant contribution to the charge form factor at large values of momentum transfer in the case of instant form kinematics. As a consequence the fact that it is possible to achieve a quantitatively satisfactory fit to the empirical charge form factor with wave function model for the conventional $q\bar{q}$ component alone does not rule out the presence of significant sea-quark components in the pion.

The present results are in line with the conclusions of Ref. [3], which were based on a phenomenological hadronic approach to the pion wave function in (light) front form kinematics. That analysis estimated the probability of the four-quark component in the pion wave function to fall within the range 10–26%. The probability for the six-quark component was estimated to be much smaller, less than 0.3%. The method of calculating the charge form factor in the constituent quark model with instant form kinematics developed here can also be readily extended to sea-quark configurations with larger numbers of sea-quark $q\bar{q}$ components. An earlier study based on the Nambu-Jona-Lasinio model indicated that within that model the sea-quark contribution would be at most 10% [4].

The configurations of the $(q\bar{q})^2$ system that are possible in the pion are described in Sec. II. Section III contains a description of the pion wave function and the different form factor contributions. The calculated results for the pion charge form factor are given in Sec. IV. Finally Sec. V contains a summarizing discussion.

II. LIGHT FLAVOR $(q\bar{q})^2$ CONFIGURATIONS IN THE PION

The lightest $(q\bar{q})^2$ component in the pion meson contains only the light flavor quarks u and d, which form the fundamental representation of the SU(2) flavor symmetry. The flavor-spin-color configurations of the qq and $\bar{q}\bar{q}$ are listed in Table I by their Young patterns.

The wave functions of the qq and $\bar{q}\bar{q}$ pairs in the flavor-spin-color-orbital space should be totally antisymmetrized, respectively. In addition, the odd parity of the pion meson requires that either a quark or an antiquark in the $qq\bar{q}\bar{q}$ component be in the P state (or higher odd-L state).

In the case where one quark is in the P state and both antiquarks are in the S state the flavor-spin-color wave functions of the qq pairs in the $(q\bar{q})^2$ component are totally symmetric while that of the $\bar{q}\bar{q}$ pairs are totally antisymmetric. This leads to four possible color singlet configurations of the

*ligb@pcu.helsinki.fi
†riska@pcu.helsinki.fi

TABLE I. The flavor-spin-color configurations of the qq and $\bar{q}\bar{q}$ pairs.

	qq	$\bar{q}\bar{q}$
$SU(2)_{flavor}$	$[2]_F, [11]_F$	$[2]_F, [11]_F$
$SU(2)_{spin}$	$[2]_S, [11]_S$	$[2]_S, [11]_S$
$SU(3)_{color}$	$[2]_C, [11]_C$	$[22]_C, [211]_C$

$(q\bar{q})^2$ component with $J^P = 0^-$:

$$\begin{aligned} &(a)\quad \{[2]_F[2]_S[2]_C\}_{qq}\{[2]_F[11]_S[22]_C\}_{\bar{q}\bar{q}},\\ &(b)\quad \{[2]_F[2]_S[2]_C\}_{qq}\{[11]_F[2]_S[22]_C\}_{\bar{q}\bar{q}},\\ &(c)\quad \{[2]_F[11]_S[11]_C\}_{qq}\{[2]_F[2]_S[211]_C\}_{\bar{q}\bar{q}},\\ &(d)\quad \{[11]_F[2]_S[11]_C\}_{qq}\{[2]_F[2]_S[211]_C\}_{\bar{q}\bar{q}}. \end{aligned} \tag{1}$$

It is natural to assume that the $(q\bar{q})^2$ configuration with the lowest energy shall have the largest probability in the pion besides that of the conventional $q\bar{q}$ component. The splitting of the energy of the $(q\bar{q})^2$ components (2) is determined by the hyperfine interaction between the quarks and the antiquarks. This interaction will here be taken to have the schematic form

$$H_I = -X \sum_{i\neq j} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c. \tag{2}$$

Here $\vec{\sigma}$ are the Pauli matrices, $\vec{\lambda}^c$ are the Gell-Mann matrices in color space, and X is a positive constant with the dimension of energy. This interaction has the same color and spin dependence as the color magnetic hyperfine interaction, which arises from single gluon exchange. The contributions to the energies of the four configurations (2) that arise from the schematic hyperfine interaction (2) can be determined by the recoupling method described in Refs. [5] and [6] and are listed in Table II.

