

Search for long lived heaviest nuclei beyond the valley of stability

P. Roy Chowdhury,^{1,*} C. Samanta,^{1,2,3,†} and D. N. Basu^{4,‡}

¹Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700 064, India

²Physics Department, Virginia Commonwealth University, Richmond, Virginia 23284-2000, USA

³Physics Department, University of Richmond, Richmond, Virginia 23173, USA

⁴Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700 064, India

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The existence of long lived superheavy nuclei (SHN) is controlled mainly by spontaneous fission and α -decay processes. According to microscopic nuclear theory, spherical shell effects at $Z = 114, 120, 126$ and $N = 184$ provide the extra stability to such SHN to have long enough lifetime to be observed. To investigate whether the so-called “stability island” could really exist around the above Z, N values, the α -decay half-lives along with the spontaneous fission and β -decay half-lives of such nuclei are studied. The α -decay half-lives of SHN with $Z = 102$ – 120 are calculated in a quantum tunneling model with DDM3Y effective nuclear interaction using Q_α values from three different mass formulas prescribed by Koura-Ueno-Tachibana-Yamada (KUTY), Myers-Swiatecki (MS), and Muntian-Hofmann-Patyk-Sobiczewski (MMM). Calculation of spontaneous fission (SF) half-lives for the same SHN are carried out using a phenomenological formula and compared with SF half-lives predicted by Smolanczuk *et al.* A possible source of discrepancy between the calculated α -decay half-lives of some nuclei and the experimental data of GSI, JINR-FLNR, RIKEN, is discussed. In the region of $Z = 106$ – 108 with $N \sim 160$ – 164 , the β -stable SHN $^{268}_{106}\text{Sg}_{162}$ is predicted to have highest α -decay half-life ($T_\alpha \sim 3.2$ h) using Q_α value from MMM. Interestingly, it is much greater than the recently measured T_α (~ 22 s) of deformed doubly magic $^{270}_{108}\text{Hs}_{162}$ nucleus. A few fission-survived long-lived SHN which are either β -stable or having large β -decay half-lives are predicted to exist near $^{294}_{110}\text{184}$, $^{293}_{110}\text{183}$, $^{296}_{112}\text{184}$, and $^{298}_{114}\text{184}$. These nuclei might decay predominantly through α -particle emission.

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I. INTRODUCTION

The theoretical studies of properties of heaviest nuclei during the past few decades have drawn considerable attention of experimentalists to investigate the existence of superheavy nuclei beyond the valley of stability. Since the macroscopic [1,2] description on the basis of liquid drop model (LDM) does not take the shell effect into account, it fails to explain the variation of fission barrier height of heavy nuclei with the increase of the fissility parameter ($\sim Z^2/A$). However, according to modern nuclear theory, hindrance to the fissioning of heavy nuclei would be enhanced due to the presence of deformed and spherical shell closures. Different semimicroscopic approaches, e.g., macroscopic-microscopic model (MMM) [3–6] and its modification [7] include pairing and nuclear shell effects [8] to reproduce the properties of ground and deformed states of nuclei. Many purely microscopic [9–13] descriptions like Hartree-Fock-Bogoliubov (HFB) model with zero range forces of Skyrme type [14,15] or finite range forces of Gogny type [16,17] and relativistic mean field (RMF) [18,19] theory predict the possible deformed and spherical neutron shell closures at $N = 162$ and $N = 184$ [20], respectively. Since strong influence of nuclear shells [21] in the region of superheavy elements might make sufficiently long lived SHN to be observed, the search for heavier elements in the natural samples was started [22–25] about 30 years ago.

Experimental investigations in finding the SHN around $Z = 107$ – 118 have been pursued mainly at three different places: Gesellschaft für Schwerionenforschung (GSI) in Darmstadt (Germany), Joint Institute for Nuclear Research (JINR) in Dubna (Russia), and RIKEN, Japan. In the beginning of the 1980’s the first observations of the elements with $Z = 107$ – 109 were made at GSI [26]. In 1994, α -decay chains were observed from nucleus $^{269}_{110}$ [27] and later on, α -decay chains from nuclides $^{271}_{110}$, $^{272}_{111}$, $^{277}_{112}$ [28–30], $^{283}_{112}$ [31] were detected at GSI.

While RIKEN claimed discovery of the $^{278}_{113}$ SHN [32,33], it also reconfirmed the α decay chains from $^{271}_{110}$ [34], $^{272}_{111}$ [35], and $^{277}_{112}$ [36]. Observations of the α decay chains of nuclei $^{294}_{118}$, $^{290-293}_{116}$, $^{288,287}_{115}$, $^{286-289}_{114}$, $^{282-284}_{113}$, $^{285,283}_{112}$ [37], $^{278-280}_{111}$, $^{273,281}_{110}$ [38,39], $^{274-276}_{109}$, $^{275}_{108}$, $^{272,270}_{107}$, $^{271}_{106}$ were reported by JINR [40–43].

Recently, the $^{270}_{108}\text{Hs}$ ($Z = 108, N = 162$) SHN has been produced in the $^{26}\text{Mg} + ^{248}\text{Cm}$ reaction [44]. According to the theoretical calculations [3,45], nucleus $^{270}_{108}\text{Hs}_{162}$ ($Z = 108$) should have the features of “deformed doubly magic” nucleus. Most of the heaviest nuclei are expected to be deformed due to partial filling of large nuclear shells by outer nucleons. Dvorak *et al.* measured the energy (E_α) of α particle emitted from $^{270}_{108}\text{Hs}_{162}$ and used the value of E_α to calculate Q_α (9.02 ± 0.03 MeV) for the α decay of $^{270}_{108}\text{Hs}_{162}$. A phenomenological formula [46] estimated the α decay half-life (~ 22 s).

Earlier, it was believed [23,47–49] that traditional spherical superheavy nuclei might form an “island of stability” centered around $^{298}_{114}\text{184}$ separated from the “peninsula” of known

*partha.roychowdhury@saha.ac.in

†chhanda.samanta@saha.ac.in; csamanta@richmond.edu

‡dnb@veccal.ernet.in

nuclei by a region of deep instability. Due to both deformed neutron shell and proton shell effects at $Z = 108$ and $N = 162$ the extension of the peninsula of known nuclei might connect the stability island of spherical superheavy nuclei around doubly magic spherical $Z = 114$ proton shell and spherical $N = 184$ neutron shell. Since fission barrier and shell effect play very important role for the existence of long lived superheavy nuclei it is crucial to determine the fission barrier and half-life of fissioning nucleus with a good accuracy. It is well known that very small barrier height against fission can break the nucleus into two fragments immediately after it is formed. The α decay of superheavy nuclei [50–56] is possible if the shell effect supplies the extra binding energy and increases the barrier height of fission. β -stable nuclei having relatively longer half-life for spontaneous fission than that for α decay indicates that dominant decay mode for such SHN might be α -decay.

