Level densities of ^{44}Sc and ^{47}Ti from different experimental techniques

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The level densities of ^{44}Sc and ^{47}Ti have been determined from measurements of particle evaporation spectra from the compound nuclear reaction ^{3}He + ^{45}Sc with an 11 MeV ^{3}He beam. The level density of ^{44}Sc has been compared to the level density obtained from an independent experimental method based on an analysis of α-γ coincidences from the transfer reaction ^{45}Sc(^{3}He,$\alpha\gamma$)^{44}Sc. The good agreement between the two experiments indicates the reliability of the level density obtained. Some level density systematics have been tested against the experimental data. New Fermi-gas level density parameters have been derived.

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I. INTRODUCTION

The nuclear level density is difficult to measure precisely because of the lack of reliable experimental techniques. The counting of discrete levels is restricted to excitation energies below about 3–5 MeV for medium mass nuclei because above this limit the levels become too close in energy to resolve. Above these energies more sophisticated methods need to be applied (see Ref. [1]). The main approach for estimating the level density above the discrete level region is to use some model-based function with parameters fitted to the density of discrete low-lying levels and the density of neutron resonances. For nuclei for which information about the neutron resonance spacing is not available, parameter systematics must be used. There are several systematics of level density parameters (mainly related to either Fermi-gas or constant temperature models) that modern computer codes utilize to calculate reaction cross sections. However, because neutron resonances are known only for a very narrow spin interval, and because the shape of the level density function is not well established, it is not yet clear how well available systematics reproduce total level densities above the discrete level region.

At this time, two experimental techniques appear to be good candidates for the systematic investigation of the total level density in nuclei above the region of discrete levels. The first one was developed at the Oslo Cyclotron Laboratory (hereafter referred to as the "Oslo method") to extract both level density and γ-strength functions from the particle-γ coincidence matrix measured from inelastic scattering (^{3}He,^{3}He$'\gamma$) and transfer (^{3}He,$\alpha\gamma$) reactions [2]. The second method uses particle evaporation spectra from compound nuclear reactions [3]. The problem is that both methods might contain intrinsic systematic uncertainties, which are difficult to estimate while remaining inside of these methods. Particularly the Oslo method suffers from normalization uncertainties because it produces only a level density function with an uncertainty factor of $A\exp(BE_x)$, where E_x is the excitation energy. The coefficients (A, B) then have to be determined from auxiliary experimental information such as neutron resonance spacing (when available) and the density of discrete levels. The Oslo method is also based on assumptions discussed below, which are possible sources of systematic uncertainties as well. The main problem with the particle evaporation technique is possible contaminations of the evaporation spectra due to multistep and/or direct reaction contributions. It could result in an incorrect slope of the obtained level density function and could cause an absolute normalization problem.

The consistency of these two experimental techniques has been confirmed in Ref. [4], where the level density of ^{56}Fe was investigated with the reaction ^{57}Fe(^{3}He,$\alpha\gamma$)^{56}Fe by the Oslo method and with the neutron evaporation spectrum from the ^{55}Mn(d,n)^{56}Fe reaction. Neutron spectra are most suitable for level density studies because neutron transmission coefficients are better known than proton and α-transmission coefficients. Moreover, the neutron channel is a preferred decay channel for the compound nucleus. This means that it is more likely that compound reactions dominate the neutron spectrum. On the other hand, it would be highly desirable to study different types of reactions for these purposes. In this work we have studied the level density from the evaporation spectra of α particles from the ^{45}Sc(^{3}He,α)^{44}Sc reaction and compared it to the level density obtained recently from the ^{45}Sc(^{3}He,$\alpha\gamma$)^{44}Sc reaction using the Oslo method. The level density of ^{47}Ti populated by the ^{45}Sc(^{3}He,p)^{47}Ti reaction has been obtained as well. Different available level density systematics have been tested.