In the case where one antiquark is in the P state and both the quarks are in the S state the wave functions of the qq pairs in flavor-spin-color space must be totally antisymmetric, while those of the $\bar{q}\bar{q}$ pairs must be totally symmetric. The possible $(q\bar{q})^2$ configurations in the pion are in this case the following:

$$\begin{aligned} &(a')\quad \{[2]_F[2]_S[11]_C\}_{qq}\{[2]_F[11]_S[211]_C\}_{\bar{q}\bar{q}},\\ &(b')\quad \{[2]_F[2]_S[11]_C\}_{qq}\{[11]_F[2]_S[211]_C\}_{\bar{q}\bar{q}},\\ &(c')\quad \{[2]_F[11]_S[2]_C\}_{qq}\{[2]_F[2]_S[22]_C\}_{\bar{q}\bar{q}},\\ &(d')\quad \{[11]_F[2]_S[2]_C\}_{qq}\{[2]_F[2]_S[22]_C\}_{\bar{q}\bar{q}}. \end{aligned} \tag{3}$$

The expectation values of the hyperfine interaction between quarks in the $(q\bar{q})^2$ configurations in (4) are listed in Table III. The results in Tables II and III show that the hyperfine interaction between quarks leads to the same energy levels

TABLE II. The expectation values $-\frac{1}{X}\langle\alpha|H_I|\alpha\rangle$ of the $(q\bar{q})^2$ configurations in Eq. (2).

α	a	b	c	d
$-\frac{1}{X}\langle\alpha\|H_I\|\alpha\rangle$	$\frac{-8}{3}$	16	$\frac{16}{3}$	0

TABLE III. The expectation values $-\frac{1}{X}\langle\alpha'|H_I|\alpha'\rangle$ of the $(q\bar{q})^2$ configurations in Eq. (4).

α'	a'	b'	c'	d'
$-\frac{1}{X}\langle\alpha'\|H_I\|\alpha'\rangle$	$\frac{16}{3}$	0	$\frac{-8}{3}$	16

of the $(q\bar{q})^2$ configurations, in which the antiquark is in the S state and the P state. This is a consequence of the fact that the hyperfine interaction is independent of the angular momentum of the constituent quarks. In the case where the antiquark is in the P state, the $(q\bar{q})^2$ configuration d' has the lowest energy, which is equal to that of the lowest energy $(q\bar{q})^2$ configuration b, in which the antiquark is in the S state. These two configurations are thus likely to constitute the most probable $(q\bar{q})^2$ components and therefore to be most significant in the structure of the pions. The roles of such configurations in the pion charge form factor are considered in the following section.

III. π^+ CHARGE FORM FACTOR

A. The form factor contribution from the $q\bar{q}$ component

The present empirical results for the charge form factor of the π^+ may be well described as a $q\bar{q}$ system in a Poincaré covariant constituent quark model with the following orbital wave function model [1]:

$$\phi(\vec{k}_1, \vec{k}_2) = \mathcal{N}_{2q} \frac{1}{\left(1 + \frac{\sum_{i=1}^2 \vec{k}_i^2}{2b^2}\right)^a}. \tag{4}$$

Here $\mathcal{N}_{2q}$ is a normalization factor, a and b are parameters and $k_i, i = 1, 2$, are the quark momenta in the rest frame of π meson ($\sum_{i=1}^2 \vec{k}_i = 0$). This wave function model is adopted here for the $q\bar{q}$ component of the pion wave function.

