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Precise root-mean-square radius of ^{4}He

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We study the world data on elastic electron-helium scattering to determine the ^{4}He charge root-mean-square radius. A precise value for this radius is needed as a reference for a number of ongoing studies in nuclear and atomic physics.

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Introduction. The charge root-mean-square (rms) radii of the helium isotopes are presently receiving considerable attention. Precise isotope shifts of ^{3}He and the unstable ^{6}He relative to ^{4}He have been measured by laser spectroscopy [1,2]; an experiment studying the isotope shift of the short-lived halo-nucleus ^{8}He has recently been completed at GANIL [3]. These isotope shift measurements provide accurate differences of the rms radii relative to ^{4}He. To deduce absolute radii, the radius of the reference nucleus ^{4}He needs to be known with the best possible precision.

The radius of the ^{4}He nucleus is also of great interest to the ongoing atomic spectroscopy measurement of $^4He^+$ performed using cooled helium ions in a Paul trap [4]. It is hoped that this experiment reaches a relative accuracy of $2 \cdot 10^{-14}$ for the 2S-1S energy difference. As compared to measurements on the hydrogen atom, precision measurements of transitions in the one-electron system He^+ allow for a more accurate test of QED, both because of the absence of hyperfine structure and the better precision of the ^{4}He charge rms potentially obtainable from electron scattering or muonic atom experiments. Data on transitions in He^+ also will be sensitive to the higher order QED terms such as the two-loop contributions, which scale like $(Z\alpha)^5$ [5] and are not yet accessible in the hydrogen atom. The interpretation of the results from this 2S-1S measurement will be limited by the accuracy the ^{4}He charge rms radius is known with.

The ^{4}He rms radius has been determined via various elastic electron scattering experiments; for a compilation see Ref. [6]. The combined data available up to 1982 were analyzed in Ref. [7], yielding the rms radius 1.676 ± 0.008 fm. The most accurate radius measurement was claimed to be the one of Carboni *et al*. [8]. From a Lamb shift experiment on muonic ^{4}He they obtained 1.673 ± 0.001 fm [9]. However, serious doubts have been expressed concerning this experiment. Two independent groups [10,11] have excluded at the pressure of 40 bars the long lifetime of the $\mu-^4He^+$ 2S-state, which would have been required for the experiment of [8] to successfully induce the 2S-2P transition; the very short lifetime found by Refs. [10] and [11] is in agreement with theoretical predictions for the rate of collisional quenching [12] and the observed evolution of the lifetime with pressure [13]. The experiment of Ref. [14] has shown that at very low pressure (0.04 bar) the lifetime of the 2S state is long enough to allow for an excitation of the 2S-2P transition with laser light. This experiment has excluded with a significance of 3.5σ the occurrence of the 2S-2P transition at the wave length claimed by Ref. [8]; accordingly, the rms radius is *not* in the interval 1.673 ± 0.0016 fm. The muonic x-ray measurement of Ref. [8] thus cannot be considered to provide a dependable ^{4}He rms radius.

In this Rapid Communication, we use the *world* data on electron scattering from ^{4}He, together with independent information from proton-^{4}He scattering, to determine a reliable and accurate rms radius.

Electron scattering data. The data base for elastic electron scattering from ^{4}He is quite extensive [15–20], reaching a maximum momentum transfer q of 8 fm^{-1}. For completeness we also mention two older experiments [21,22], which, however, are not accurate enough to contribute significantly.

The experiment of greatest interest to a determination of the rms radius is the one of Ottermann *et al*. [20] who, using gas targets for helium and hydrogen, measured cross section ratios in the range of $q = 0.5$–2.0 fm^{-1}, with a systematic error of order 0.7%. This experiment has the lowest uncertainties and covers the q region 0.8–1.4 fm^{-1}, which has the greatest sensitivity to the rms radius. The data of von Gunten [19], although also very precise, are measured at very low momentum transfer where the finite-size effect in the form factor is still very small.

For the present analysis, we have converted the helium cross sections measured relative to the proton to absolute cross sections employing a modern fit of the world data on e-p scattering [23], established by taking into account also for the proton the nonnegligible Coulomb distortion effects [24].

Constraint on density at large radii. The tail of the charge density at large radii gives a large contribution to the rms radius, as a consequence of the r^4 weight in the $\langle r^2\rangle$ integral. The density at radii larger than 1.9 fm, where it has fallen to less than 10% of the central value, contributes more than 55% to $\langle r^2\rangle$. The uncertainty on the density in this region accordingly is responsible for much of the final error bar on the rms radius. If the density at large radii can be constrained using additional knowledge, this can greatly benefit the accuracy of the rms radius.