In our previous works [51,54,57–59] we showed the applicability of the microscopic calculation in predicting the α decay half-lives of SHN from a direct comparison with the experimental data [40–42]. However, as a number of SHN were predicted to have relatively large α decay half-lives, it is necessary to find out whether those SHN would survive the fission [60] and β -decay. In such cases those nuclei can be detected in the laboratory through α decay. This work explores the possibility of finding long lived SHN by comparing the calculated α decay half-lives (T_α) with available theoretical spontaneous fission (SF) half-lives [61,62], calculated β -decay half-lives (T_β) [63] and the experimental data on SF. The α decay half-lives of SHN with $Z = 102$ – 120 are calculated in a quantum tunneling model with DDM3Y effective nuclear interaction using Q_α values from three different mass formulas.

A brief outline of the methodology of the present calculation is presented in Sec. II. Spontaneous fission half-lives from both phenomenological [64,65] and microscopic approach [61,62] are given in Sec. III. Finally, in Sec. IV, results and discussions and in Sec. V, summary and conclusion are presented.

II. FORMALISM

The α decay half-lives are calculated in the framework of quantum mechanical tunneling of an α particle from a parent nucleus [57] and the results are shown in Figs. 1–3. The details of calculation of the α decay half-lives of superheavy nuclei were described in our earlier works [57–59]. The required nuclear interaction potentials are calculated by double folding the density distribution functions of the α particle and the daughter nucleus with density dependent M3Y effective interaction. The microscopic α -nucleus potential thus obtained, along with the Coulomb interaction potential and the minimum centrifugal barrier required for the spin-parity conservation, form the potential barrier. The spin-parity conservation condition in a decay process is fulfilled if and only if

$$\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 + \mathbf{I}, \quad \pi = \pi_1 \cdot \pi_2 \cdot (-1)^l, \quad (1)$$

where \mathbf{J} , \mathbf{J}_1 , and \mathbf{J}_2 are the spins of the parent, daughter, and emitted nuclei, respectively, π , π_1 , and π_2 are the parities

of the parent, daughter, and emitted nuclei, respectively, and \mathbf{I} is the orbital angular momentum carried away in the process. This conservation law, thus, forces a minimum angular momentum to be carried away in the decay process. Consequently, contribution of the angular momentum gives rise to a centrifugal barrier

$$V_l = \hbar^2 l(l + 1)/(2\mu R^2), \quad (2)$$

where μ is the reduced mass of the daughter and emitted nuclei system and R is the distance between them.

The half-lives of α disintegration processes are calculated using the WKB approximation for barrier penetrability. Spherical charge distributions have been used for calculating the Coulomb interaction potentials. Most of the experimental Q -values of α decay (Q_α) are obtained from experiments done in GSI, Germany and JINR, Dubna. For theoretical Q -values, mass formulas from KUTY [66], Myers-Swiatecki [67], and Muntian *et al.* [7,68,69] are used.

The experimental decay Q values (Q_{ex}) have been obtained from the measured α particle kinetic energies E_α using the following expression:

$$Q_{\text{ex}} = \left(\frac{A_p}{A_p - 4} \right) E_\alpha + (65.3Z_p^{7/5} - 80.0Z_p^{2/5}) 10^{-6} \text{ MeV}, \quad (3)$$

where the first term is the standard recoil correction and the second term is an electron shielding correction in a systematic

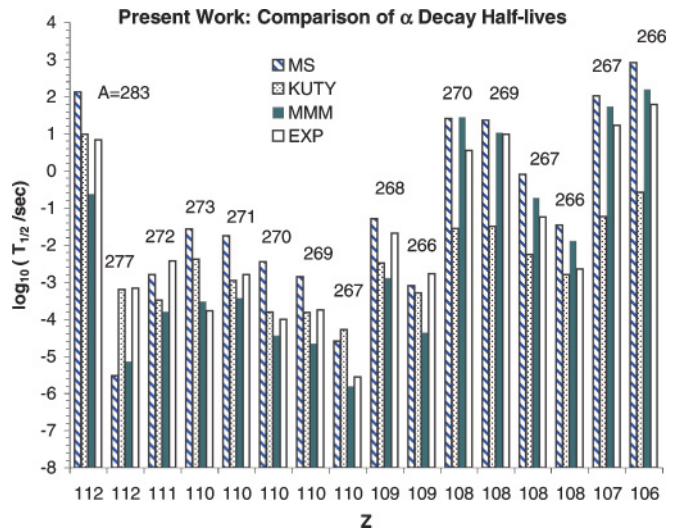


FIG. 1. (Color online) Plots of α decay half-life [$\log_{10}(T_{1/2}/\text{sec})$] in logarithmic scale versus proton number Z for different mass number A (indicated on top of each column) using zero angular momentum transfer ($\ell = 0$). (a) bar coded columns are theoretical half lives (T_α M3Y Q-MS) in WKB frame work with DDM3Y interaction and [$Q_{\text{th}}^{\text{MS}}$] from Myers-Swiatecki mass formula, (b) columns filled with dots (T_α M3Y Q-K) are in the same framework but with [$Q_{\text{th}}^{\text{KUTY}}$] from Koura-Tachibana-Ueno-Yamada mass estimates, (c) solid columns are theoretical half lives (T_α M3Y Q-M) in WKB frame work with DDM3Y interaction and [Q_{th}^{M}] from Muntian-Patyk-Hofmann-Sobiczewski mass formula, (d) hollow columns are experimental α decay half lives (T_α Expt). Experimental errors are given in Table I.