In the impulse approximation the contribution of the $q\bar{q}$ component to the pion charge form factor is obtained as the matrix element of the electric current density operator between the initial and final states in the Breit frames as

$$F_\pi^{(q\bar{q})}(Q^2) = \int d^3\vec{p}_2 \sqrt{J_2 J_2'} S_e(\vec{p}_1, \vec{p}_1')\, \phi(\vec{k}_1, \vec{k}_2)\phi(\vec{k}_1', \vec{k}_2'). \tag{5}$$

The initial and final momenta of the constituents in their respective Breit frames are denoted $\vec{p}_i$ and $\vec{p}_i'$, respectively ($\vec{p}_1' = \vec{p}_1 + \vec{Q}$ and $\vec{p}_2 = \vec{p}_2'$).

If the momentum transfer $\vec{Q}$ is taken to define the z axis, the relations in instant form kinematics between the momenta in the Breit frames and the rest frames are

$$\begin{aligned} \vec{p}_{i\perp} &= \vec{k}_{i\perp} = \vec{k}_{i\perp}' = \vec{p}_{i\perp}',\\ p_{i\parallel} &= v_0 k_{i\parallel} + v_\parallel\, \omega_i,\\ p_{i\parallel}' &= v_0' k_{i\parallel}' + v_\parallel' \omega_i',\\ E_i &= v_\parallel k_{i\parallel} + v_0 \omega_i,\\ E_i' &= v_\parallel' k_{i\parallel}' + v_0' \omega_i'. \end{aligned} \tag{6}$$

Here the energy components are defined as

$$\omega_i = \sqrt{\vec{k}_i^2 + m^2}, \quad \omega_i' = \sqrt{\vec{k}_i'^2 + m^2},$$
$$E_i = \sqrt{\vec{p}_i^2 + m^2}, \quad E_i' = \sqrt{\vec{p}_i'^2 + m^2}. \tag{7}$$

In these relations m denotes the constituent mass and $v = \{v_0, \vec{0}_\perp, v_\parallel\}$ and $v' = \{v_0', \vec{0}_\perp, v_\parallel'\}$ denote the constituent boost velocities in the initial and final states. These satisfy the constraint $v^2 = v'^2 = -1$.

In instant form kinematics the boost velocities may be defined as [7]

$$v_\parallel = -\frac{Q}{2\sum_{i=1}^n \omega_i},$$
$$v_\parallel' = \frac{Q}{2\sum_{i=1}^n \omega_i'}. \tag{8}$$

Here n represents the number of constituents.

The Jacobian that is induced by the transformation between the rest frame and the Breit frame of the meson is in the case of the $q\bar{q}$ component obtained as [8]

$$J_2 = \frac{\omega_2}{E_2}\left(1 - v_\parallel \frac{k_{1\parallel}}{E_1}\right). \tag{9}$$

The expression for the corresponding final state Jacobian J_2' (5) is obtained by replacement of the arguments by the corresponding primed coordinates.

The electric current density operator in Eq. (5) is taken to be the charge component of the Dirac current for pointlike constituents, γ_μ:

$$S_e(\vec{p}, \vec{p}') = \sqrt{1 + \frac{Q^2}{4M_\pi^2}}\sqrt{\frac{(E'+m)(E+m)}{4E'E}} \times \left\{1 + \frac{\vec{p}'\cdot\vec{p}}{(E'+m)(E+m)}\right\}. \tag{10}$$

The corresponding current is conserved, as the constituents are on the mass shell in covariant quantum mechanics [7].

The expressions for the boost velocities of the constituents (8) reveal that their magnitudes fall with increasing number of constituents n, if the constituent mass is constant. Given that form factors fall with increasing momentum transfer Q^2, it follows that, at sufficiently large Q^2, the wave function component that contains the largest number of constituents will give the largest contribution to the form factor. This feature is explicit in instant form kinematics. It has a natural physical interpretation in that the form factor describes the probability that the system stays bound upon absorption of the momentum transfer Q. The relative probability for this to happen is smaller if few constituents absorb the momentum transfer than if many can share it so that the fractional momentum transfer per constituent is smaller.

