Coupling between α-condensed states and normal cluster states

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We have studied the α-condensed states of ^{16}O and ^{20}Ne based on a microscopic α-cluster model. This was performed by introducing a Monte Carlo technique for the description of the THSR (Tohsaki Horiuchi Schuck Röpke) wave function, which is called the "virtual THSR" wave function. Earlier microscopic calculations pointed out the possibility of the existence of four-α-cluster condensed states. Here, in addition to studying the four-α case, we also studied the case of five-α particles, for which the states are shown to be stable around the threshold energy even after taking into account the coupling effect between normal cluster states with ^{16}O+α configurations.

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In the last couple of years, α condensation in atomic nuclei has attracted increased interest [1–6]. Although the effect of antisymmetrization changes the intrinsic wave functions, it is basically considered that all the α clusters in an α-condensed state occupy the same 0s orbital, which has a spatially extended distribution [2]. This situation is well expressed by introducing the so-called THSR (Tohsaki Horiuchi Schuck Röpke) wave function, in which the oscillator parameter is large, which is completely different from the normal 0s orbital for each nucleon. The most plausible candidates for α-condensed states are the ground state of ^{8}Be (2α) and the second 0^+ state of ^{12}C$^*(3\alpha)$ at $E_x = 7.65$ MeV around the $N\alpha$ threshold energies. The squared overlap between the wave function of a microscopic cluster model and the THSR wave function has been found to be more than 90% for ^{12}C [3,4], which suggests that the single THSR wave function is a good approximation for the description of the condensed state. Furthermore, a candidate for the 4α-condensed state around the threshold energy in ^{16}O has been studied from both theoretical and experimental sides [1,3,7].

Recently, the research into α-condensed states moved onto the second stage. For instance, the possibility of α-condensed states around a core in heavier nuclei has been suggested [8–11]. The coherent emission of α-condensed states from the compound nucleus in heavier nuclei has been reported [8], which leads us to the hypothesis that α-condensed states can be formed not only in ^{8}Be and ^{12}C but also in heavier nuclei with some core. Using the ^{28}Si+^{24}Mg reaction, the compound states of ^{52}Fe have been populated, and the ^{8}Be(0_1^+) and ^{12}C(0_2^+) emissions from these states have been observed, which are much enhanced compared to the sequential α emission. From a statistical model point of view, it is natural to consider that the emitted second 0^+ state of ^{12}C (or the ground state of ^{8}Be) is formed inside the Coulomb barrier of the compound nucleus. The enhancement of the emission of the condensed states could be due to the lowering of the effective Coulomb barrier for the condensed states [9], since the kinetic energy of the emitted ^{12}C in coincidence with γ emission has been observed to be much smaller than the energy sum of three α's in the sequential 3α emission.

To study the possibility of the α-condensed states of heavier nuclei from the theoretical side, we have introduced a Monte Carlo technique for the description of the THSR wave function, which is called the "virtual THSR" wave function [12]. The calculations have been performed for ^{24}Mg and ^{28}Si, and it has been found that α-condensed states are predicted around the ^{16}O core at the ^{16}O+$N\alpha$ threshold energy. In this brief report, as the next step using the virtual THSR wave function, we show the possibility of the 4α- and 5α-condensed states around the $N\alpha$ threshold energies. Also, we have calculated the coupling between the normal cluster states and the condensed states for ^{20}Ne and shown the stability of the condensed state. Until now, the low-lying states of ^{20}Ne have been known to be well described by models with the ^{16}O+α configurations [13], and the coupling effect between the α-condensed states and normal cluster states are taken into account by diagonalizing the Hamiltonian matrix.