II. EXPERIMENTS AND METHODS

A. The Oslo method

At the Oslo Cyclotron Laboratory, a measurement of the ^{45}Sc(^{3}He,$\alpha\gamma$)^{44}Sc reaction with a 38 MeV ^{3}He beam was performed. The self-supporting natural target of 99.9% ^{45}Sc had a thickness of 3.4 mg/cm^2. Eight Si ΔE-E telescopes were arranged close to the target at an angle of 45° relative to the beam. The γ-ray detector CACTUS [5], consisting of 28 collimated NaI crystals with a total efficiency of 15%

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surrounded the target and particle detectors. The experimental setup enabled particle-γ coincidence measurements of the reaction ^{45}Sc(^{3}He,$\alpha\gamma$)^{44}Sc. The experiment ran for about 5 days, with a typical beam current of $\sim$1 nA.

The essential part of the analysis of particle-γ coincidences is the extraction of first-generation spectra $P(E_x, E_\gamma)$ at each excitation energy bin E_x, which is the initial excitation energy of the γ transitions. The corresponding technique is described in Ref. [6]. The first generation matrix $P(E_x, E_\gamma)$ can be decomposed into a level density $\rho(E_x - E_\gamma)$ and γ-transmission function $T(E_\gamma)$ as

$$P(E_x, E_\gamma) \propto \rho(E_x - E_\gamma)T(E_\gamma). \tag{1}$$

The details of this particular experiment and its analysis are described in Ref. [7]. Here we would like to outline the important assumptions behind this decomposition.

(i) The γ decay from each excitation energy bin and the spin population within the bin are independent of how the levels were populated; whether directly by the reaction or by γ decay from higher-lying states.
(ii) The γ-strength function does not depend on the excitation energies of either initial or final states, it depends only on the γ energy.

It is difficult to estimate how large the possible violations of the assumptions are and how they affect the final results. Special concern is caused by the possible temperature dependence of the γ-strength function suggested in theoretical work [8] which would mean a violation of the second assumption.

B. Level density from evaporation spectra

To obtain an independent result on the level density of ^{44}Sc, we measured the α-particle evaporation spectrum from the ^{45}Sc(^{3}He,α)^{44}Sc reaction. The proton spectrum was also studied, which allowed us to investigate the level density of the residual ^{47}Ti nucleus. The experiment was performed with an 11-MeV ^{3}He beam from the tandem accelerator of the Ohio University Edwards Accelerator Laboratory. Proton and α-particle spectra were measured with a charged-particle spectrometer [1]. Seven 2-m time-of-flight tubes with Si detectors placed at the end were set up at angles ranging from 22.5° up to 157.5°. The masses of the charged particles were determined by measuring both the energy deposited in the Si detectors and the time of flight. The mass resolution was sufficient to resolve protons, deuterons, ^{3}He/^{3}H, and α particles.

The cross section of outgoing particles resulting from compound nucleus decay can be calculated in the framework of the Hauser-Feshbach (HF) model [9], according to which

$$\frac{d\sigma}{d\varepsilon_b}(\varepsilon_a, \varepsilon_b) = \sum_{J\pi^c} \sigma^{\mathrm{CN}}(\varepsilon_a) \frac{\sum_{I\pi^r} \Gamma_b(U, J, \pi^c, E, I, \pi^r)\rho_b(E, I, \pi^r)}{\Gamma(U, J, \pi^c)}, \tag{2}$$

with

$$\Gamma(U, J, \pi^c) = \sum_{b'} \Bigg(\sum_k \Gamma_{b'}\left(U, J, \pi^c, E_k, I_k, \pi^r_k\right) + \sum_{I'\pi^{r'}} \int_{E_c}^{U-B_{b'}} dE'\, \Gamma_{b'} \times (U, J, \pi^c, E', I', \pi^{r'})\, \rho_{b'}(E', I', \pi^{r'}) \Bigg). \tag{3}$$

