Monopole and quadrupole interactions in binding energies of *sd*-shell nuclei

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It is demonstrated by shell-model calculations of *sd*-shell nuclei that the binding energies are dominated by the monopole part of nucleon-nucleon interaction. The monopole component, which comes mainly from the two-body interaction in triplet-even channel, can be renormalized into one-body single-particle energies. It is also shown that the proton-neutron quadrupole interaction causes a sizable gain of binding energy by coherent configuration mixing of quadrupole deformation.

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I. INTRODUCTION

Nuclear binding energies were systematically studied and simulated by a semi-empirical mass formula in 1930s by Weizsäcker and Bethe [1,2]. The mass formula consists of volume, surface, and Coulomb energy terms, which are based on the liquid-drop model of nuclei, and of symmetry and pairing energy terms, which take into account properties of nuclear forces. Various models have been employed for systematic mass predictions of nuclei throughout the nuclear chart. They are mostly macroscopic + microscopic models, and some others are based on microscopic model calculations [3–7]. It has been found, however, that root-mean-square deviations of the latter are considerably larger than those of the macroscopic + microscopic models, and there are still problems to be solved for reliable predictions of nuclear masses from basic nucleon-nucleon interactions [8].

Nuclear shell model, which was conceived in late 1940s by Mayer and Jensen who introduced a spin-orbit interaction in order to account for observed magic numbers [9–11], has been applied successfully to light and medium-light nuclei. Especially, empirical effective interactions have been widely used in *p*-shell nuclei [12] and *sd*-shell nuclei [13,14]. These effective interactions were obtained by a least-squares fit, by treating single-particle energies and two-body matrix elements as free parameters, so as to reproduce binding energies and excitation energies of a number of observed states which are presumably described by the respective full $0\hbar\omega$ model space. In the *sd* shell, semi-empirical effective interactions were analyzed and compared with typical *G*-matrix effective interactions [15]. In the next major shell, the *pf* shell, such an optimization of two-body interaction matrix elements was hard to do, and a *G*-matrix effective interaction [16] has been used with modifications of the monopole strengths, shifts of single-particle energies and minor changes of some two-body matrix elements [17,18]. The modified interaction, named KB3, has been employed for $f_{7/2}$-shell nuclei. The monopole shift between the spin-orbit partners, $f_{7/2}$ and $f_{5/2}$, and the single-particle energy difference between these orbits are responsible for successful description of magnetic dipole excitations [19,20], double beta decay of ^{48}Ca [21], and a detailed analysis of the effective coupling constant of g_A for Gamow-Teller transitions in $A = 41$–50 nuclei [22]. The monopole shift between $f_{7/2}$ and $p_{3/2}$ is important for reproducing low-lying energy spectra, and the KB3 interaction has been used in systematic spectroscopic studies on $A = 48$ [23] and $A = 47, 49$ nuclei [24], back-bending phenomena [23,25], and binding energies [26], and, with a KB3G interaction with a shell-gap readjustment, $A = 50$, 51, and 52 isobaric chains [27]. A least-squares fit procedure has later been used to obtain empirical effective interactions in the *pf* shell, the FPD6 interaction [28], and the GXPF1 interaction [29], where a special attention has been paid to the monopole interaction.

The importance of the monopole interaction was first emphasized by Bansal and French [30,31], who derived a simple equation for average energies of one-hole-many-particle states by introducing isoscalar and isovector monopole interactions. In the shell model, a decisive role of the monopole has been recognized for successful description of nuclear structure and later in the context of shell evolution by many authors, in addition to those given in the previous paragraph, including Schiffer and True [32], Blomqvist and Rydström [33], Pittel *et al.* [34], Zuker and his collaborators [35–39], Brown [40], Otsuka and his collaborators [41–44], Grawe [45,46], and Umeya and Muto [47–51]. It is noted that closed shell energies are written in terms of single-particle energies and the monopole strength. This indicates that single-particle energies of an orbit with respect to different cores are related to each other by the monopole interaction [33,46].

Neutron-rich nuclei have attracted much attention to the monopole interaction as the major driving force of shell structure evolution. Otsuka *et al.* suggested that the spin-isospin $(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})(\boldsymbol{\tau}\cdot\boldsymbol{\tau})$ component of nucleon-nucleon interaction between spin-orbit partners, $j = \ell \pm \frac{1}{2}$, is responsible for the shell evolution [42]. But, Umeya and Muto clearly showed later, by employing the spin-tensor decomposition, that the monopole interaction is dominated by triplet-even central

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components of shell-model effective interactions [47,48,51]. These triplet-even components are attributed to the medium-range central interaction and second order effects of tensor interaction by channel coupling between, for example, $^3S_1$ and $^3D_1$. It has also been shown that the tensor interaction plays a minor but important role for shell structure evolution by shell model [52–54] and also by mean-field model calculations [55–60]. This effect appears clearly when nucleons are filling a large-j orbit in heavy nuclei. This is the first order contribution of tensor forces, and it is part of the monopole interaction.

The purpose of this study is to investigate mainly the monopole interaction from a view point of binding energy of nuclei, by making use of shell-model calculations of sd-shell nuclei with the USD interaction which is known to reproduce experimental binding energies [13,14]. We perform three kinds of decompositions of two-body effective interaction: (i) a decomposition into proton-proton, neutron-neutron and proton-neutron interactions, (ii) a multipole expansion, which clearly defines the monopole interaction, (iii) the spin-tensor decomposition which separates contributions from central, spin-orbit, and tensor interactions in spin/isospin triplet and singlet channels. These decomposition methods are useful for the investigation of empirical and/or effective interactions, and, in particular, the spin-tensor decomposition was used in the analyses of sd-shell empirical interactions [15].

The present paper consists as follows. The nuclear Hamiltonian is briefly explained in Sec. II. The Hamiltonian is described in the second quantized form in the jj-coupling scheme. Section III is devoted to the decompositions of two-body interaction in a general form, with an emphasis on the monopole interaction. Numerical results for sd-shell nuclei are shown in Sec. IV, according to the three kinds of decompositions of the effective nucleon-nucleon interaction. We will show that the energy gains due to the nuclear force are dominated by the monopole interaction, that the monopole component of two-body interaction can be renormalized into one-body single-particle energies, and that the first order contribution of tensor force can be seen in the evolution of single-particle energies. We also discuss the role of quadrupole components of the proton-neutron interaction on binding energies. Conclusion of this study is given in Sec. V.