B. The form factor contribution from the $(q\bar{q})^2$ component

In the case where both of the antiquarks in the $(q\bar{q})^2$ component are in the S state, the wave function of the $(q\bar{q})^2$ component, which has the symmetry configuration b in Eq. (2), and which is expected to have the lowest energy, may be expressed as

$$|\pi^+\rangle_S = -\frac{1}{\sqrt{2}}\, uu(\bar{d}\bar{u} - \bar{u}\bar{d})\,\{[\mathbf{1}_S \otimes \mathbf{1}_S']_1 \otimes \mathbf{1}_X\}_{0^-} \times \{\mathbf{6}_C \otimes \bar{\mathbf{6}}_C\}_{1_C}\,\Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4). \tag{11}$$

Here $\vec{k}_i, i = 1\ldots4$, are the momenta of the constituent quarks in the rest frame of the pion ($\sum_{i=1}^4 \vec{k}_i = 0$). The spin triplet combinations of the qq and $\bar{q}\bar{q}$ pairs are denoted $\mathbf{1}_S$ and $\mathbf{1}_S'$, respectively. These combine with the P-state qq pairs to the total quantum numbers of pion $J^P = 0^-$. The spherical harmonic for the qq pairs in the $(q\bar{q})^2$ component is defined as

$$\mathbf{1}_{X,m} = \xi_{1m}, \quad \vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{k}_1 - \vec{k}_2), \tag{12}$$

where ξ_{1m} ($m = -1, 0, 1$) are the spherical components of $\vec{\xi}_1$. In Eq. (11) we have denoted the Young pattern representations of the color states of qq and $\bar{q}\bar{q}$ pairs in Table I with their corresponding dimensions (**6**).

The orbital wave function of the $(q\bar{q})^2$ component is taken to have the form

$$\Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) = \mathcal{N}_{4q}\frac{1}{\left(1 + \frac{\sum_{i=1}^4 \vec{k}_i^2}{2B^2}\right)^{A+1}}, \tag{13}$$

where $\mathcal{N}_{4q}$ is a normalization factor.

Then the contribution to the π^+ charge form factor from the $qq\bar{q}\bar{q}$ component in the case where one of the antiquarks is in the S state may be written as

$$F_{\pi^+}^S(Q^2) = \tfrac{4}{3}A^S(Q^2) - \tfrac{1}{3}B^S(Q^2). \tag{14}$$

Here the terms A and B are defined as

$$A^S(Q^2) = \frac{1}{3}\int d^3\vec{p}_2 d^3\vec{p}_3 d^3\vec{p}_4 \sqrt{J_4(1)J_4'(1)}\, S_e(\vec{p}_1, \vec{p}_1')\vec{\xi}_1\cdot\vec{\xi}_1' \times \Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)\Phi(\vec{k}_1', \vec{k}_2', \vec{k}_3', \vec{k}_4'), \tag{15}$$

$$B^S(Q^2) = \frac{1}{3}\int d^3\vec{p}_1 d^3\vec{p}_2 d^3\vec{p}_3 \sqrt{J_4(4)J_4'(4)}\, S_e(\vec{p}_4, \vec{p}_4')\vec{\xi}_1\cdot\vec{\xi}_1' \times \Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)\Phi(\vec{k}_1', \vec{k}_2', \vec{k}_3', \vec{k}_4'). \tag{16}$$

Here $A^S(Q^2)$ represents the matrix element where a photon couples to the first quark in the $(q\bar{q})^2$ component, while $B^S(Q^2)$ represents the matrix element where the photon couples to the antiquark (the fourth constituent).