Here $\sigma^{\mathrm{CN}}(\varepsilon_a)$ is the fusion cross section, ε_a and ε_b are energies of relative motion for incoming and outgoing channels ($\varepsilon_b = U - E_k - B_b$, where B_b is the separation energy of particle b from the compound nucleus), Γ_b is the transmission coefficient of outgoing particles, and the quantities (U, J, π^c) and (E, I, π^r) are the energy, angular momentum, and parity of the compound and residual nuclei, respectively. The energy E_c is the continuum edge, above which levels are modeled using a level density parametrization. For energies below E_c, the known excitation energies, spins, and parities of discrete levels are used. In practice E_c is determined by the available spectroscopic data in the literature. It follows from Eq. (3) that the cross section is determined by both transmission coefficients of outgoing particles and the level density of the residual nucleus $\rho_b(E, I, \pi)$. It is believed that transmission coefficients are known with sufficient accuracy near the line of stability because they can be obtained from optical model potentials, which are usually based on experimental data for elastic scattering and total cross sections in the corresponding outgoing channel. Transmission coefficients obtained from different systematics of optical model parameters do not differ by more that 15–20 % from each other in our region of interest (1–15 MeV of outgoing particles). The uncertainties in level densities are much larger. Therefore, the HF model can be used to improve level densities by comparing experimental and calculated particle evaporation spectra. Details and assumptions of this procedure are described in Refs. [3] and [10]. The code HF2002 [11] was used for calculations of spectra from compound nuclear reactions.

The main uncertainty of this method comes from contributions of noncompound mechanisms of a nuclear reaction including direct, multistep direct, and multistep compound. They correspond to different stages of nucleon-nucleon interactions inside the projectile + target nuclear system until complete equilibrium is achieved. The measurement of the energy distribution of outgoing particles at backward angles reduces considerably the contribution from noncompound reactions, but does not guarantee their complete elimination. Therefore, the systematic uncertainties connected to the presence of noncompound reaction contributions can be estimated only by applying different experimental techniques directed to measure the level density of the same nucleus.

By comparing data obtained from the Oslo method with data extracted from particle evaporation spectra one can estimate possible systematic uncertainties pertaining to these methods.

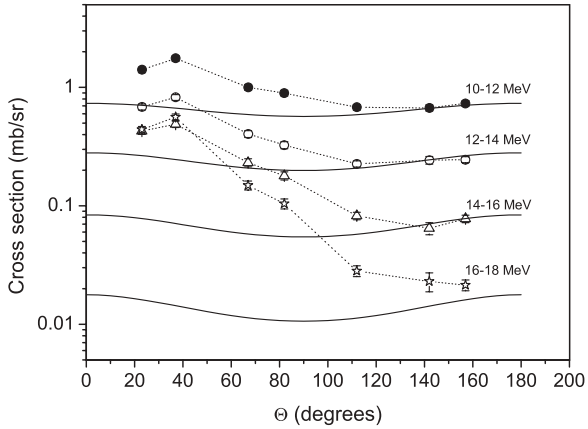

FIG. 1. Angular distributions of α particles in the c.m. system for different energy intervals. The points are experimental data and the solid lines are HF calculations normalized to match the experimental points at backward angles for low energy particles.

III. RESULTS

To investigate the reaction mechanism, the proton and α-particle angular distributions were measured (Figs. 1 and 2). In the figures the particle energies are restricted to ensure that only first stage particles emitted immediately from the compound nucleus ^{48}V can contribute. For compound nuclear reactions the HF calculation predicts a symmetric angular distribution of the cross section with respect to 90° in the center of mass system. The present measurement exhibits forward-peaked distributions for both protons and α particles. However, it is important to note that for lower energy α particles, the angular distribution starts to follow the calculated curve at ≈115° and beyond. For higher energy particles the asymmetry is stronger. For α particles in the energy interval 16–18 MeV, i.e., for those populating the discrete levels of ^{44}Sc, the angular distribution does not agree with calculations even at backward angles. This means that high energy α particles contain contributions from noncompound reactions even at backward angles. From this analysis, it is possible to conclude that the α-particle spectra measured at backward

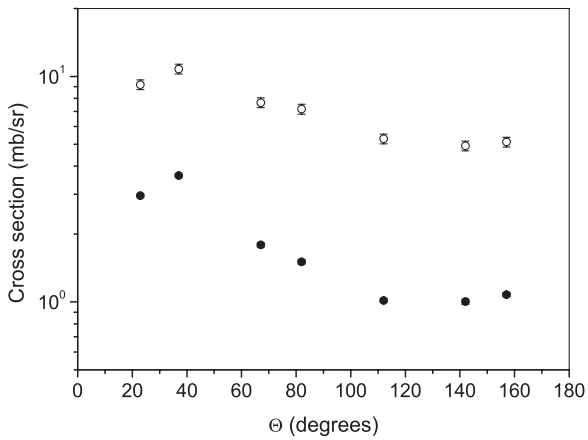

FIG. 2. Angular distributions of protons (open circles) and α particles (solid circles) in the c.m. system.