II. HAMILTONIAN AND BINDING ENERGY

The nuclear Hamiltonian is assumed to consist of one- and two-body interactions,

$$H = \sum_{jm} \varepsilon_j^{\rm core} a_{jm}^\dagger a_{jm} + \sum_{\alpha\alpha' JM} \langle 2\alpha | V | 2\alpha' \rangle_J A^\dagger(2\alpha JM) A(2\alpha' JM), \quad (1)$$

which applies to valence nucleons, and the core energy is omitted in this Hamiltonian. $\varepsilon_j^{\rm core}$ is the single-particle energy with respective to the core, and it is a sum of kinetic energy and interaction energy of the valence nucleon and the core nucleons. $a_{jm}^\dagger$ and a_{jm} are the creation and annihilation operators, respectively, of a nucleon in a single-particle state with angular momentum j and its projection m. The sum of combinations jm runs over all valence single-particle states in the model space. The second term of Eq. (1) represents two-body interaction. $A^\dagger(2\alpha JM)$ creates an antisymmetrized two-particle state with the coupled angular momentum J and its z-component M, and the corresponding annihilation operator is denoted by $A(2\alpha JM)$. The additional quantum number α distinguishes different two-particle configurations, such as jj' and j^2, and the sums of α and α' run over all possible configurations that are allowed in the model space. When there are three single-particle orbits, j_1, j_2, and j_3, the configuration quantum number α takes

$$\alpha = j_1^2, \quad j_1 j_2, \quad j_1 j_3, \quad j_2^2, \quad j_2 j_3 \quad \text{and} \quad j_3^2.$$

The Coulomb interaction is not included in the Hamiltonian, and an energy eigenstate has a good isospin quantum number, in addition to the angular momentum and parity, J^π.

The eigenvalue equation for the nuclear Hamiltonian is written as

$$H\left|J_\nu^\pi\right\rangle = E\left|J_\nu^\pi\right\rangle. \quad (2)$$

J^π is the spin-parity, and ν labels eigenstates with the same spin-parity in the same nucleus. The binding energy of an eigenstate J_ν^π is defined here by

$$B = -E. \quad (3)$$

The binding energy thus defined includes neither the core energy nor the Coulomb energy. When the two-body interaction, the second term of Eq. (1), is decomposed into n terms

$$V = \sum_{k=1}^{n} V_k, \quad (4)$$

the binding energy is given by a sum of single-particle energies and the corresponding two-body interaction components,

$$B = B(\text{spe}) + \sum_{k=1}^{n} B(k). \quad (5)$$

The component $B(k)$ of the binding energy is evaluated by the expectation value of V_k,

$$B(k) = -\left\langle J_\nu^\pi \middle| V_k \middle| J_\nu^\pi \right\rangle, \quad (6)$$

where $|J_\nu^\pi\rangle$ is obtained by solving the eigenvalue equation, Eq. (2), with the full Hamiltonian.

III. DECOMPOSITIONS OF TWO-BODY INTERACTION

A. *pp*-, *nn*- and *pn*-interactions

A simple decomposition of the two-body interaction is that into three-types of interactions, i.e., proton-proton (pp), neutron-neutron (nn), and proton-neutron (pn) interactions,

$$V = V_{pp} + V_{nn} + V_{pn}, \quad (7)$$

and the binding energy of an energy eigenstate is then expressed as a sum of four terms,

$$B = B(\text{spe}) + B(V_{pp}) + B(V_{nn}) + B(V_{pn}). \quad (8)$$

Two-particle matrix elements of the like-nucleon interactions are expressed in isospin formalism as $T = 1$ elements,

$$\langle 2\alpha|V_{pp/nn}|2\alpha'\rangle_J = \langle 2\alpha|V|2\alpha'\rangle_{J,T=1}. \quad (9)$$

Matrix elements of the proton-neutron interaction are written in terms of $T = 0$ and $T = 1$ matrix elements as

$$\langle j_p j_n|V_{pn}|j'_p j'_n\rangle_J = \frac{\sqrt{(1+\delta_{j_p j_n})(1+\delta_{j'_p j'_n})}}{2} \times (\langle j_p j_n|V|j'_p j'_n\rangle_{J,T=1} + \langle j_p j_n|V|j'_p j'_n\rangle_{J,T=0}), \quad (10)$$

where the difference is considered in the normalization of two-particle states between the proton-neutron scheme and isospin scheme, and $\delta_{j_p j_n}$ is unity when the proton and neutron single-particle orbits, j_p and j_n, have the same set of quantum numbers.

B. Multipole expansion and monopole interaction

A two-body interaction can be expanded into multipole components. Denoting the expansion as

$$V = \sum_k V^{(k)}, \quad (11)$$

the binding energy is accordingly given by a sum of the respective contributions,

$$B = B(\text{spe}) + \sum_k B(V^{(k)}). \quad (12)$$

In the following, the multipole expansion is shown in some detail for proton-neutron interaction and like-nucleon interaction separately, with an emphasis on the monopole component.

1. Proton-neutron interaction

The proton-neutron interaction can be written as expanded into multipoles,

$$V_{pn} = \sum_{j_p j_n j'_p j'_n JM} \langle j_p j_n|V_{pn}|j'_p j'_n\rangle_J A^\dagger(j_p j_n JM) A(j'_p j'_n JM). \quad (13)$$

Normalized proton-neutron states are created by

$$A^\dagger(j_p j_n JM) = \left[a^\dagger_{j_p} \otimes a^\dagger_{j_n}\right]^{(J)}_M, \quad (14)$$

where the square bracket denotes angular-momentum coupling to form a spherical tensor of rank-J. The annihilation operator $A(j'_p j'_n JM)$, which is the Hermitian conjugate of $A^\dagger(j'_p j'_n JM)$, is written as

$$A(j'_p j'_n JM) = (-1)^{1+J-M}\left[\tilde{a}_{j'_p} \otimes \tilde{a}_{j'_n}\right]^{(J)}_{-M}, \quad (15)$$

where $\tilde{a}_{jm} = (-1)^{j-m} a_{j\,-m}$ are modified annihilation operators, and the phase guarantees that both $a^\dagger_{jm}$ and $\tilde{a}_{jm}$ are m components of spherical tensor operators of rank-j. The proton-neutron interaction is expanded into multipoles by changing the order of the neutron creation and proton annihilation operators, $a_{j'_p} a^\dagger_{j_n} = -a^\dagger_{j_n} a_{j'_p}$, and angular-momentum recoupling,

$$V_{pn} = \sum_k V^{(k)}_{pn} \quad (16)$$

with

$$V^{(k)}_{pn} = \sum_{j_p j_n j'_p j'_n} f^{(k)}(j_p j_n, j'_p j'_n)\, u^{(k)}(j_p, j'_p) \cdot u^{(k)}(j_n, j'_n), \quad (17)$$