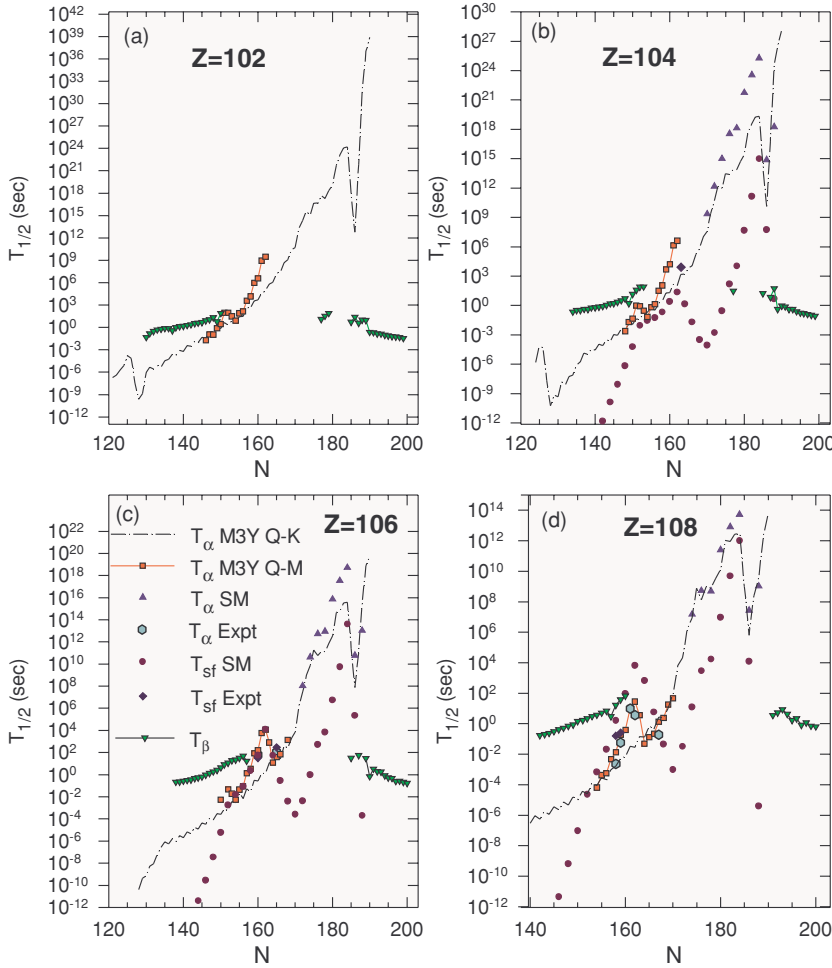


FIG. 2. (Color online) Variation of α decay and fission half-lives with neutron number for elements (a) $Z = 102$, (b) $Z = 104$, (c) $Z = 106$, (d) $Z = 108$ are shown. For all plots the following symbols are used: Dash-dotted line (T_α M3Y Q-K) and continuous line with square symbols (T_α M3Y Q-M) represent α decay half-lives calculation using Q-values from KUTY (Q-K) and Muntian *et al.* (Q-M) respectively in this work. Triangle symbol (T_α SM) represents α -decay half-lives predicted within a microscopic framework [61, 62]. Hexagon symbol (T_α Expt) represents measured α decay half-lives. Solid circle (Tsf SM) represents fission half-lives predicted by microscopic calculation. Diamond symbol (Tsf expt) represents measured fission half-lives for some nuclei. Line-inverted triangle (T_β) shows β decay half-lives predicted in Ref. [63].

manner as suggested by Perlman and Rasmussen [70]. A_p and Z_p are mass number and atomic number of parent nucleus, respectively.

The theoretical decay Q values Q_{th} have been obtained from theoretical estimates for the atomic mass excesses [7, 66–69] using the following relationship:

$$Q_{th} = M - (M_\alpha + M_d) = \Delta M - (\Delta M_\alpha + \Delta M_d), \quad (4)$$

which if positive allows the decay, where M , M_α , M_d and ΔM , ΔM_α , ΔM_d are the atomic masses and the atomic mass excesses of the parent nucleus, the emitted α particle, and the residual daughter nucleus, respectively, all expressed in the units of energy. As Q_α -value appears inside the exponential integral as well as in denominator of the expression [57,58] of α decay half-lives, the entire calculation is very sensitive to the Q-values.

III. PHENOMENOLOGICAL AND MICROSCOPIC CALCULATIONS FOR SPONTANEOUS FISSION HALF-LIVES

Spontaneous fission of heavy nuclei was first observed by Flerov and Petrijak in 1940 [71] from the ^{238}U nucleus.

Spontaneous fission and α decay are the main decay modes [72–74] of superheavy nuclei. β -decay could be another possible decay mode for the superheavy nuclei lying beyond the β -stability line of the nuclear chart. However, since the β -decay proceeds via weak interaction, the process is slow and less favored and energy involved (released) is also less compared to spontaneous fission and α decay which proceed quickly via strong interaction making these processes more probable. For heaviest nuclei, the mutual repulsion of electric charge is higher than surface energy of the nucleus arising from the short range nuclear forces. On the basis of liquid drop model, Bohr and Wheeler [75] described the mechanism of nuclear fission and established a limit for $Z^2/A \sim 48$ for spontaneous fission. Beyond this limit nuclei are unstable against spontaneous fission. According to this fact, no nucleus beyond $Z \sim 100$ can exist due to very small fission barrier. Both theoretical and experimental investigations on superheavy nuclei (SHN) support the fact that a bound superheavy nucleus can be formed only due to shell effects.

A simple semi-empirical formula on spontaneous fission half-lives for even-even, odd A and odd-odd nuclei in the ground state was proposed by Swiatecki in 1955 [76]. By including the deviation of experimental ground state masses from a smooth reference surface based on liquid drop model,

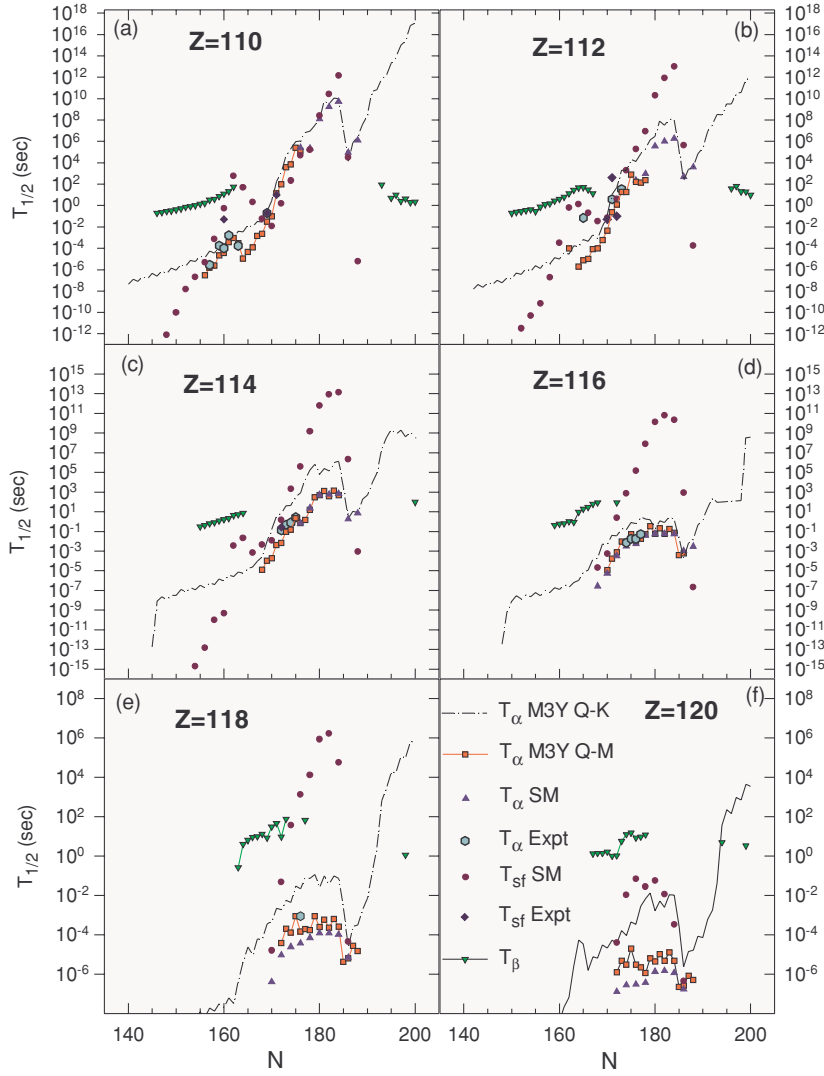


FIG. 3. (Color online) Same as Fig. 2 for elements (a) $Z = 110$, (b) $Z = 112$, (c) $Z = 114$, (d) $Z = 116$, (e) $Z = 118$, (f) $Z = 120$.