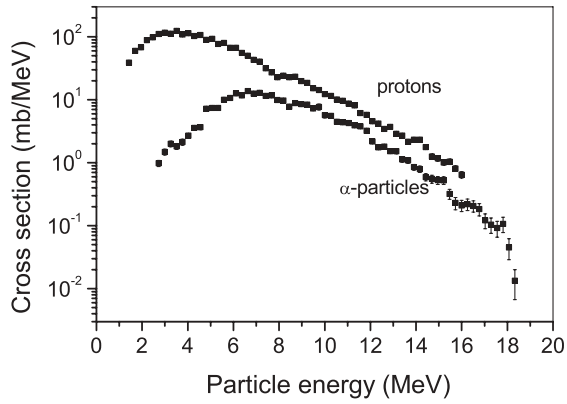

FIG. 3. Experimental energy spectra of protons and α particles measured at 157° with respect to the beam line.

angles can be used for extracting level densities but only in the energy region 10–16 MeV, which corresponds to excitation energies of the residual ^{44}Sc nucleus between 2 and 8 MeV. We could not make a similar analysis for protons because the thickness of our detectors (1000 and 1500 μm) was not sufficient to stop protons with energies greater than 10 and 15 MeV, respectively (note that the maximum proton energy from this reaction is 21 MeV). Therefore, the proton angular distribution is integrated over all energies and is presented in Fig. 2 along with integrated distribution for α particles. The similarity of these distributions indicates that the compound mechanism is the main mechanism determining both proton and α-particle spectra measured at backward angles.

The energy dependence of proton and α-particle cross sections measured at 157° with respect to the beam axis are shown in Fig. 3. The level densities for both ^{44}Sc (populated by α particles) and ^{47}Ti (populated by protons) nuclei were obtained by the method described in Ref. [3] and in our previous article [1]: a level density model is chosen for the calculation of the differential cross section of Eq. (3). The parameters of the model are then adjusted to reproduce the experimental spectra as closely as possible. The input level density is improved by binwise renormalization according to the expression

$$\rho_b(E, I, \pi) = \rho_b(E, I, \pi)_{\text{input}} \frac{(d\sigma/d\varepsilon_b)_{\text{meas}}}{(d\sigma/d\varepsilon_b)_{\text{calc}}}. \qquad (4)$$

To get the absolute normalization, information about the level density of discrete levels is used.

The level densities of ^{47}Ti and ^{44}Sc extracted from proton and α-particle evaporation spectra are shown in Fig. 4. The level density of ^{44}Sc extracted from the Oslo experiment is presented for comparison.

The absolute normalization of the level density for ^{44}Sc has been obtained by matching the Oslo level density to the density of discrete levels in the low energy region and by matching the slope of the Oslo level density to the slope of the level density obtained from the particle evaporation spectrum. One can see the good agreement between the shapes of the level densities from two types of experiments. The absolute normalization of the level density for ^{47}Ti was obtained from the ratio of α/proton cross sections of the ^{3}He + ^{45}Sc reaction.

TABLE I. Ratio of experimental and model level densities at different excitation energies E_x. The bottom line shows the comparison of the experimental and calculated ratios of α-particle and proton cross sections.