The original THSR wave function for the α-condensed state has the form

$$\begin{aligned}\Phi &= \int d\vec{R}_1 d\vec{R}_2 \cdots d\vec{R}_n \\ &\quad \mathcal{A} G_1(\vec{R}_1) G_2(\vec{R}_2) G_3(\vec{R}_3) \cdots G_n(\vec{R}_n) \\ &\quad \times \exp\left[-\left(\vec{R}_1^2 + \vec{R}_2^2 + \vec{R}_3^2 \cdots \vec{R}_n^2\right)/\sigma^2 \right] \\ &= \mathcal{A} \prod_{i=1}^{n} \int d\vec{R}_i G_i(\vec{R}_i) \exp\left[-\vec{R}_i^2/\sigma^2\right], \end{aligned} \tag{1}$$

where $\mathcal{A}$, $G_i(\vec{R}_i)$, and σ are the antisymmetrizer, the wave function for the ith α cluster centered at $\vec{R}_i$, and the oscillator parameter of the α condensation, respectively. The four nucleons (proton spin-up, proton spin-down, neutron spin-up, and neutron spin-down) in the ith α cluster share the common spatial part of the wave function ($\exp[-\nu(\vec{r} - \vec{R}_i)^2]$) with a Gaussian form.

To simplify this wave function, we introduce the virtual THSR wave function in the following way [12]:

$$\Psi = \sum_{k=1}^{m} P^{\pi} P^{J}_{MK} \Psi_k, \tag{2}$$

$$\Psi_k = [\mathcal{A} G_1(\vec{R}_1) G_2(\vec{R}_2) G_3(\vec{R}_3) \cdots G_n(\vec{R}_n)]_k. \tag{3}$$

Here, the integral over the Gaussian center parameters $\{\vec{R}_i\}$ in the original THSR wave function [in Eq. (1)] is replaced by the sum of many Slater determinants. The Gaussian center parameters $\{\vec{R}_i\}$ are randomly generated by the weight function W with a Gaussian shape:

$$W(\vec{R}_i) \propto \exp\left[-\vec{R}_i^2/\sigma^2\right]. \tag{4}$$

With increasing ensemble number, the distribution of $\{\vec{R}_i\}$ approaches a Gaussian with σ width parameter. Thus, it can be considered that the integration in the original THSR wave function [see Eq. (1)] is performed by using a Monte Carlo technique for the virtual THSR wave function, and the wave function agrees with the original THSR wave function when the number of Slater determinants [m in Eq. (2)] increases.

The projection onto good parity (P^{π}) and angular momentum (P^{J}_{MK}) is performed numerically. Here, π is positive parity and $J = M = K = 0$. If we have a sufficiently large number of basis states, the wave function approaches the eigenstate of angular momentum and parity (0^+), and these projections are not necessary. Therefore, the mentioned projections are needed only for the purpose of reducing the number of basis states. The number of mesh points for the Euler angle integral is $16^3 = 4096$.

In addition to the α-condensed states, the normal cluster states in ^{20}Ne, which are known to describe well the properties of the low-lying states, have been taken into account. We introduce an ^{16}O+α model for ^{20}Ne where ^{16}O is described as a tetrahedron configuration of four α clusters with a relative distance of 1 fm, and the relative distance between the last α cluster and ^{16}O is taken from 1 to 5 fm with a 1 fm step (five basis states).

In this case the Hamiltonian operator $\hat{H}$ has the form

$$\hat{H} = \sum_{i=1}^{A} \hat{t}_i - \hat{T}_{\text{c.m.}} + \sum_{i>j}^{A} \hat{v}_{ij}, \tag{5}$$

where $\hat{t}_i$ is the kinetic energy of ith nucleon, and the center-of-mass kinetic energy $\hat{T}_{\text{c.m.}}$ is exactly removed. Here, the two-body interaction $\hat{v}_{ij}$ includes the central part and the Coulomb part. We use the following Volkov No. 2 effective N-N potential [14]:

$$V(r) = (W - M P^{\sigma} P^{\tau}) \sum_{k=1,2} V_k \exp\left(-r^2/c_k^2\right), \tag{6}$$

where $W = 1 - M$ (M is the Majorana exchange parameter). It is known that although $M \sim 0.6$ reproduces the α-α scattering phase shift, larger M values are needed for the structure calculation beyond ^{12}C, and here M is chosen to be 0.63.