where

$$f^{(k)}(j_p j_n, j'_p j'_n) = \sum_J (-1)^{j_p + j'_n - J}(2k+1)(2J+1) \times W(j_p j_n j'_p j'_n; Jk)\langle j_p j_n|V_{pn}|j'_p j'_n\rangle_J \quad (18)$$

are strengths of the multipole components. In Eq. (17), $u^{(k)}(j_p, j'_p)$ and $u^{(k)}(j_n, j'_n)$ are unit-tensor operators of rank-k, which act on proton and neutron systems, respectively. They are defined by

$$u^{(k)}(j, j') = \frac{(-1)^{2j}}{\sqrt{2k+1}}[a^\dagger_j \otimes \tilde{a}_{j'}]^{(k)}, \quad (19)$$

and the dot denotes an inner product of two unit-tensor operators with the same rank. The rank k can take integer values which satisfy

$$\max(|j_p - j'_p|, |j_n - j'_n|) \leqslant k \leqslant \min(j_p + j'_p, j_n + j'_n).$$

The $k = 0$ component of the multipole expansion, Eq. (17), is the monopole interaction. The rank-0 unit-tensor operator can be formed only when $j = j'$,

$$u^{(0)}(j, j') = \delta_{jj'}\frac{1}{\sqrt{2j+1}}\hat{N}_j, \quad (20)$$

where

$$\hat{N}_j = \sum_{m=-j}^{j} a^\dagger_{jm} a_{jm} \quad (21)$$

is the number operator of the single-particle orbit j. It is thus obvious that the monopole interaction appears only for diagonal two-particle matrix elements with $j_p = j'_p$ and $j_n = j'_n$. Then, the monopole strength is written as

$$f^{(0)}(j_p j_n, j_p j_n) = \frac{\sum_J (2J+1)\langle j_p j_n|V_{pn}|j_p j_n\rangle_J}{\sqrt{(2j_p+1)(2j_n+1)}} \quad (22)$$

by substituting the Racah coefficient with $k = 0$. The monopole component of the proton-neutron interaction is thus reduced to

$$V^{(0)}_{pn} = \sum_{j_p j_n} \Delta\varepsilon_{j_p j_n}\, \hat{N}_{j_p} \hat{N}_{j_n}, \quad (23)$$

where $\Delta\varepsilon_{j_p j_n}$ is defined by

$$\Delta\varepsilon_{j_p j_n} = \frac{\sum_J (2J+1)\langle j_p j_n|V_{pn}|j_p j_n\rangle_J}{(2j_p+1)(2j_n+1)}. \quad (24)$$

The factor in the denominator is the number of states which are allowed from the coupling of j_p and j_n,

$$(2j_p+1)(2j_n+1) = \sum_{J=|j_p-j_n|}^{j_p+j_n} (2J+1). \tag{25}$$

The $\Delta\varepsilon_{j_pj_n}$ is thus the J-average of the proton-neutron interaction matrix elements. When the J-averaged value is substituted in Eq. (18) as $\langle j_pj_n|V_{pn}|j_pj_n\rangle_J = \Delta\varepsilon_{j_pj_n}$ for the diagonal matrix elements, and otherwise zero, $\langle j_pj_n|V_{pn}|j_p'j_n'\rangle_J = 0$ for $j_p \neq j_p'$ and/or $j_n \neq j_n'$, all multipoles except $k=0$ vanish because of a property of Racah coefficients. The monopole proton-neutron interaction can be written as

$$V_{pn}^{(0)} = \sum_{j_pj_nJM} \Delta\varepsilon_{j_pj_n} A^\dagger(j_pj_nJM)A(j_pj_nJM). \tag{26}$$

This expression is also useful for the evaluation of monopole contributions to binding energy by the expectation value, Eq. (6).

It is noted that, in Sec. IV C, $\Delta\varepsilon_{j_pj_n}$ will appear in single-particle energies which incorporate interactions between valence nucleons.

2. Like-nucleon interaction

The proton-proton interaction can be expanded into multipoles in the same way,

$$\begin{aligned} V_{pp} &= \sum_{[j_1j_2][j_1'j_2']JM} \langle j_1j_2|V_{pp}|j_1'j_2'\rangle_J A^\dagger(j_1j_2JM)A(j_1'j_2'JM) \\ &= \sum_k V_{pp}^{(k)}, \end{aligned} \tag{27}$$

where the sum is taken over all possible combinations of j_1j_2 and those of $j_1'j_2'$. The multipole expansion for like-nucleon interaction is a little complicated, since two-particle creation operators are written by a product of single-particle creation operators as

$$A^\dagger(jj'JM) = \begin{cases} [a_j^\dagger \otimes a_{j'}^\dagger]_M^{(J)} & (j \neq j') \\ \dfrac{1-(-1)^{2j-J}}{2\sqrt{2}} [a_j^\dagger \otimes a_j^\dagger]_M^{(J)} & (j = j') \end{cases} \tag{28}$$

due to the different normalizations of two-particle states between $j \neq j'$ and $j = j'$.

The monopole component with $k=0$ is given by

$$V_{pp}^{(0)} = \sum_{[jj']} \left[\frac{1}{(1+\delta_{jj'})} \Delta\varepsilon_{jj'} \hat{N}_j \hat{N}_{j'} - \delta_{jj'} \frac{1}{2} \Delta\varepsilon_{jj'} \hat{N}_j \right], \tag{29}$$

where

$$\Delta\varepsilon_{jj'} = \frac{\sum_J (1-\delta_{jj'}(-1)^{2j-J})(2J+1)\langle jj'|V_{pp}|jj'\rangle_J}{(2j+1)(2j'+1)}. \tag{30}$$

In contrast to the proton-neutron interaction, the second term of Eq. (29) appears as a part of the $k=0$ interaction, when the anticommutation relation does not vanish. The $\Delta\varepsilon_{jj'}$ for $j \neq j'$ is the same form as Eq. (24) of the proton-neutron interaction. However, in cases of $j = j'$, only even-J values are allowed because of antisymmetrization, and an additional factor of two is multiplied compared to cases of $j \neq j'$. Further, the factor in the denominator does not coincide with the number of two-particle states allowed for the j^2 configuration, which is $j(2j+1)$ instead of $(2j+1)^2$. Because of this, the monopole component of like-nucleon interactions cannot be expressed in the same form as Eq. (26) of the proton-neutron interaction. In fact, the substitution of $\langle j^2|V_{pp}|j^2\rangle_J = \Delta\varepsilon_{jj}$ for even-J values into the right side of Eq. (30) does not reproduce $\Delta\varepsilon_{jj}$ on the left side. The inconsistency is cured, if we define

$$\Delta\varepsilon'_{jj'} = \begin{cases} \Delta\varepsilon_{jj'} & (j \neq j') \\ \dfrac{2j+1}{2j} \Delta\varepsilon_{jj} & (j = j'), \end{cases} \tag{31}$$

where, in the second case, the factor of two in the denominator compensates the same factor in the definition of $\Delta\varepsilon_{jj}$. Then, the monopole component can be written as

$$V_{pp}^{(0)} = \sum_{[jj']JM} \Delta\varepsilon'_{jj'} A^\dagger(jj'JM)A(jj'JM), \tag{32}$$

and both operators of Eqs. (29) and (32) give the same expectation value in Eq. (6).