At large radii, outside the range of the nucleon-nucleus potential, the proton 1S radial wave function falls like a Whittaker function $W_{-\eta,1/2}(2\kappa r')/r'$, with κ and η depending on the proton removal energy and $r' = r\, m_A/m_{A-1}$. This shape of the proton wave function can be used to constrain the fitted charge density (for corrections see below).

For the special case of ^{4}He, we know not only the *shape* of the density but also the *absolute value*. The world data on elastic scattering of protons from ^{4}He have been analyzed by Plattner *et al*. [25] using Forward Dispersion Relations (FDR). This analysis, which in addition to data uses as input only the singularity structure of the p-^{4}He scattering amplitude, determined the residual of the nearest pole, due to the proton exchange amplitude involving the p-^{3}H configuration. This residual, which is known with $\sim 5\%$ accuracy, fixes the *absolute* (point) density of protons in the large-radius tail of ^{4}He, i.e., the constant multiplying $W^2_{-\eta,1/2}(2\kappa r')/r'^2$.

To proceed from the point density to the charge density, we use a point density calculated in a Woods-Saxon (WS) potential well with parameters adjusted to fit the large-radius point density from FDR and, at smaller radii, the GFMC (Greens Function Monte Carlo) density of Pieper and Wiringa [26]. This density is folded with a modern proton charge distribution to produce the large-radius charge density. For the neutron we use the same WS potential, with depth slightly adjusted to get the correct neutron removal energy. This is good enough an approximation for the isoscalar nucleus ^{4}He given the small contribution of neutrons to the helium charge density. Various test have shown that the resulting charge density at large radii is not sensitive to the procedure employed.

Results of fit of data. The electron scattering data have been fit using a "model-independent" SOG (Sum-of-Gaussians) expansion for the charge density [27]. The cross section data have been calculated using a phase-shift code that solves the Dirac equation for the electrons in the electrostatic potential of the charge density. The data given in terms of Born form factors [17] (with Coulomb corrections already performed) are compared to the corresponding PWIA (Plane Wave Impulse Approximation) form factors.

In the fit, the values of the charge density from FDR for radii $r > 2.4$ fm, where the density has fallen to less than 3% of its central value, are included as data points, with the quoted error bar of $\pm 5\%$.

For the fit, the random error bars of the data are used, and the uncertainty of derived quantities is calculated using the error matrix. The systematic errors, mainly normalization uncertainties of the cross sections, are taken into account by changing the individual data sets by their systematic error, refitting, and adding quadratically the resulting changes in the quantities of interest. This yields a conservative estimate for the systematic errors. Random and systematic errors are added in quadrature.

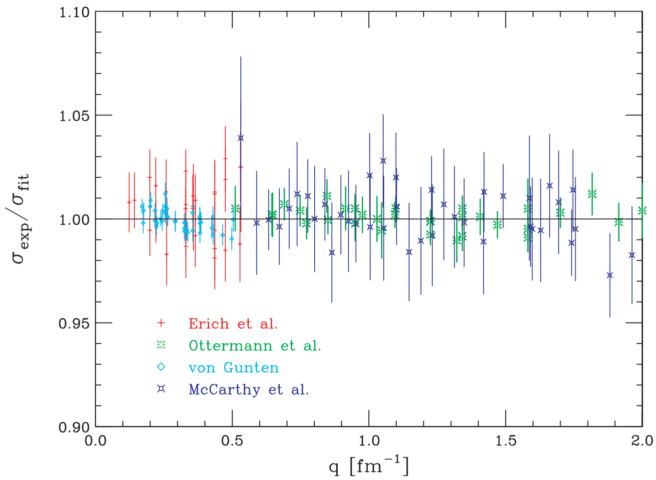

FIG. 1. (Color online) Ratio of experimental and fit ^{4}He(e, e) cross sections.

The quality of the fit of the data is very satisfactory, with a χ^2 of 133 for 168 degrees of freedom. Figure 1 shows on a very expanded scale the ratio data/fit in the q range of interest. The agreement of the fit density with the FDR charge density is also perfect.

The resulting rms radius amounts to 1.681 ± 0.004 fm, where the uncertainty covers both statistical and systematic errors. As compared to the radius previously extracted from electron scattering [7], the uncertainty is reduced by a factor of two. This is due to the more accurate data, in particular to the ones of Ref. [20] that have become available in the meantime. Relative to the previous value of 1.676 ± 0.008 fm the radius has moved up by 1/2 the error bar. This is a consequence of the additional data and the fact that the tail charge density has increased somewhat due to the folding of the FDR tail with a more modern proton charge distribution (corresponding to a larger proton charge rms radius [23]).

Because of the FDR constraint on the density at large radii, the uncertainty of the rms radius, ± 0.004 fm, is very small; the charge rms radius given above actually is the most precise rms radius of any nucleus determined via elastic electron scattering.

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