The Jacobians for the transformations between the corresponding Breit frames and the rest frames are

$$J_4(1) = \frac{\omega_2\omega_3\omega_4}{E_2E_3E_4}\left(1 - v_\parallel\frac{k_{1\parallel}}{E_1}\right), \tag{17}$$

$$J_4(4) = \frac{\omega_1\omega_2\omega_3}{E_1E_2E_3}\left(1 - v_\parallel\frac{k_{4\parallel}}{E_4}\right). \tag{18}$$

As in the case of Eq. (5), the primed variables in Eqs. (15) and (16) represent the final states variables that correspond to the initial state variables without primes. Here we do not take into account the Wigner rotation of the spin axis that is caused by the boosts, as its consequences are numerically insignificant for momentum transfers below 10 GeV^2 [7,9]. Comparison of the expressions for the Jacobians (9) and (18) for the case of two and four constituents, respectively, makes it clear how to generalize these expressions to the case of n constituents.

In the case where one antiquark is in the P state and both quarks are in the ground state, the wave function of the $(q\bar{q})^2$ component with the lowest energy, which has the symmetry configuration d' in Eq. (4), may be obtained from that in the case where one of the quarks is in the P state (11) by the replacements

$$u \leftrightarrow -\bar{d} \quad d \leftrightarrow \bar{u}. \tag{19}$$

The explicit expression is then

$$|\pi^+\rangle_P = \frac{1}{\sqrt{2}}(ud - du)\bar{d}\bar{d}\,\{[\mathbf{1}_S \otimes \mathbf{1}'_S]_1 \otimes \mathbf{1}'_x\}_{0^-} \times \{\mathbf{6}_C \otimes \bar{\mathbf{6}}_C\}_{1_C}\,\Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4). \tag{20}$$

Here the spherical harmonic for the $\bar{q}\bar{q}$ pair is denoted as

$$\mathbf{1}'_{x,m} = \xi_{3m}, \quad \vec{\xi}_3 = \frac{1}{\sqrt{2}}(\vec{k}_3 - \vec{k}_4). \tag{21}$$

The explicit expression for the π^+ charge form factor in the case where one antiquark is in the P state is

$$F^P_{\pi^+}(Q^2) = \tfrac{1}{3}A^P(Q^2) + \tfrac{2}{3}B^P(Q^2). \tag{22}$$

Here the orbital integrals are defined as

$$A^P(Q^2) = \frac{1}{3}\int d^3\vec{p}_2 d^3\vec{p}_3 d^3\vec{p}_4 \sqrt{J_1(1)J'_1(1)}S_e(\vec{p}_1, \vec{p}'_1)\vec{\xi}_3\cdot\vec{\xi}'_3 \times \Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)\Phi(\vec{k}'_1, \vec{k}'_2, \vec{k}'_3, \vec{k}'_4), \tag{23}$$

$$B^P(Q^2) = \frac{1}{3}\int d^3\vec{p}_1 d^3\vec{p}_2 d^3\vec{p}_3 \sqrt{J_4(4)J'_4(4)}S_e(\vec{p}_4, \vec{p}'_4)\vec{\xi}_3\cdot\vec{\xi}'_3 \times \Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)\Phi(\vec{k}'_1, \vec{k}'_2, \vec{k}'_3, \vec{k}'_4). \tag{24}$$

The symmetrical form of the expressions (11) and (20) has the consequence that

$$A^P(Q^2) = B^S(Q^2), \quad B^P(Q^2) = A^S(Q^2). \tag{25}$$

Because of the symmetry structure of the spin-flavor-color state of the $(q\bar{q})^2$ components, there is no contribution of off-diagonal $(q\bar{q})^2 \to q\bar{q}$ transition matrix elements to the pion charge form factor. In the case of the nucleons the contribution of such transition matrix elements to the form factors are much larger than that of the corresponding diagonal matrix elements [8].

IV. RESULTS

To investigate a possible role of the $(q\bar{q})^2$ component in the form factor of the π^+ meson, the wave function parameters are chosen so that the combined contribution of the $q\bar{q}$ and the $(q\bar{q})^2$ components yields a form factor that agrees with the empirical one under the assumption of a probability for the latter component of 10%. The corresponding wave function parameters a, b (4) and A, B (13) are listed in Table IV. The constituent quark mass was taken to be 120 MeV, which is close to the value required to describe the nucleon form factors in instant form kinematics with a wave function of corresponding

TABLE IV. The model parameters.