The neutron-neutron interaction can be expanded into multipole interactions in the same form as the proton-proton interaction, just by replacing V_{pp} with V_{nn}.

C. Central, spin-orbit, and tensor interactions

The two-body interaction can be decomposed into central, spin-orbit, and tensor interactions. The decomposition procedure was briefly mentioned in Ref. [12] and in detail in Refs. [47,51]. It is based on the assumption, first, that the radial wave functions are identical for spin-orbit partners, $j = \ell \pm \frac{1}{2}$, which allows transformation from jj-coupling to LS-coupling of a product of two-particle wave functions, and, secondly, that radial potentials involved in the nuclear interaction are functions of only the relative coordinate $r = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$ of the two interacting nucleons.

The central interaction can be written as

$$\begin{aligned} V_{\rm C} = {} & V_{\rm SE}(r)\Pi_\sigma^S\Pi_\tau^T + V_{\rm TO}(r)\Pi_\sigma^T\Pi_\tau^T \\ & + V_{\rm SO}(r)\Pi_\sigma^S\Pi_\tau^S + V_{\rm TE}(r)\Pi_\sigma^T\Pi_\tau^S, \end{aligned} \tag{33}$$

where

$$\Pi_\sigma^S = \frac{1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}, \quad \Pi_\sigma^T = \frac{3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}, \tag{34}$$

$$\Pi_\tau^S = \frac{1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4}, \quad \Pi_\tau^T = \frac{3+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4} \tag{35}$$

are projection operators onto spin/isospin singlet (S) and triplet (T) parts of the two-particle wave functions. The spin-orbit and tensor interactions survive only between spin-triplet states, and they consist of two components,

$$V_{\rm LS} = V_{\rm LSE}(r)\boldsymbol{L}\cdot\boldsymbol{S}\Pi_\tau^S + V_{\rm LSO}(r)\boldsymbol{L}\cdot\boldsymbol{S}\Pi_\tau^T, \tag{36}$$

$$V_{\rm TN} = V_{\rm TNE}(r)S_{12}\Pi_\tau^S + V_{\rm TNO}(r)S_{12}\Pi_\tau^T, \tag{37}$$

where $\boldsymbol{L}$ is the orbital angular momentum of the relative motion, and $\boldsymbol{S} = (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)/2$. The tensor operator S_{12} is defined by

$$S_{12} = (\hat{\boldsymbol{r}} \cdot \boldsymbol{\sigma}_1)(\hat{\boldsymbol{r}} \cdot \boldsymbol{\sigma}_2) - \tfrac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) = [\hat{\boldsymbol{r}} \otimes \hat{\boldsymbol{r}}\,]^{(2)} \cdot [\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2\,]^{(2)}, \quad (38)$$

where $\hat{\boldsymbol{r}}$ is the unit vector along the relative coordinate $\boldsymbol{r}$.

The effective interactions which have been utilized in shell-model calculations, including the USD interaction designed for $(sd)^{A-16}$-configuration calculations, are not provided in a potential form but by two-particle matrix elements numerically. However, they can be decomposed into the components mentioned above, first, by transforming the given jj-coupling matrix elements to LS-coupling matrix elements. Then, central, spin-orbit and tensor contributions are separated by employing different spherical-tensor structure of them and Racah algebra. The central interaction involves no spin operator or in a form of $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, which are scalars, i.e., rank-0 spherical tensors. The spin-orbit interaction has the structure $V(r)\boldsymbol{L} \cdot \boldsymbol{S}$, where $V(r)\boldsymbol{L}$ and $\boldsymbol{S}$ are rank-1 spherical tensor operators in ordinary and spin spaces, respectively, and the tensor operator, Eq. (37), is, clearly from the second line of Eq. (38), the inner product of rank-2 operators.

The assumption of the same radial wave function for $j = \ell \pm \frac{1}{2}$ orbits, in the present case $d_{5/2}$ and $d_{3/2}$, and the three interactions, Eqs. (33), (36), and (37), reduces the number of degrees of freedom. In sd-shell, the central, spin-orbit and tensor interactions are described by 45 matrix elements in LS-coupling, while there are 63 independent matrix elements in jj-coupling scheme. The remaining matrix elements are those between spin-singlet and spin-triplet states, and the selection rule for spin quantum numbers is obtained by spin structure of $\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2$, which may be called an antisymmetric spin-orbit (ALS) interaction, see Ref. [28]. There are nine those matrix elements in LS-coupling in each isospin channel, $T = 0$ and $T = 1$. The appearance of this kind of interaction matrix elements can be attributed to a small difference between $d_{5/2}$ and $d_{3/2}$ radial wave functions. On the other hand, ALS components in shell-model effective interactions may be derived from a three-body force by averaging over one nucleon in the core. In total, there are ten components; four central components, two spin-orbit components, two tensor components, and two antisymmetric spin-orbit components.

IV. RESULTS AND DISCUSSION

Shell-model calculations have been performed for sd-shell nuclei. The model space consists of three single-particle orbits, $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$, on the inert ^{16}O core. The USD interaction [13,14] is adopted for the nucleon-nucleon interaction, and Coulomb interaction is neglected. The USD interaction was designed so as to reproduce binding energies and energy spectra of sd-shell nuclei in the $(sd)^n$ configurations on the ^{16}O core. It reproduces binding energies very well except for nuclei with large neutron (proton) excess.