Swiatecki successfully reproduced the experimental data with his semi-empirical formula [76]. Microscopic calculation of spontaneous fission half-lives is very difficult due to both the complexity of the fission process and the uncertainty of the height and shape of the fission barrier [77]. Ren and Xu [64,65] generalized the formulas of spontaneous fission half-lives of even-even nuclei in their ground state to both, the case of odd nuclei and the case of fission isomers [65]. The spontaneous fission half-lives of odd- A nuclei and of odd-odd nuclei in the ground state were calculated by using the generalized form of the Swiatecki's formula and by a new formula where the blocking effect of unpaired nucleon on the half-lives were taken into account. By introducing a blocking factor or a generalized seniority in the formulas of the half-lives of even-even nuclei, the experimental fission half-lives of odd- A nuclei and of odd-odd nuclei with $Z = 90$ to $Z = 108$ were reasonably reproduced with the same parameters used in ground state of even-even nuclei.

In the present work, spontaneous fission half-lives for both neutron deficient and neutron rich isotopes of elements $Z = 102$ – 120 have been calculated using the following formula

[65]:

$$\log_{10}(T_{1/2}/\text{yr}) = 21.08 + c_1 \frac{(Z - 90 - v)}{A} + c_2 \frac{(Z - 90 - v)^2}{A} + c_3 \frac{(Z - 90 - v)^3}{A} + c_4 \frac{(Z - 90 - v)}{A} (N - Z - 52)^2, \quad (5)$$

where $c_1 = -548.825021$, $c_2 = -5.359139$, $c_3 = 0.767379$, $c_4 = -4.282220$, $v = 0$ for the spontaneous fission of even-even nuclei and $v = 2$ for odd- A and odd-odd nuclei. The seniority number v was introduced for taking the blocking effect of unpaired nucleon on the transfer of many nucleon-pairs during the fission process. A variation of SF half-lives calculated by this formula [Eq. (5)] with increasing neutron number for different elements from $Z = 102$ to $Z = 120$ are shown in Figs. 4 and 5 for comparison with the α decay and SF half-lives predicted by our calculation and by Smolanczuk *et al.* [61,62], respectively. Moreover, in this work, spontaneous fission half-lives calculated in a dynamical

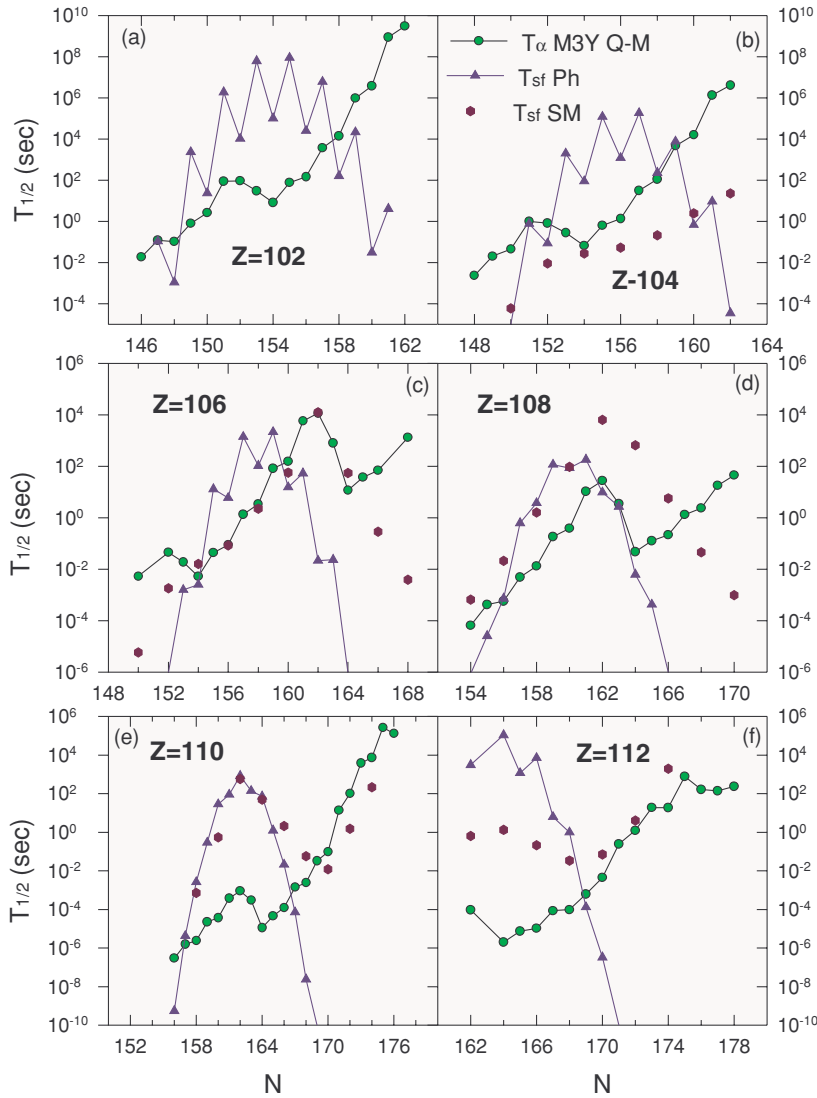


FIG. 4. (Color online) Plots for variation of phenomenological [64,65] and microscopically [61,62] calculated fission half lives with neutron numbers for (a) $Z = 102$, (b) $Z = 104$, (c) $Z = 106$, (d) $Z = 108$, (e) $Z = 110$, (f) $Z = 112$. The corresponding α decay half-lives from the present calculation are also shown for comparison. Continuous line with solid circle (T_{α} M3Y Q-M) represents α decay half-lives predicted by this work using Q value from Muntian *et al.* [7,68,69]. Continuous line with solid triangle (Tsf Ph) represents spontaneous fission half-lives predicted by phenomenological calculation. Solid dark circle (Tsf SM) shows the spontaneous fission half-lives predicted by microscopic calculation.

approach using macroscopic-microscopic method (MMM) of Refs. [61,62] are also used (Figs. 2–5) to find out the “island” where the predicted SHE could survive the fission.