Nucleus	E_x (MeV)	$\rho^{\exp}/\rho^{\text{model}}$			
		FG [12]	HFBCS [15]	GC [14]	CT [12]
^{47}Ti	5.5	0.78(16)	0.92(19)	1.52(31)	1.19(24)
	6.5	0.83(17)	1.14(23)	1.73(35)	1.18(24)
	7.5	0.69(14)	1.12(23)	1.53(31)	0.92(19)
	8.5	0.58(12)	1.10(22)	1.34(27)	0.70(14)
	9.5	0.60(12)	1.30(26)	1.40(28)	0.63(12)
	10.5	0.53(10)	1.29(26)	1.20(24)	0.45(9)
	11.5	0.49(10)	1.35(27)	1.06(21)	0.36(7)
^{44}Sc	2.5	1.41(30)	0.67(14)	1.50(31)	1.53(32)
	3.5	1.16(24)	0.61(13)	1.46(30)	1.27(26)
	4.5	1.00(20)	0.56(11)	1.42(29)	1.04(21)
	5.5	0.91(18)	0.56(11)	1.43(29)	0.88(18)
	6.5	0.93(19)	0.60(12)	1.55(31)	0.81(16)
		$\frac{\sigma_\alpha^{\exp}/\sigma_p^{\exp}}{\sigma_\alpha^{\text{cal}}/\sigma_p^{\text{cal}}}$			
		1.6(2)	0.5(2)	1.1(2)	1.2(2)

The experimental level densities have been compared to some level density models widely used in modern HF computer codes. These prescriptions are based on the Fermi gas (FG) model, the constant temperature (CT) model with parameters from the recent compilation of Ref. [12], and the Gilbert-Cameron (GC) formula [13]. Parameter systematics are obtained mainly on the basis of available information about the level density in the region of discrete levels and neutron resonances. For the GC model the Fermi-gas level density parameter a was calculated according to the Ignatyuk systematics [14] while parameters of the constant temperature part of the GC formula were obtained from the fit to discrete levels. We also tested the level density calculations based on the Hartree-Fock-BCS approach (HFBCS) [15] recommended by the RIPL data base [16]. Table I shows the ratio between experimental and model level densities at different excitation energies. It shows also how well HF calculations reproduce the ratio of α and proton cross sections. This is an important issue because this ratio gives an additional constraint on level densities of residual nuclei. The conclusion is that the level density of ^{47}Ti is best reproduced with the HFBCS model but the FG systematics fit better for ^{44}Sc. No single model with parameters from systematics reproduces level densities of both nuclei equally well. However, HF calculations with GC and CT models reproduce well the ratio of α and proton cross sections.

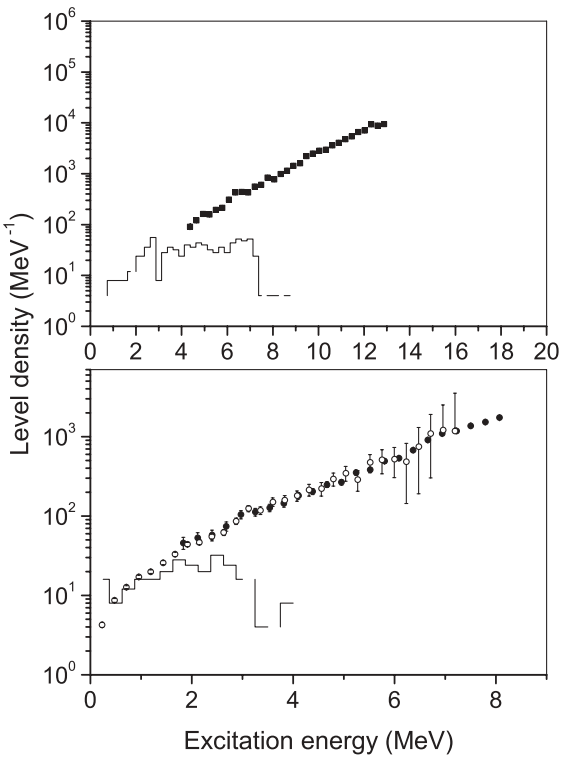

FIG. 4. Level densities of ^{47}Ti (upper panel) and ^{44}Sc (lower panel) nuclei obtained from proton and α-particle energy spectra, respectively. Black points are data from particle evaporation spectra. Open points are data from the Oslo experiment [7]. The histograms represent level densities from the counting of discrete levels.