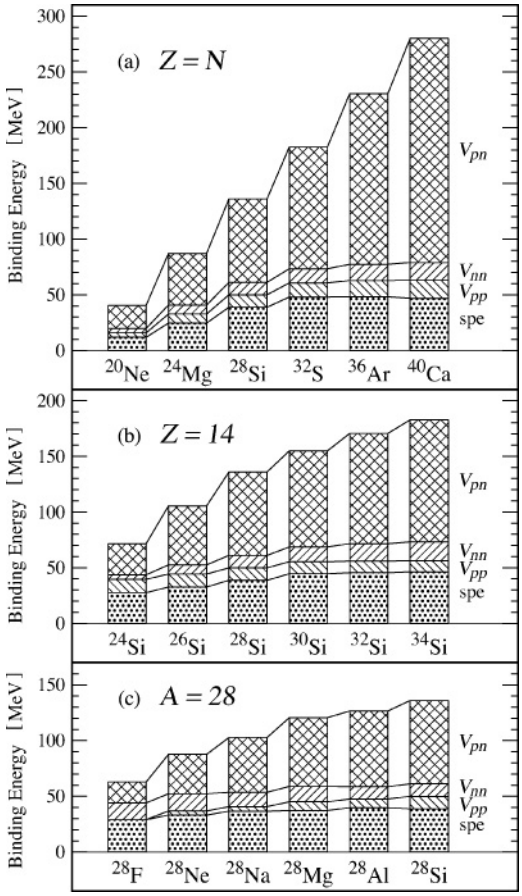

FIG. 1. The decomposition of the binding energies according to Eq. (7) for ground states of (a) even-even $Z = N$ nuclei, (b) even-N silicon isotopes, and (c) $A = 28$ isobars with $9 \leqslant Z \leqslant 14$ in the sd-shell.

A. Attraction of the *pn*-interaction

The decomposition of binding energies according to Eq. (7) is shown in Fig. 1 for the ground states of three chains of nuclei. For the $Z = N$ nuclei, the binding energy increases proportionally with the number of valence nucleons from 4(^{20}Ne) to 24(^{40}Ca). The increase is mostly due to that of the pn-interaction contribution. The proton-neutron interaction is on average much more attractive than the like-nucleon interactions. The single-particle-energy contribution does not increase with the valence nucleon number. This is because nucleons tend to occupy higher-lying orbits with the increase of mass number. For the $Z = N$ nuclei with $A - 16 = 4n$ (n = 1–6) valence nucleons, the number of pairs of protons (neutrons) is $n(2n - 1)$ and that of proton-neutron pairs is $4n^2$. However, the binding energy increases almost linearly with the mass number.

Binding energies of silicon isotopes are shown in the middle panel of Fig. 1. The linearity with mass number seems to be broken, especially on the left side of ^{28}Si. The energy gain due to the proton-proton interaction stays at an almost constant value for the isotopes with the same atomic number $Z = 14$, while that due to the neutron-neutron interaction increases with neutron number but not beyond ^{28}Si. The breaking of linearity is mainly due to the proton-neutron interaction for the nuclei

with $N - Z$ changes from -4 to 6, depending on the distance from the symmetric nucleus ^{28}Si.

Figure 1(c) shows binding energies of $A = 28$ isobars with $9 \leqslant Z \leqslant 14$. Those on neutron deficient side, $19 \geqslant Z \geqslant 14$, are symmetric with respect to the $Z = N$ nucleus ^{28}Si, since the full $(sd)^{12}$ configurations are adopted with the nuclear Hamiltonian which conserves isospin and does not include the Coulomb energy. The binding energies show a parabolic feature around the symmetric ^{28}Si, which has the largest binding energy, and this represents the symmetry energy of the semi-empirical Weizsäcker-Bethe mass formula. The parabolic shape is mostly due to that of the proton-neutron interaction. The pairing effect is also seen between even-even and odd-odd nuclei.

B. Monopole interaction

The multipole decomposition is alternatively made. The binding energies are separated into single-particle energies, the monopole component ($k = 0$) and the sum of higher multipole components ($k \neq 0$). The result for the same chains of nuclei as shown in Fig. 1 is shown in Fig. 2. The monopole interaction is the largest component of the binding energy in most of nuclei. The increase of the monopole contribution makes the binding energy larger with mass number toward the single- and double-closed shell, and the closed-shell nucleus ^{40}Ca has no contributions from higher multipoles ($k \neq 0$) of the two-body interaction. This can be understood as follows.

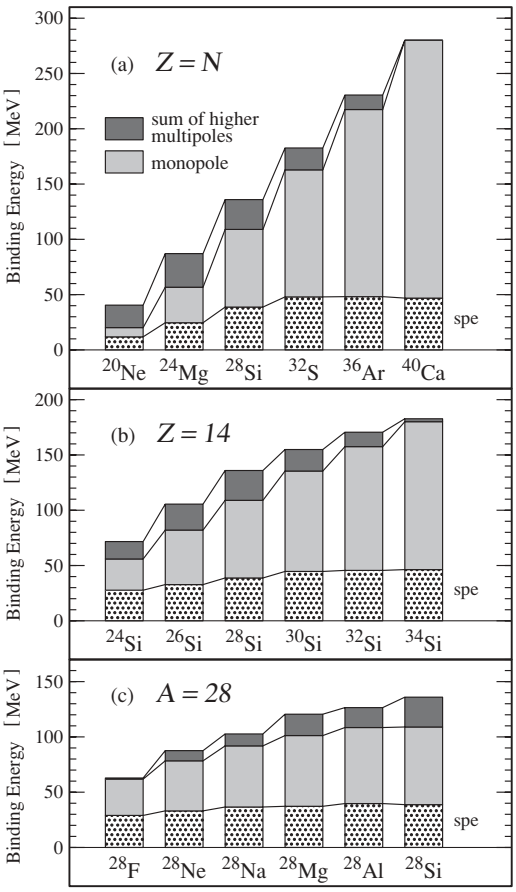

FIG. 2. The decomposition of the binding energies into contributions coming from single-particle energies, monopole interaction $k = 0$ and a sum of higher multipoles, for ground states of (a) even-even $Z = N$ nuclei, (b) even-N silicon isotopes, and (c) $A = 28$ isobars with $9 \leqslant Z \leqslant 14$ in the sd-shell.