IV. RESULTS AND DISCUSSION

In this paper, our main aim is to check whether the predicted SHN do really survive against the spontaneous fission (SF). Gamow-Teller β -decay half-lives (T_{β} in Figs. 2 and 3) obtained from a microscopic quasiparticle random phase approximation with single particle levels by Moller, Nix, and Kratz [63] has also been used in this work to check the possibility of β -decay of SHN. In earlier works [51,54,57–59], we were able to well reproduce the available experimentally measured α decay half-lives following the semiclassical quantum tunneling method with double folded density dependent effective M3Y interaction. In Ref. [58], T_{α} of about 314 nuclei were predicted using Q_{α} values from MMM (Q_M) [7,68,69] and modified liquid droplet model of Myers-Swiatecki [67] to show the necessity of more accurate mass formula in determining the Q_{α} -values with a good

accuracy at least correct upto 10 KeV for heaviest nuclei. But the spontaneous fission survivability of those nuclei was not checked in our previous work. For this reason, although some SHN ($Z = 102$ – 120) are long-lived against α -decay [57,58], they may not live long if they have shorter SF half-lives. For example, at $N = 162$ the magnitudes of T_{α} using Q_M for the elements No ($Z = 102$), Rf ($Z = 104$), Sg ($Z = 106$), Hs ($Z = 108$) are 3.06×10^9 s, 4.10×10^6 s, 1.16×10^4 s, 28 s, respectively [58]. If SF half-lives (T_{SF}) of such nuclei are about the same order or relatively longer than their respective T_{α} , only then can a significant fraction of nuclei survive fission and decay by α emission. In such cases α -decay chain followed by SF of one of the product nuclei may be observed. Therefore, accurate estimation of T_{SF} of SHN is essential to know whether predicted long-lived SHN against α emission do really exist against SF.

Gupta and Burrows [78] summarized the measured ground state values of spontaneous fission and α decay half-lives with Q_{α} for heaviest nuclei having mass number $A = 266$ – 294 . These values were taken from experimental measurements carried out at GSI, Germany, and JINR, Dubna. In Table I, a comparison between experimentally measured and calculated

TABLE I. Comparisons between observed and theoretical (this work) α -decay half lives using measured Q_α . The experimental α -decay half lives ($T_{1/2}^{\text{EXP}}$) are taken from Ref. [78] except the values with single (*) and double (**) asterisk symbols which are taken from Ref. [31] and Ref. [44] respectively.

Parent A_Z	Exptl Q -value Q_{ex} (MeV)	Half-lives (Exp) $T_{1/2}^{\text{EXP}}$	This work ($\ell = 0$) $T_{1/2}^{\text{DDM3Y}}[Q_{\text{ex}}]$	This work ($\ell \neq 0$) $T_{1/2}^{\text{DDM3Y}}[Q_{\text{ex}}]$	ℓ
$^{283}_{112}$	9.704 ± 0.015	$*6.9^{+6.9}_{-2.3}$ s	$4.67^{+0.49}_{-0.44}$ s		
$^{277}_{112}$	11.594 ± 0.055	$0.69^{+0.69}_{-0.24}$ ms	93^{+29}_{-23} μ s	$0.59^{+0.19}_{-0.14}$ ms	4
$^{272}_{111}$	11.150 ± 0.035	$3.8^{+1.4}_{-0.8}$ ms	$1.31^{+0.27}_{-0.22}$ ms		
$^{273}_{110}$	11.37 ± 0.05	$0.17^{+0.17}_{-0.06}$ ms	75^{+21}_{-17} μ s	$0.13^{+0.04}_{-0.03}$ ms	2
$^{271}_{110}$	10.899 ± 0.020	$1.63^{+0.44}_{-0.29}$ ms	930^{+110}_{-90} μ s	$1.15^{+0.14}_{-0.12}$ ms	1
$^{270}_{110}$	11.20 ± 0.05	$0.10^{+0.14}_{-0.04}$ ms	$0.083^{+0.024}_{-0.019}$ ms	$0.10^{+0.03}_{-0.02}$ ms	1
$^{269}_{110}$	11.58 ± 0.07	179^{+245}_{-66} μ s	28^{+13}_{-8} μ s	88^{+36}_{-26} μ s	3
$^{267}_{110}$	12.28 ± 0.11	$2.8^{+13.3}_{-1.2}$ μ s	$1.1^{+0.8}_{-0.4}$ μ s		
$^{268}_{109}$	10.486 ± 0.035	21^{+8}_{-5} ms	$12.3^{+2.8}_{-2.2}$ ms		
$^{266}_{109}$	10.996 ± 0.025	$1.7^{+1.8}_{-1.6}$ ms	750^{+110}_{-90} μ s	$1.3^{+0.2}_{-0.1}$ ms	2
$^{270}_{108}$	$9.30^{+0.07}_{-0.03}$	$3.6^{+0.8}_{-1.4}$ s	$1.36^{+0.30}_{-0.51}$ s		
** $^{270}_{108}$	9.02 ± 0.03	**22 s	$9.53^{+2.24}_{-1.86}$ s		
$^{269}_{108}$	9.315 ± 0.022	$9.7^{+9.7}_{-3.3}$ s	$3.19^{+0.52}_{-0.43}$ s		
$^{267}_{108}$	9.978 ± 0.020	58^{+23}_{-14} ms	$45.5^{+5.8}_{-5.2}$ ms		
$^{266}_{108}$	10.336 ± 0.020	$2.3^{+1.3}_{-0.6}$ ms	$2.24^{+0.27}_{-0.25}$ ms		
$^{267}_{107}$	8.96 ± 0.30	17^{+14}_{-6} s	12^{+93}_{-11} s		
$^{266}_{106}$	8.88 ± 0.03	62^{+166}_{-44} s	$4.89^{+1.20}_{-0.93}$ s	16^{+4}_{-3} s	3

half-lives using DDM3Y effective nucleon-nucleon (NN) interaction in a WKB framework is shown for 16 superheavy nuclei. Most of these nuclei (12 out of 16) were not addressed in our earlier works [51,54,57–59]. In Fig. 1 the theoretical Q_α -values from different mass formulas prescribed by MS [67], KUTY [66], Muntian *et al.* (MMM) [7,68,69] are employed to calculate α -decay half-lives of the same 16 SHN presented in Table I. The measured Q_α -values are used to calculate α decay half-lives ($T_{1/2}^{\text{DDM3Y}}[Q_{\text{ex}}]$) in Table I. Half-lives calculated in the present work agree reasonably well with experimental data for most of the superheavy nuclei. Nuclei for which spin-parities of parent and daughter nuclei are not known, zero angular momenta ($\ell = 0$) transfers are used. Since $\ell = 0$ gives minimum centrifugal barrier, probability of α -tunneling increases and consequently half-life decreases. But, for some nuclei like $^{277}_{112}$, $^{273}_{110}$, $^{269}_{110}$, $^{266}_{106}$, etc., angular momenta carried by α particles may not be zero. For these nuclei, calculated half-lives are too small compared to measured values. In particular, for $^{277}_{112}$ calculated value (~ 0.093 ms) is less than the measured one (0.69 ms) by one order of magnitude. This discrepancy can be removed only if one can assume nonzero orbital angular momentum transfer in the α decay process. In our calculation, $\ell = 4$ gives the better agreement of predicted half-life for $^{277}_{112}$ nucleus with measured value. It is therefore important to determine the proper spin-parity of parent and daughter nuclei to enable correct centrifugal barrier for α decay half-life calculation. Although, higher ℓ -values are shown for some nuclei for better agreement with the experimental result, but more experiments

on the same nuclei (shown in Table I) with higher statistics are needed for reconfirmation of the data and measured α decay energies since reaction cross sections are of the order of picobarns (pb).