$(q\bar{q})^2$	m_q (MeV)	b (MeV)	B (MeV)	a	A
10%	120	190	100	2.3	1.8
20%	120	190	110	2.3	2.0
40%	120	190	139	2.3	2.21
50%	120	190	143	2.3	2.25

form [7]. The wave function parameters for the $q\bar{q}$ and the $(q\bar{q})^2$ components were chosen such that the empirical mean square radius for the pion, $r_\pi^2 = 0.44$ fm^2, was recovered.

The calculated result for the π^+ charge form factor is shown in Fig. 1 for the case where both antiquarks are in the S state in the $(q\bar{q})^2$ component. The result indicates that above 1 GeV^2 with these parameters the main form factor contribution arises from the smaller $(q\bar{q})^2$ component. That this should be so is, in fact, quite natural, as in the case of elastic form factors, the form factor falloff with momentum should depend on Q^2 divided by the square of the number of involved constituents. In this case the contribution of the $(q\bar{q})^2$ component is very small (and in fact negative): -0.03 fm^2. The sign of this contribution depends on the parameter values.

The corresponding calculated results for the π^+ charge form factor for the case where one of the antiquarks is in the P state are shown in Fig. 2. These results are, in fact, very similar to those obtained in the former case, where both antiquarks are in the S state, the main difference being a slightly larger magnitude for the contribution to the mean square radius from the $(q\bar{q})^2$ component in the present case (-0.04 fm^2).

In Fig. 3 the form factor is shown as obtained for different values of the probability for the $(q\bar{q})^2$ component. These results were obtained by only slight variation of the two parameters in the wave function of the $(q\bar{q})^2$ component (13). These results show that the present empirical data on the pion

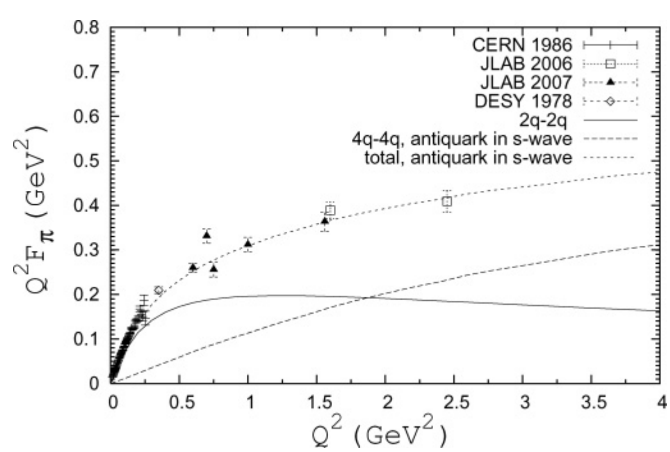

FIG. 1. The π^+ charge form factor obtained with a 10% $(q\bar{q})^2$ component probability with both antiquarks in the S state. Solid line, the contribution from the $q\bar{q}$ component; dashed line, the contribution from the $(q\bar{q})^2$ component; dotted line, the result combining the contributions from the $q\bar{q}$ and the $(q\bar{q})^2$ components. The data sets CERN 1986, DESY 1978, JLAB 2006, and JLAB 2007 are taken from Refs. [10–13], respectively. The data point at $Q^2 = 1.60$ GeV^2 from Ref. [13] has been shifted for better visibility.