Consider a closed-configuration state, where a proton orbit j_p and a neutron orbit j_n are fully filled, the $(2j_p + 1)$ protons are coupled to $J_p^\pi = 0^+$ and the $(2j_n + 1)$ neutrons are coupled to $J_n^\pi = 0^+$,

$$|\mathrm{cc}\rangle = \left|j_p^{2j_p+1}(0_p^+),\, j_n^{2j_n+1}(0_n^+);\, J^\pi = 0^+\right\rangle. \tag{39}$$

For the proton configuration, the matrix element of V_{pp} is given by

$$\left\langle j_p^{2j_p+1}(0_p^+)\middle|V_{pp}\middle|j_p^{2j_p+1}(0_p^+)\right\rangle = \sum_J (2J+1)\left\langle j_p^2\middle|V_{pp}\middle|j_p^2\right\rangle_J, \tag{40}$$

which is equivalent to the expectation value of the monopole component of V_{pp}. The matrix element of V_{nn} for the neutron configuration is written in the same form. Further, of the proton-neutron interaction, only the monopole component $V_{pn}^{(0)}$ survives, since both protons and neutrons are coupled to angular momentum zero, respectively. Therefore, the expectation value of the monopole components,

$$V^{(0)} = V_{pp}^{(0)} + V_{nn}^{(0)} + V_{pn}^{(0)}, \tag{41}$$

for the closed-configuration state is written, by substituting expectation values of number operators in Eq. (29), as

$$\begin{aligned}\langle \mathrm{cc}\,|V^{(0)}|\mathrm{cc}\rangle = {} & \tfrac{1}{2}(2j_p+1)^2\Delta\varepsilon_{j_pj_p} + \tfrac{1}{2}(2j_n+1)^2\Delta\varepsilon_{j_nj_n} \\ & + (2j_p+1)(2j_n+1)\Delta\varepsilon_{j_pj_n}.\end{aligned} \tag{42}$$

When the j_p-orbit is occupied by $(2j_p + 1)$ protons but the j_n-orbit is partly filled, the pp- and pn-interactions contribute only through the monopole components, $V_{pp}^{(0)}$ and $V_{pn}^{(0)}$. Even if both single-particle orbits have some vacancy, there arises a considerable energy gain from the monopole components.

It is, however, noted that the monopole component survives only between the same shell-model basis states, the diagonal elements of the Hamiltonian matrix, since the basis states are eigenstates of the number operators. The monopole interaction, therefore, does not induce configuration mixing. The latter is caused by higher multipoles. Large energy gains due to the higher multipoles appear in the symmetric nuclei on the $Z = N$ line, and the largest value is obtained for ^{24}Mg. This will be discussed in some detail in the subsection of quadrupole interaction, Sec. IV E.

The core nucleus ^{16}O of the sd-shell calculation consists of closed $0s_{1/2}$ and $0p_{3/2}$-$0p_{1/2}$ shells. The observed binding energy of ^{16}O, 127.619 MeV, includes the kinetic energies of the sixteen nucleons and the Coulomb energy. Here, we estimate the binding energy due to the nuclear force, according to an extensive Hartree-Fock + BCS calculation [4]. They calculated not only binding energies but also kinetic and Coulomb energies separately for a number of nuclei over the nuclear chart, adopting the Skyrme SIII force and the three-dimensional Cartesian-mesh representation which allows any type of deformation. For the ^{16}O nucleus, the kinetic energy amounts to 114.643 MeV (proton) + 116.576 MeV

(neutron) = 231.219 MeV, and the Coulomb energy to 16.848 MeV, both contributing in negative sign to the binding energy. The binding energy is calculated, with the correction for the finite mesh size, to be 128.099 MeV, which should be compared with the experimental value. Based on these values, the binding energy of ^{16}O due to the nuclear force is estimated to be 376 MeV. This is much larger than the observed binding energy, and it is clear from the discussion given above that the energy gain is exclusively due to the monopole components. In addition, the single-particle energies with respect to the core, $\varepsilon_j^{\rm core}$, consist of kinetic energy and two-body interaction between the valence nucleon and the core nucleons. Since the single-particle orbits in the core are fully occupied, again only the monopole components contribute to $\varepsilon_j^{\rm core}$. Taking into account the core and single-particle energies with respect to the core, we conclude that binding energies of nuclei are dominated, much more than seen from Fig. 2, by the attraction of the monopole components of the two-body interaction, with a small additional gain due to configuration mixing by the higher multipoles.

C. Single-particle energies

The single-particle energy ε_j of a nucleus is defined, according to French [31] and Baranger [61], by spectroscopic strength distributions of one-nucleon stripping and pick-up reactions, and this leads to an expression with a anticommutator and a commutator as

$$\varepsilon_j = \frac{1}{2j+1} \sum_{m=-j}^{j} \langle J_\nu^\pi | \{[a_{jm}, H], a_{jm}^\dagger\} | J_\nu^\pi \rangle. \tag{43}$$

Substituting the nuclear Hamiltonian, single-particle energies are written [47,48,51] as

$$\varepsilon_j = \varepsilon_j^{\rm core} + \varepsilon_j^{\rm val}, \tag{44}$$

where the second term $\varepsilon_j^{\rm val}$ represents the interaction between the valence nucleons, and they are given by

$$\varepsilon_{j_p}^{\rm val} = \langle J_\nu^\pi | \left(\sum_{j_p'} \Delta\varepsilon_{j_p j_p'} \hat{N}_{j_p'} + \sum_{j_n} \Delta\varepsilon_{j_p j_n} \hat{N}_{j_n} \right) | J_\nu^\pi \rangle,$$
$$\varepsilon_{j_n}^{\rm val} = \langle J_\nu^\pi | \left(\sum_{j_n'} \Delta\varepsilon_{j_n j_n'} \hat{N}_{j_n'} + \sum_{j_p} \Delta\varepsilon_{j_p j_n} \hat{N}_{j_p} \right) | J_\nu^\pi \rangle, \tag{45}$$

for proton and neutron single-particle orbits, respectively. Here, $\Delta\varepsilon_{j_p j_p'}$ and $\Delta\varepsilon_{j_n j_n'}$ are defined by Eq. (30), and $\Delta\varepsilon_{j_p j_n}$ by Eq. (24). The single-particle energies, ε_j, are state dependent, since the single-particle energies are defined with the expectation values of number operator.

The single-particle energy has an intimate relation to the monopole components of two-body interaction. For the simple closed-configuration state of Eq. (39), the expectation value of the monopole components, Eq. (42), can be rewritten as

$$\begin{aligned} 2\langle {\rm cc}|V^{(0)}|{\rm cc}\rangle &= (2j_p+1)\big((2j_p+1)\Delta\varepsilon_{j_p j_p} + (2j_n+1)\Delta\varepsilon_{j_p j_n}\big) \\ &\quad + (2j_n+1)\big((2j_n+1)\Delta\varepsilon_{j_n j_n} + (2j_p+1)\Delta\varepsilon_{j_p j_n}\big) \\ &= (2j_p+1)\varepsilon_{j_p}^{\rm val} + (2j_n+1)\varepsilon_{j_n}^{\rm val} \end{aligned} \tag{46}$$

which is identical to the expectation value of the single-particle energies due to the valence-nucleon interaction. For the present $(sd)^{A-16}$-model, we define here the ratio of the sum of single-particle energies (those with respect to the core are subtracted) to the monopole contribution multiplied by a factor of 2,