In this work, existing SF calculation with a microscopic approach [61,62] have been used to find the region of long lived fission survived nuclei in the SHN region of the nuclear chart. In Figs. 2 and 3, comparisons between calculated T_α , T_β , and T_{SF} are shown only for even- Z elements with proton number $Z = 102$ – 120 since T_α and T_{SF} calculations of Refs. [61,62] are valid only for even-even nuclei. A few available observed data for both T_α and T_{SF} are also shown for most of the elements in the plots of Figs. 2 and 3. For $Z = 104$, the highest value of T_α ($\sim 4.1 \times 10^6$ s) according to calculation of this work using Q_M appears around $N = 162$ where much smaller value of T_{SF} calculated in Ref. [61] (~ 23 s) makes this nucleus $^{266}_{104}162$ unstable against SF. Therefore, if synthesis of this nucleus is possible in the present-day setup, substantial fraction of $^{266}_{104}162$ would undergo SF within a few seconds. For $Z = 102$, no such calculation on SF is available in Refs. [61,62]. Since calculated T_{SF} and Q_M values both are based on MMM, α decay half-life calculations using DDM3Y effective interaction with Q_M (T_α M3Y Q-M) is preferably chosen to compare with T_{SF} . In addition to that, calculation within the same framework using Q_{KUTY} have also been presented in Figs. 1–3.

In the case of Sg ($Z = 106$) isotopes, DDM3Y with Q_M predicts that the longest T_α ($\sim 1.16 \times 10^4$ s ~ 3.2 h) would be at $N = 162$. It is comparable to T_{SF} (3.5 h) of Ref. [61].

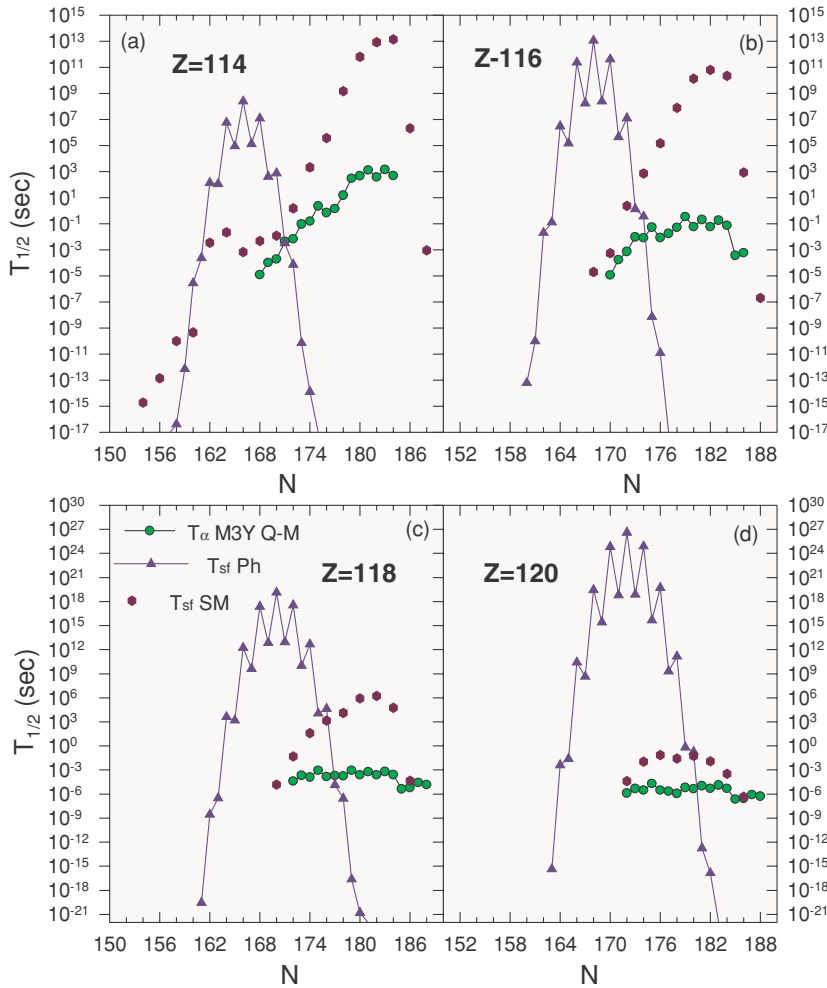


FIG. 5. (Color online) Same as Fig. 4 for (a) $Z = 114$, (b) $Z = 116$, (c) $Z = 118$, (d) $Z = 120$.

Therefore, α decay channel is one of the dominant decay mode of this $^{268}\text{Sg}_{162}$ nucleus. If it is produced in the laboratory, it may be observed only for few hours (lifetime ~ 1.68 h) since both SF and α decay half-lives are small. It is to be noted that in the present work, lifetime of some SHN (either β -stable or have large T_{β}) are predicted by considering SF and α decay half lives only. Incidentally, $^{264}\text{No}_{162}$, $^{266}\text{Rf}_{162}$, $^{268}\text{Sg}_{162}$, and $^{270}\text{Hs}_{162}$ are known to be either β -stable or have very large T_{β} [63]. Q_{α} values using KUTY mass formula are not always very much reliable since it does not reproduce all the observed Q -values with good accuracy. But, this mass formula can be used to locate the region of possible existence of long-lived SHN where Q -values from other mass formula are not available. It may be pointed out that the use of Q_{KUTY} in our calculation shows reasonable agreement for several nuclei in Fig. 1.

As Q_M values are not available for more neutron rich isotopes of Sg, using Q_{KUTY} in present calculation predicts $T_{\alpha} \sim 3.71 \times 10^{15}$ s $\sim 1.18 \times 10^8$ y at $N = 184$. The SF half-life of $^{290}\text{Sg}_{184}$ is less ($T_{\text{SF}} \sim 4.07 \times 10^{13}$ s $\sim 1.3 \times 10^6$ y) than T_{α} . Although T_{β} of this nucleus is not specified but it is expected to be large in Ref. [63]. Hence, if synthesized, $^{290}\text{Sg}_{184}$ is expected to have a long enough lifetime ($\tau \sim 1.27 \times 10^6$ y) to be observed in the laboratory. But it is still smaller than the age of earth $\sim 4.5 \times 10^9$ y by three orders

of magnitude. Calculated T_{α} using Q_M (28 s) for $^{270}\text{Hs}_{162}$ is less than T_{SF} (~ 1.8 hr) and therefore α decay chain of such nucleus is expected to be observed. This prediction from our calculation is in good agreement with recently observed [44] doubly magic deformed nucleus $^{270}\text{Hs}_{162}$. The present calculation using experimental Q -value gives $T_{\alpha} \sim 9.53$ s (Table I).