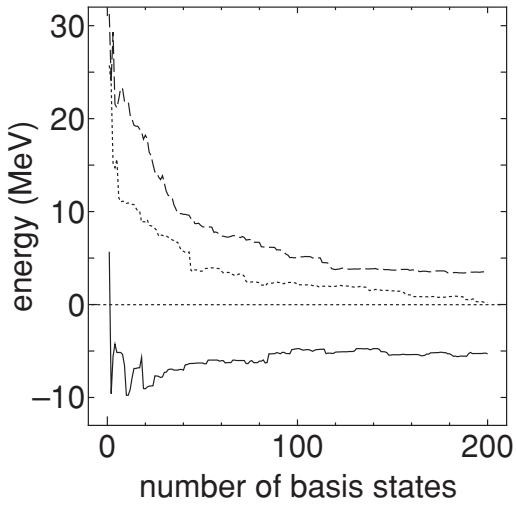

FIG. 1. Energy convergence of ^{16}O (4α) as a function of the number of basis states [m in Eq. (2)] measured from the 4α threshold. The solid, dotted, and dashed lines correspond to the α-condensation parameter [σ in Eq. (4)] equal to 2, 3, and 4 fm, respectively. The dotted line at 0 MeV parallel to the horizontal axis shows the threshold energy of 4α.

The application of the virtual THSR wave function for two-α(^{8}Be) and three-α (^{12}C) cases was shown in our previous article [12], and the virtual THSR wave function with $\sigma = 3 \sim 4$ fm is found to give a reasonable root-mean-square radius for the second 0^+ of ^{12}C compared with that obtained by other approaches. Here, the results for the four- and five-α cases are presented. The energy convergence of ^{16}O (4α) as a function of the number of basis states measured from the 4α threshold is shown in Fig. 1. Here, the size parameter of the Gaussian wave function for each nucleon is set to $\nu = 0.23$ fm^{-2}. All lines converge at the 4α threshold energy and the α-condensed nature is well expressed by the present method. The possibility of the 4α-condensed state was originally pointed out by the α-condensation models [1,3]. Recent experimental results suggest a candidate for the 4α-condensed state at $E_x = 13.6 \pm 0.2$ MeV with a width of 0.6 ± 0.2 MeV [7] just below the 4α threshold ($E_x = 14.44$ MeV).

Next the study is further extended to the 5α case. The energy convergence of ^{20}Ne (5α) as a function of the number of basis states measured from the 5α threshold is shown in Fig. 2. Here, the size parameter of the Gaussian wave function for each nucleon is set to $\nu = 0.18$ fm^{-2}. The solid and dotted lines converge at the 5α threshold energy; however, a large number of basis states are needed in the case of $\sigma = 4$ fm.

The 0^+ energy levels of ^{20}Ne are shown in Fig. 3. The levels in (a) are the results of the ^{16}O+α model space (five basis states) and those in (b) are after adding the virtual THSR wave functions with three different condensation parameters ($\sigma = 2, 3, 4$ fm). It is clearly seen that one new state (the fourth state) appears around the 5α threshold after adding the α-condensed basis states. The gaslike nature can be confirmed by calculating the large rms radius of the fourth 0^+ state (3.5 fm). Since the state is below the 5α threshold (dotted

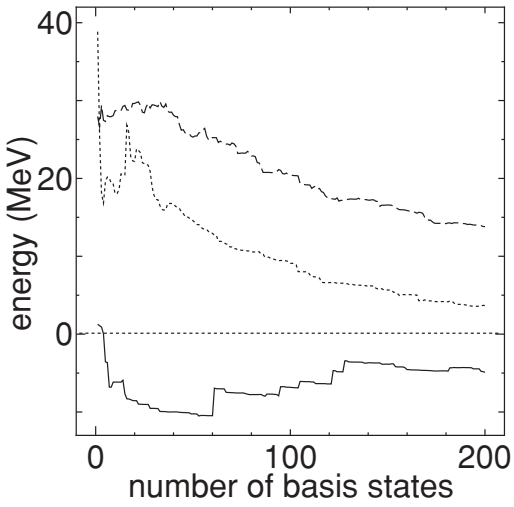

FIG. 2. Energy convergence of ^{20}Ne (5α) as a function of m in Eq. (2) measured from the 5α threshold. The solid, dotted, and dashed lines correspond to the α-condensation parameter [σ in Eq. (4)] equal to 2, 3, and 4 fm, respectively. The dotted line at 0 MeV shows the threshold energy of 5α.