$$R = \frac{\langle J_\nu^\pi | \sum_j \varepsilon_j^{\rm val} \hat{N}_j | J_\nu^\pi \rangle}{2\langle J_\nu^\pi | V^{(0)} | J_\nu^\pi \rangle}, \tag{47}$$

where the sum in the numerator is taken over all valence orbits, and $V^{(0)}$ represents the sum of monopole components of the *pp*-, *nn*- and *pn*-interactions, Eq. (41). The $\varepsilon_j^{\rm val}$'s already contain the expectation value of the number operator, see Eq. (45). On the other hand, the monopole components, Eqs. (23) and (29), are written by $\Delta\varepsilon_{jj'}$ and number operator in linear and quadratic forms. The ratio is exactly $R = 1$ for the closed-shell nucleus ^{40}Ca, where the following equality for expectation values of number operator holds:

$$\langle J_\nu^\pi | \hat{N}_j \hat{N}_{j'} | J_\nu^\pi \rangle = \langle J_\nu^\pi | \hat{N}_j | J_\nu^\pi \rangle \langle J_\nu^\pi | \hat{N}_{j'} | J_\nu^\pi \rangle. \tag{48}$$

It is noted that shell-model basis states which we usually take are eigenstates of the number operator of an orbit, $\hat{N}_j$, but an energy eigenstate $|J_\nu^\pi\rangle$ is not. In other words, the number operator $\hat{N}_j$ can have matrix elements between different energy eigenstates. The equality, Eq. (48), is thus violated for open-shell nuclei. But the violation is generally very small. For example, the ratios are evaluated for $Z = N$ nuclei to be $R = 1.035$ (^{20}Ne), 0.994 (^{24}Mg), 1.009 (^{28}Si), 1.004 (^{32}S), and 1.000 (^{36}Ar). The approximate equality is a result of the fact that the equation obtained by the sum over j' on both sides of Eq. (48) is always satisfied for any single-particle orbit j. The tiny deviations come from differences of $\Delta\varepsilon_{jj'}$ which are multiplied before summing over j' when we calculate single-particle energies ε_j. We thus conclude that the monopole components of the two-body interaction are almost completely renormalized into the single-particle energies.

The result for the closed-shell nucleus should be compared with the energy expectation value of the Hartree-Fock calculation [65],

$$E^{\rm HF} = \sum_i \varepsilon_i - \sum_{i<j} v_{ij,ij}. \tag{49}$$

The Hartree-Fock method assumes that the nuclear wave function is described by a single determinant of single-particle wave functions. This corresponds to the closed-shell which is expressed by a single basis state with no configuration mixing. The above expression states that the energy expectation value is not the sum of single-particle energies. The second term is the sum of two-particle matrix elements, and it is the monopole components of the two-body interaction. In the shell model, the monopole value is a half of the sum of single-particle energies, and the energy of the closed-shell is equal to the monopole expectation value.

The single-particle energies are calculated not only with the USD interaction [13,14] but also with new USDA, USDB and renormalized *G*-matrix (RGSD) interactions [62]. The new USDA and USDB interactions give the single-particle energies very close to those of the USD interaction, generally within 0.2 MeV with a few exceptionally large deviations

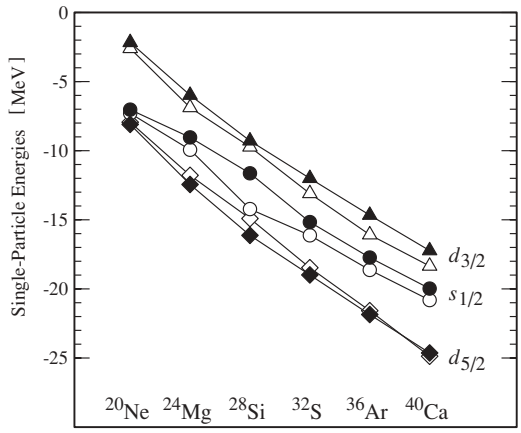

FIG. 3. Single-particle energies of the three orbits in sd-shell for 0^+ ground states of even-even $Z = N$ nuclei, calculated with the USD interaction (filled) and the renormalized G-matrix interaction (open).

of about 0.4 MeV. However, the calculated binding energies of the closed-shell nucleus ^{40}Ca are in good agreement with one another, $B(^{40}\text{Ca}) = 280.102$ MeV (USD), 280.183 MeV (USDA), and 279.961 MeV (USDB). On the other hand, the RGSD interaction yields a binding energy of $B(^{40}\text{Ca}) = 287.659$ MeV with the single-particle energies with respect to the core from the USD interaction. Such a large discrepancy is naturally due to those of monopole components, and the calculated single-particle energies for the ground states of $Z = N$ nuclei are compared in Fig. 3. In the $Z = N$ nuclei, the single-particle spectrum of protons is identical to that of neutrons in the present calculation with the Hamiltonian which conserves the isospin. The RGSD interaction gives single-particle energies of $d_{3/2}$ and $s_{1/2}$ orbits systematically smaller than those of the USD interaction, especially for the $s_{1/2}$ orbit at the middle of sd-shell. The degeneracy of the $d_{5/2}$ and $s_{1/2}$ orbits at ^{28}Si would cause a large amount of configuration mixing in the two orbits, in a contrast to the empirical USD, USDA and USDB interactions which yield a large gap of about 5 MeV between them. The degeneracy can be traced back to a strong attraction between $d_{5/2}$ and $s_{1/2}$ of the RGSD proton-neutron interaction, $\Delta\varepsilon_{d_{5/2}s_{1/2}} = -0.352$ MeV, compared to the corresponding value $\Delta\varepsilon_{d_{5/2}s_{1/2}} = +0.104$ MeV of the USD interaction. The monopole strength $\Delta\varepsilon_{d_{5/2}d_{3/2}}$ is also more attractive in the RGSD interaction by about 0.3 MeV than the USD interaction.

It is a problem to be resolved that effective interaction theories, such as G-matrix method, do not predict correct monopole components of two-body interaction. This is the reason why the monopole shifts have been introduced to the G-matrix effective interaction to define the KB3 interaction in the pf shell, and, on the other hand, empirical effective interactions have been utilized if available. One of the alternatives of G-matrix is the unitary-model-operator approach (UMOA), where much effort has been paid for an accurate numerical calculation of the short-range repulsion of the bare nucleon-nucleon interaction [63] and the Pauli exclusion operator has been treated exactly [64]. An improvement has been achieved in agreement with experimental data of single-particle energies, but there remain some deviations. The origin of the discrepancies in the monopole interactions might be found in three-body forces [39]. The problem becomes more serious when we go to heavier mass region, where empirical effective interactions are not available and we have to rely on predictions of effective interaction theories.