From Figs. 2 and 3 it is seen that the extra stability effect of neutron shell at $N = 162$ almost disappears with the increase of atomic number and T_{α} becomes of the order of millisecond to microsecond for the elements having $Z \geq 110$. On the other hand, in more neutron rich side around $N = 184$ of elements $Z = 110, 112, 114$ theoretical T_{α} using Q_{KUTY} in present calculation are of the order of 10^{10} s, 10^8 s, 10^6 s, respectively, which are much less than theoretical T_{SF} of the order of 10^{12} s, 10^{13} s, 10^{13} s, respectively. It must be noted that $^{296}112_{184}$, $^{298}114_{184}$ are β -stable whereas $^{294}110_{184}$ is predicted to have large T_{β} in Ref. [63] due to its very small positive Q_{β} -value. β -stable nuclei and those with very large T_{β} are not shown in Figs. 2 and 3.

Beyond the $Z = 114$ peak value of the T_{α} plot (Fig. 3) around $N = 184$ suddenly reduces showing a possible signature of spherical proton shell at $Z = 114$. According to the present calculation, $^{298}114_{184}$ the so-called doubly magic spherical superheavy nucleus predicted by nuclear structure

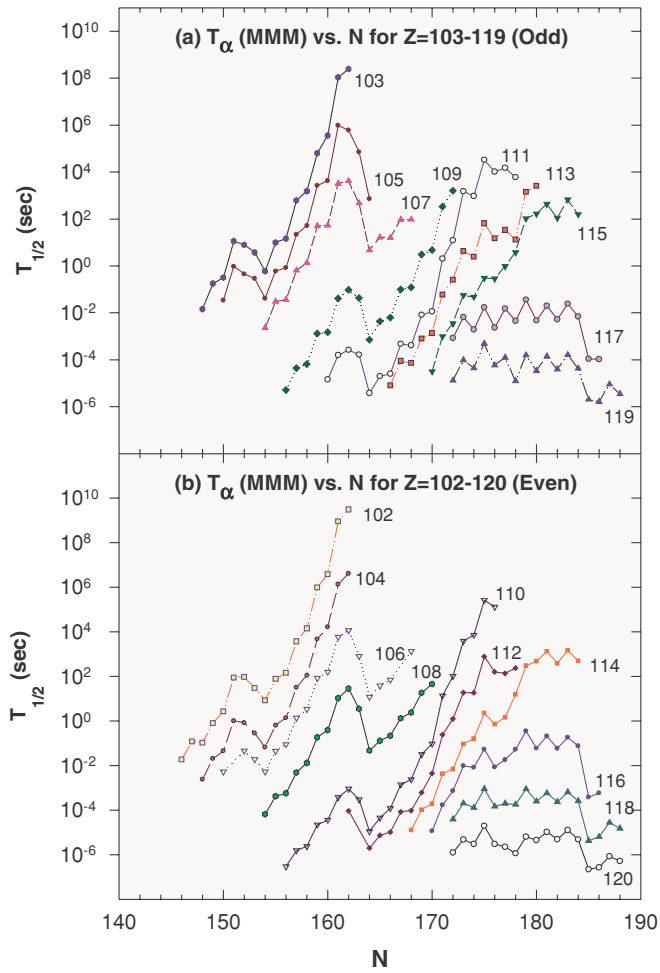


FIG. 6. (Color online) Variation of α decay half-lives (T_α) predicted by present calculation with neutron number for (a) odd-Z and (b) even-Z elements from $Z = 102-120$. Each graphs show the abrupt reduction of T_α after $N = 162$ as the possible signature of shell effect. Similar reductions of T_α after $N = 184$ are also shown by the elements for which Q_α values are available from MMM calculation.

theory in the middle of 1960, has T_α values of the order of 10^6 s and 5×10^2 s using Q_{KUTY} and Q_M , respectively, which are much less than $T_{SF} \sim 10^{13}$ s.

In Figs. 4 and 5, three curves describing spontaneous fission and α -decay half-lives are shown in each graph except $Z = 102$ for which calculation of spontaneous fission half-lives in Refs. [61,62], represented by Tsf SM in Figs. 4 and 5, are not available. For the isotopes of elements $Z = 106, 108$ the values of Tsf SM become of the order of 1 ms near $N = 170$. However, the values of Tsf SM are comparable to the calculated α decay half-lives of this work (T_α M3Y Q-M) around $N = 154$ to 164 for Sg. In case of hassium isotopes ($Z = 108, N = 156-163$), Tsf SM $> T_\alpha$ M3Y Q-M reveals the fact that isotopes of Hs within this range may undergo α decay with a longest half-life of the order of few seconds ($\sim 10-30$ s). This is in good agreement with the recent experimental observation of α decay from the nucleus $^{270}\text{Hs}_{162}$. But spontaneous fission half-lives calculated using Eq. (5) [64,65] fall rapidly around $N = 160-170$ for

elements having $Z = 102-112$. In Fig. 5, SF half-life values for neutron rich isotopes of elements having $Z = 114, 116, 118, 120$ are less than 10^{-15} s near $N = 180$. For $Z = 114$ only, this calculation matches with microscopic results for some isotopes having $N = 160-170$. This calculation contradicts the following two facts: (i) Experimentally measured value of T_α for $^{283}\text{112}_{171}$ is ~ 6.9 s (see Table I) whereas, SF half-life for this nucleus calculated by using Eq. (5) is extremely low ($\sim 10^{-11}$ s) indicating immediate fissioning of the nucleus. (ii) Since T_{SF} by Ren and Xu rapidly falls around $N = 160-170$ (Fig. 4) and $N = 180$ (Fig. 5) for $Z = 102-112$ and $Z = 114-120$, respectively, it could not explain the predicted extra shell stability of SHN at $N = 184$ for heavier elements.

From the present calculation of T_α using Q_{KUTY} it seems that longer T_α might be observed for more neutron rich side ($Z \geq 116, N > 190$) due to possible existence of neutron shell closure. On the contrary, from the trend of SF plots for $Z = 116, 118, 120$ it appears that lowering of T_{SF} might destroy the existence of such neutron rich ($N > 190$) superheavy isotopes of elements $Z = 116, 118, 120$. Hence, the presence of long-lived SHN with neutron shell closure beyond $N = 184$ may be ruled out. However, it is an important task to determine the SF and α decay half-lives for $N > 184$ to confirm whether there is any possibility of neutron shell closure beyond $N = 184$ for heaviest elements ($Z \geq 116$).

In two graphs of Fig. 6, using Q_M values in this calculation, variation of α decay with neutron number for both odd-Z and even-Z elements having $Z = 102-120$ are shown. The plots of both graphs (a) and (b) of Fig. 6 clearly show peaks of T_α -values around $N = 162$ and $N = 184$ for all elements with $Z = 102-120$, which possibly indicates the neutron shell closure at $N = 162$ and 184. This is in good agreement with the present-day knowledge of microscopic theory.