To improve the level density prescription for these nuclei, we used the FG model with free parameters a and δ to fit the experimental level densities. The rigid-body spin cutoff parameter was adopted for this fit. The parameters we obtained are $a = 5.13$ MeV^{-1} and $\delta = -2.91$ MeV for ^{44}Sc and $a = 5.06$ MeV^{-1} and $\delta = -1.95$ MeV for ^{47}Ti. These parameters can be compared to parameters from systematics [12]: $a = 5.68$ MeV^{-1} and $\delta = -2.064$ MeV for ^{44}Sc and $a = 5.99$ MeV^{-1} and $\delta = -0.738$ MeV for ^{47}Ti. Discrepancies in corresponding level densities are shown in Table I. The systematics of Ref. [12] agree with the experimental level density of the ^{44}Sc nucleus but are off by a factor of 1.3–2 for ^{47}Ti. However, in the case of ^{47}Ti, level density parameters

from our experiment agree better with ones determined in Ref. [12] on the basis of a fit to low-lying discrete levels and neutron resonance spacings ($a = 5.14(30)$ MeV^{-1} and $\delta = -1.35(74)$ MeV).

It should be noted that the drawback of all available level density systematics is that all of them use the neutron resonance spacing as a main source for the experimental level density at the neutron binding energy. The problem is that the neutron resonances are known within a narrow spin interval and the spin cutoff parameter must be used to calculate the total level density, which actually affects the reaction cross section calculations. The lack of experimental information on the spin cutoff parameter above the discrete level region introduces additional uncertainties in the calculation of reaction cross sections and can cause deviations from our experimental data (see Table I). An alternative option would be to establish a level density systematic based on experimental data on total level densities. There was an attempt [17] to establish the systematic based on particle evaporation spectra. About 50 nuclei from the $A = 10$–70 region have been analyzed. However, because of large discrepancies in level density parameters from different experiments, no good systematic regularity has been found.

The consistency between experimental level densities obtained from the Oslo method and particle evaporation spectra supports the underlying assumption of the Oslo method. It shows that the statistical mechanism is the major mechanism of γ decay following α-particle emission in the ^{45}Sc(^{3}He,$\alpha\gamma$) reaction. The spin of levels populated by either α particles or γ transitions does not seem to be much different. Also, the uncertainties due to the possible temperature dependence of the γ-strength function are small enough to not affect the final level density obtained by the Oslo method. All of these results indicate that the Oslo method, within its limitations, is a reliable tool for studying nuclear level densities.

The method based on particle energy spectra may suffer from systematic uncertainties connected to contributions of noncompound reaction mechanisms. These contributions depend on the type of reaction used as well as on the angle at which the spectra are measured. Backward angles allow one to reduce the contribution from noncompound reactions considerably but do not eliminate completely this effect, especially for high energy particles. The measurement of the angular distribution is an important tool in the analysis helping to determine the angle and energy ranges to be used for the level density determination.

IV. CONCLUSION

The level density of ^{44}Sc has been obtained from two independent experiments by using two different methods. These are the Oslo method based on the analysis of particle-γ coincidences from the ^{45}Sc(^{3}He,$\alpha\gamma$) reaction and the method based on the analysis of particle spectra from the compound nuclear reaction ^{45}Sc(^{3}He,α). Both methods produce the level densities that are in good agreement with each other. It has been shown that possible systematic uncertainties of the Oslo method resulting from underlying assumptions are negligible and do not cause any serious problems. The α particles from the ^{45}Sc(^{3}He,α) compound reaction measured at backward angles can be used to extract the level density of the corresponding residual nucleus. The angular distribution is an important factor in determining the range of energies of outgoing particles where the compound reaction mechanism is dominant.

The level density of ^{47}Ti has been obtained from the proton evaporation spectrum of the ^{45}Sc(^{3}He,p) reaction. Both ^{44}Sc and ^{47}Ti experimental level densities have been compared to several level density models. Despite the fact that some of these models reproduce experimental data well for one of these nuclei, none of the models seem to fit experimental data for both of them. The deviation from the best fit can be as large as 50%. New Fermi-gas level density parameters have been obtained.

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