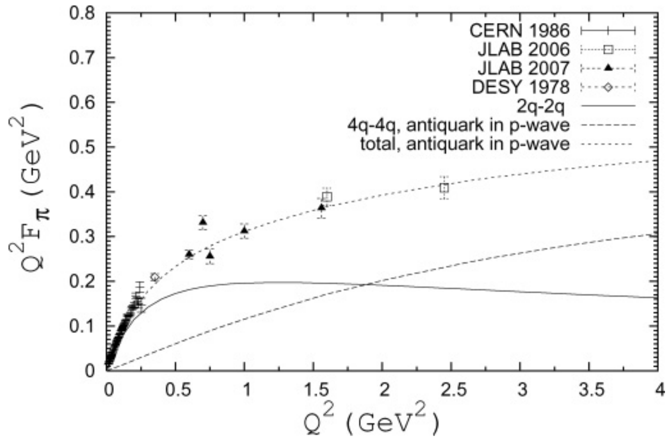

FIG. 2. The π^+ charge form factor with a 10% probability for the $(q\bar{q})^2$ component with one antiquark in the P state. The labeling of the curves is the same as that in Fig. 1.

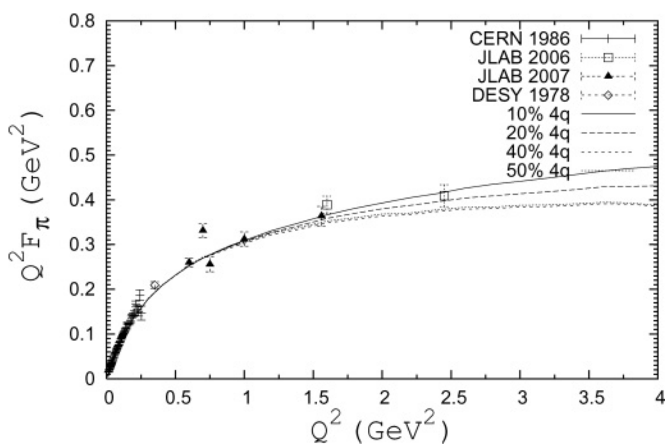

FIG. 3. The π^+ charge form factor obtained with 10–50% $(q\bar{q})^2$ component probabilities with both antiquarks in the S state. The data sets CERN 1986, DESY 1978, JLAB 2006, and JLAB 2007 are taken from Refs. [10–13], respectively. The data point at $Q^2 = 1.60$ GeV^2 from Ref. [13] has been shifted for better visibility.

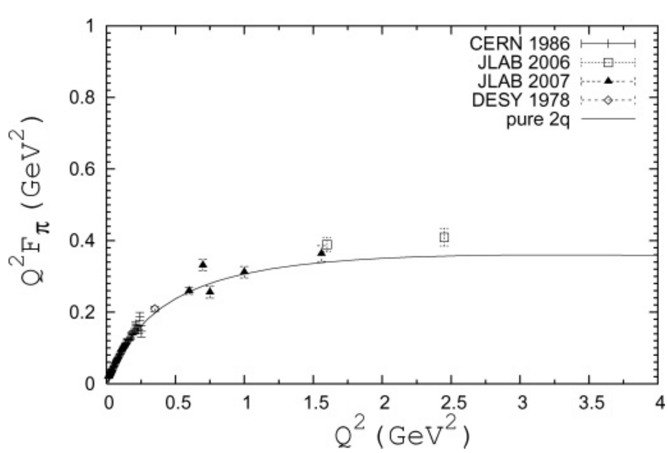

FIG. 4. The π^+ charge form factor for the pure $q\bar{q}$ model. The labeling of the curves is the same as that in Fig. 1.

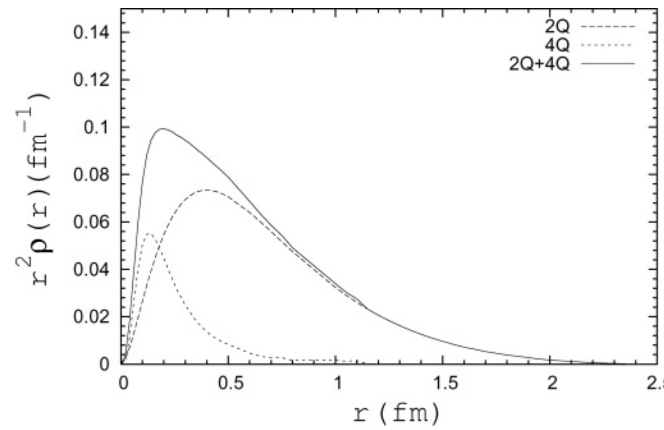

FIG. 5. The "charge density" of the π^+ for a $(q\bar{q})^2$ component with 20% probability. The contributions from the $q\bar{q}$ and $(q\bar{q})^2$ components are denoted $2Q$ and $4Q$, respectively.

charge form factor can allow for a $(q\bar{q})^2$ component probability of up to 50%.