line in Fig. 3), the solution is not the continuum state of five α clusters. Also, although the state is above the ^{16}O+α threshold, the state is missing in Fig. 3(a) within the ^{16}O+α model space, and it appears only after adding the virtual THSR wave function in Fig. 3(b). The virtual THSR wave function does not contain the asymptotic wave function of ^{16}O+α, since all the α clusters are treated democratically, thus the solution is not the continuum wave function of ^{16}O+α. The discussion above suggests the possibility that the state might have been observed as a resonance state. Of course, it would be further necessary to incorporate the ^{12}C+α+α model space when making a precise comparison with experiments. However, from the present analysis, we can conclude that the α-condensed state survives

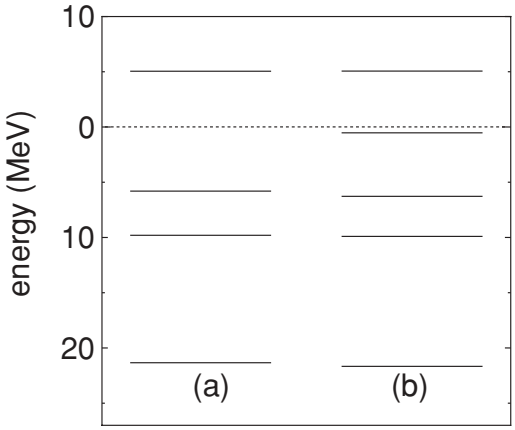

FIG. 3. 0^+ energy levels of ^{20}Ne measured from the 5α threshold. (a) ^{16}O+α model space. (b) Results after adding the α-condensed states with $\sigma = 2, 3, 4$ fm.

TABLE I. Squared overlap between the wave function for each 0^+ state of ^{20}Ne [Fig. 2(b)] and the virtual THSR wave function with $\sigma = 2, 3, 4$ fm.

State	$\sigma = 2$ fm	$\sigma = 3$ fm	$\sigma = 4$ fm
0^+_1	0.19	0.03	0.00
0^+_2	0.02	0.01	0.00
0^+_3	0.09	0.04	0.01
0^+_4	0.68	0.51	0.13
0^+_5	0.00	0.04	0.03

after imposing the coupling condition with the most important cluster configuration of ^{16}O+α.

The squared overlap between the wave function for each 0^+ state of ^{20}Ne [Fig. 3(b)] and the virtual THSR wave function with $\sigma = 2, 3, 4$ fm is shown in Table I. We can confirm that the fourth 0^+ state has a large component of the condensed state with a relatively large σ value (0.51 with $\sigma = 3$ fm). Also, the ground state has a squared overlap with the virtual THSR wave function when the σ value gets small (0.19 with $\sigma = 2$ fm). This is because the limit of $\sigma = 0$ corresponds to the SU(3) limit of the wave function, where all the Gaussian center parameters of the five α clusters approach the origin and the wave function corresponds to the lowest configuration of the shell model due to the antisymmetrization effect.

It is basically possible to study the coupling effect between the virtual THSR wave function and normal cluster states of ^{16}O as well. However, using the traditional effective interactions for the cluster models, it has been known to be difficult to reproduce simultaneously the binding energies of ^{12}C and ^{16}O [15]. Because of this problem, the threshold energies of ^{12}C+α (experimentally $E_x = 7.16$ MeV) and four α's (experimentally $E_x = 14.44$ MeV) are much closer than experimental ones, thus a mixing of these two components occurs.

We have studied the coupling effect between the α-condensed states and normal cluster states of ^{16}O+α in ^{20}Ne based on a microscopic α-cluster model. This is performed by introducing a Monte Carlo technique for the description of the THSR wave function, which is called the virtual THSR wave function. It is shown that the α-condensed states appear around the 5α threshold energy. The possibility of a 4α-condensed state has been already pointed out by the α-condensation models; however, now the study has been extended to the 5α case, and the state is shown to be stable even after imposing the coupling condition with the normal cluster states. As a next step, coupling with the shell model states will be taken into account, since the cluster-shell competition has been known to be important, especially in the ground state of ^{20}Ne [16]. Furthermore, the analysis of the 6α and 7α cases based on the same approach is ongoing.

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