V. SUMMARY AND CONCLUSION

The natural existence of superheavy nuclei is limited primarily by spontaneous α decay and spontaneous fission processes. A SHN in spite of having longer α decay half-lives may undergo immediate spontaneous fission if the latter has a low half-life. On the other hand, SF stable SHN may have shorter ($\sim 1 \mu\text{s}$ or less) α decay half-lives. In both the cases, such SHN may not be observed even if they are synthesized in the present day laboratory setup. The main aim of this work is to find out the fission-survived long lived SHN. In fact, if SHN have high degree of stability against both α decay and SF, we would be able to observe them if produced in the laboratory provided those SHN are not far away from β -stability line. We have calculated the α decay half-lives of SHN in quantum tunneling method with microscopic NN potential using Q -values from different mass formulas and compared them with the β -decay and SF half-lives to find the long lived SHN.

The highlights of observations made in this work are summarized as follows:

(i) Among all three mass formulas, Q_α -values used from MMM model (Q_M) [7,68,69] in the present method, reproduces the observed data reasonably well (see Fig. 1), but

nonavailability of Q_α -values in more neutron rich side limits its usage. Therefore, for a higher Z region, the mass formula of KUTY, which extends up to $Z = 130$, has been considered.

(ii) Although $^{266}\text{Rf}_{162}$ nucleus has relatively longer T_α half-life ($\sim 4.1 \times 10^6$ s ~ 47.5 d using Q_M) but it is unstable against SF with Tsf SM ~ 23 s only (Fig. 2).

(iii) The mass formula of KUTY predicts α decay lifetime of the $^{290}\text{Sg}_{184}$ nucleus to $\sim 10^8$ y whereas Tsf SM $\sim 10^6$ y makes the lifetime ($\sim 10^6$ y) of this nucleus very long but still smaller than the age of the earth ($\sim 4.5 \times 10^9$ y) by three orders of magnitude. This nucleus is either β -stable or might have very large T_β according to the calculations of Ref. [63].

(iv) The larger deviations between calculated and experimentally measured α decay half-lives are observed in case of only few nuclei such as $^{277}\text{112}$ which may be due to higher minimum orbital angular momenta carried away by α particles for spin-parity conservation. Inaccuracy in the measurement of T_α of $^{277}\text{112}$ nucleus due to very low count rate also may not be ruled out.

(v) Using the formulation based on the liquid drop model in Refs. [64,65] SF half-lives calculations have also been done for higher Z elements in this work. Results shown in the plots of Figs. 4 and 5 do not match with SF half-lives calculations in a microscopic approach of Refs. [61,62] for both neutron rich and neutron deficient isotopes of heaviest elements with $Z = 104$ – 120 . The half-life value using this phenomenological prescription also contradicts the observed α -decay from $^{283}\text{112}_{171}$ nucleus with measured $T_\alpha \sim 6.9$ s.

(vi) It is evident from the present T_α calculations that the effect of the deformed neutron shell closure at $N = 162$ will be insignificant beyond $Z = 108$ as T_α goes on decreasing with increasing atomic number. For $N = 162$ isotopes of elements having $Z = 110, 112$, values of T_{SF} are 9.8 m, 0.63 s, respectively and T_α using Q_M are of the order of milliseconds (~ 1 ms) and microseconds ($\sim 93 \mu\text{s}$) (see plots of Fig. 3), respectively, i.e., the values of T_α go on decreasing more rapidly than the corresponding SF half-lives with increasing atomic number.

(vii) Calculated T_α using Q_{KUTY} predicts almost β -stable long lived SHN around $^{294}\text{110}_{184}$, $^{296}\text{112}_{184}$, $^{298}\text{114}_{184}$ with T_α of the order of ~ 311 y, ~ 3.10 y, ~ 17 d, respectively, which are much less than their T_{SF} ($\sim 4.48 \times 10^4$ y, $\sim 3.09 \times 10^5$ y, $\sim 4.38 \times 10^5$ y, respectively) values. Hence the dominant decay mode of the above nuclei and their immediate neighbors is expected to have α decay mode. T_α value of $^{293}\text{110}_{183}$ is about 352 y which is slightly greater than that for $^{294}\text{110}_{184}$ nucleus. The SF half-life of this nucleus is not found in

Refs. [61,62]. For $^{292}\text{108}_{184}$ nucleus since T_α ($\sim 9.6 \times 10^4$ y using Q_{KUTY}) value is comparable to its T_{SF} ($\sim 3.2 \times 10^4$ y), this nucleus is one of the possible members of stability island. The exact values of T_β of $^{293}\text{110}_{183}$ and $^{292}\text{108}_{184}$ nuclei are not shown in Ref. [63] but predicted to be large.

(viii) Using Q_{KUTY} values in the present calculation shows longer T_α -values for neutron rich ($N > 190$) isotopes of $Z = 116, 118$, and 120 indicating a possible neutron shell closure next to $N = 184$ might occur. On the contrary, calculated SF half-lives (Fig. 3) of Refs. [61,62] show a trend of lowering of T_{SF} (< 1 ms to $1 \mu\text{s}$) for neutron rich isotopes of those elements which indicates the higher probability of SF of such SHN in this region. However, more accurate determination of fission barrier and their corresponding half-lives are essential to predict long lived SHN in the region of very high atomic number.

(ix) It may be pointed out that the calculation of T_α is very sensitive to Q_α -values, and none of the mass formula used here cover the entire mass range with extreme accuracy. Therefore, a better mass estimate covering the wide range of superheavy masses with a good accuracy is necessary.

In summary, we find that the possibility of existence of SHN above $Z = 114$ with considerable life time is very low. Although $Z = 120, 124, 126$ with $N = 184$ might form spherical-doubly-magic nuclei and survive fission [79], they would undergo α decay within microseconds. A small “island/peninsula” might survive fission and β -decay but undergo α decay in the region $Z = 106$ – 108 , $N \sim 160$ – 164 . Interestingly, in this region the β -stable SHN $Z = 106$, $N = 162$ has the highest α decay half-life ~ 3.2 h (Fig. 2, using Q_M) that is much greater than the recently discovered deformed-doubly-magic SHN ^{270}Hs (measured $T_\alpha \sim 22$ s). Thus a search for this long-lived SHN $^{268}\text{Sg}_{162}$ can be pursued. Similarly, the nucleus with $Z = 110$, $N = 183$ appears to be near the center of a possible “magic island” ($Z = 104$ – 116 , $N \sim 176$ – 186) with α decay half-life ~ 352 y (Fig. 3, using Q_{KUTY}) which is greater than that of the doubly-magic SHN $Z = 114$, $N = 184$ ($T_\alpha \sim 17$ d). Since the SHN $^{290}\text{Sg}_{184}$ has T_α and T_{SF} values $\sim 10^8$ y (Fig. 2, using Q_{KUTY}), and $\sim 10^6$ y, respectively, it might have longer lifetime in comparison to other superheavy nuclei. However, for both $^{293}\text{Ds}_{183}$ and $^{290}\text{Sg}_{184}$ nuclei, β -decay might be another possible decay mode with large T_β values. Only future experiments can confirm this. Finally, the experimental investigations to detect the α -cascade can be pursued on $^{294}\text{110}_{184}$, $^{293}\text{110}_{183}$, $^{296}\text{112}_{184}$, and $^{298}\text{114}_{184}$ nuclei which are expected to decay predominantly through α particle emission.

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