For comparison the form factor that is obtained for the pure $q\bar{q}$ quark model for the pion is shown in Fig. 4. These results were obtained with the constituent quark mass value 80 MeV and with the parameters $a = 2.0$ and $b = 198$ MeV in the $q\bar{q}$ wave function model (4). This shows that the pion charge form factor may be described with such a simple model wave function in instant form kinematics, a result that was noted for the case of front form kinematics in Ref. [1].

V. CONCLUSIONS

The results of this study show that the present data for the charge form factor of the charged pions may be described almost as well with inclusion of sea-quark configurations with probabilities up to at least 50% in the covariant constituent quark model with instant form kinematics as without such configurations. The results in Fig. 3 do nevertheless indicate a slight preference for the smaller admixtures of 10–20% of sea-quark configurations. This conclusion is similar to that obtained in the hadronic approach with front form kinematics previously [3].

In this phenomenological study the parameters in the wave function model were determined by a fit to the empirical form factor. When the sea-quark component is included in the form factor, that component will give the dominant contribution to the form factor at sufficiently large Q^2, which dominates over that from the $q\bar{q}$ component, however small its probability. The $(q\bar{q})^2$ component corresponds to structures that have shorter range than the basic $q\bar{q}$ component. This is illustrated in Fig. 5, where the charge density contributions from the $q\bar{q}$ and the $(q\bar{q})^2$ components, along with their sum, are shown for the case in which the probability of the latter component is 20%. If the probability of the $(q\bar{q})^2$ component is taken to be larger, the peak in the profile $r^2\rho(r)$ moves toward that of the $q\bar{q}$ component.

The fact that the sea-quark contribution gives the largest contribution to the form factor at (sufficiently) large values of momentum transfer is a consequence of the fact that the magnitude of the energy denominator in the expressions for the boost velocities for the constituents (8) grows with the number of constituents. This implies that the momentum transfer is shared by the largest number of constituents and, in effect, to a smaller relative momentum transfer per constituent. Because the form factor is a monotonically falling function of momentum transfer, the consequence is that the largest contribution at large Q^2 is given by the component with the largest number of constituents.

[1] F. Coester and W. N. Polyzou, Phys. Rev. C **71**, 028202 (2005).
[2] C. Helminen and D. O. Riska, Nucl. Phys. **A699**, 624 (2002).
[3] A. Szczurek, H. Holtmann, and J. Speth, Nucl. Phys. **A605**, 496 (1996).
[4] R. H. Lemmer and R. Tegen, Nucl. Phys. **A593**, 315 (1995).
[5] F. Close, *An Introduction to Quarks and Partons* (Academic Press, London, 1979).
[6] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977).
[7] B. Juliá-Díaz, D. O. Riska, and F. Coester, Phys. Rev. C **69**, 035212 (2004).
[8] Q. B. Li and D. O. Riska, Nucl. Phys. **A791**, 406 (2007).
[9] F. Coester and D. O. Riska, Nucl. Phys. **A728**, 439 (2003).
[10] S. R. Amendolia *et al.*, Nucl. Phys. **B277**, 168 (1986).
[11] H. Ackermann *et al.*, Nucl. Phys. **B137**, 294 (1978).
[12] T. Horn *et al.*, Phys. Rev. Lett. **97**, 192001 (2006).
[13] V. Tadevosyan *et al.*, Phys. Rev. C **75**, 055205 (2007).