D. Triplet-even channel attraction

The binding energies due to the monopole interaction are decomposed into ten components, i.e., four central components (SE, TO, SO, and TE), two spin-orbit components (LSO and LSE), two tensor components (TNO and TNE), and two antisymmetric LS components (ALSO and ALSE). The decompositions for the three chains of nuclei are shown in Fig. 4. The contributions of non-central interactions are summed, and the sum is shown on the top for each nucleus. The monopole interaction is dominated by the central components. The even-channel contributions are positive and the odd-channel contributions are negative. This result reflects the basic feature common to realistic bare NN interactions, even-channel potentials are attractive and odd-channel potentials are repulsive. The SE potential is the most attractive in the bare

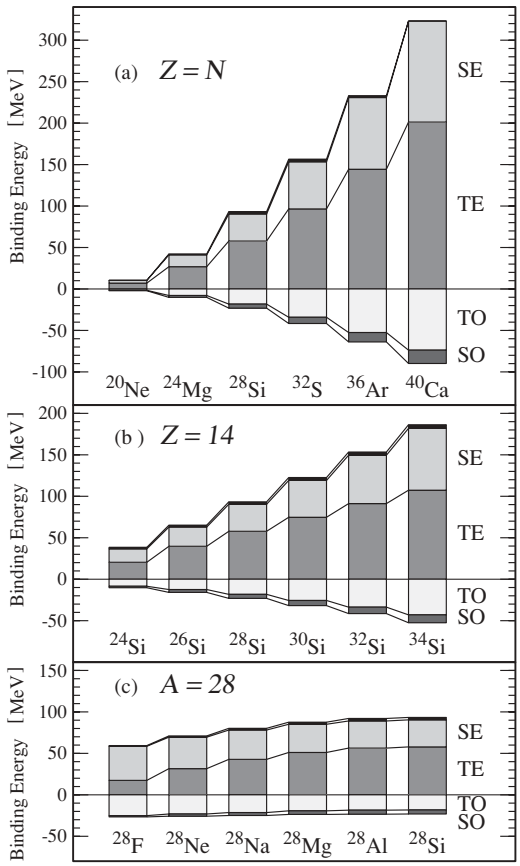

FIG. 4. The decomposition of the monopole interaction $k = 0$ into the four components of the central interaction (TE, SE, TO, and SO) and a sum of noncentral interactions, calculated for ground states of (a) even-even $Z = N$ nuclei, (b) even-N silicon isotopes, and (c) $A = 28$ isobars with $9 \leqslant Z \leqslant 14$ in the sd-shell.

interactions, but the TE component shares the largest fraction of the binding energies calculated with the effective USD interaction. This is due to the second-order tensor correlations that, in the TE channel, the tensor interaction allows coupling such as between $^3S_1$ and $^3D_1$, and this enhances the attraction of the TE central part of the empirical interaction. Figure 4 further indicates that the repulsive TO component cancels the attractive SE component to a large extent, and the net binding due to the $T = 1$ interaction remains to be small. On the other hand, the repulsion of the SO component is much smaller than the attraction of the TE component. The SO interaction has a small $(2S + 1)(2T + 1)$ weighting compared with the TO interaction. In the bare G matrix, the SO interaction is actually quite strongly repulsive while the TO interaction is much weaker. In the renormalized effective interaction or the empirical effective interaction, the TO interaction becomes more repulsive. The binding energies are, therefore, due mainly to $T = 0$ central interaction, in particular, to the attraction in the triplet-even channel. This has been shown numerically in Refs. [47,48,51] for the effective interactions in the light-mass region.

Recently, it has been pointed out that the tensor force plays an important role as the second mechanism of shell evolution. This is the first order effect of the tensor interaction. In the sd-shell, the tensor force has no effect on the $s_{1/2}$ orbit, and the monopole strengths due to the tensor component for the spin-orbit partners, $d_{5/2}$ and $d_{3/2}$, satisfy the following equation:

$$\begin{aligned} 6 \cdot 4 \Delta\varepsilon_{d_{5/2}d_{3/2}}(\mathrm{TN}) &= -6^2 \Delta\varepsilon_{d_{5/2}d_{5/2}}(\mathrm{TN}) \\ &= -4^2 \Delta\varepsilon_{d_{3/2}d_{3/2}}(\mathrm{TN}). \end{aligned} \quad (50)$$

With the USD interaction, the value of the equation for the triplet-odd tensor component of the like-nucleon interaction is

$$6 \cdot 4\, \Delta\varepsilon_{d_{5/2}d_{3/2}}(\mathrm{TNO}) = -2.110\ \mathrm{MeV},$$

and those for the triplet-odd ($T = 1$) and triplet-even ($T = 0$) components are

$$\begin{aligned} 6 \cdot 4\, \Delta\varepsilon_{d_{5/2}d_{3/2}}(\mathrm{TNO}) &= -1.055\ \mathrm{MeV}, \\ 6 \cdot 4\, \Delta\varepsilon_{d_{5/2}d_{3/2}}(\mathrm{TNE}) &= -3.589\ \mathrm{MeV}, \end{aligned}$$

respectively, for the proton-neutron interaction. When a tensor force is associated with the isospin product $\tau_1 \cdot \tau_2$, such as the one-pion exchange potential, the ratio of $T = 0$ to $T = 1$ strengths is restricted to three. In the empirical USD interaction, the ratio is larger than three, indicating a stronger $T = 0$ (triplet-even) tensor component.

Single-particle energies in the sd-shell are calculated with and without the tensor components, while the wave functions of energy eigenstates are constructed with the full Hamiltonian. The calculated results are shown in Fig. 5. In the $Z = N$ nuclei in going from the beginning to the middle of the shell, where nucleons are filling mainly the $d_{5/2}$ orbit, the tensor force slightly reduces the binding energy of the $d_{5/2}$ orbit while it raises the $d_{3/2}$ orbit. This is because of the repulsion of the tensor force between $d_{5/2}$ nucleons, $\Delta\varepsilon_{d_{5/2}d_{5/2}}(\mathrm{TN}) > 0$, but it is attractive between the spin-orbit partners, $\Delta\varepsilon_{d_{5/2}d_{3/2}}(\mathrm{TN}) < 0$. From the middle of the shell, nucleons fill $s_{1/2}$ and $d_{3/2}$ orbits, the contributions of the tensor force are reduced, and they finally vanish in the closed-shell nucleus ^{40}Ca because of

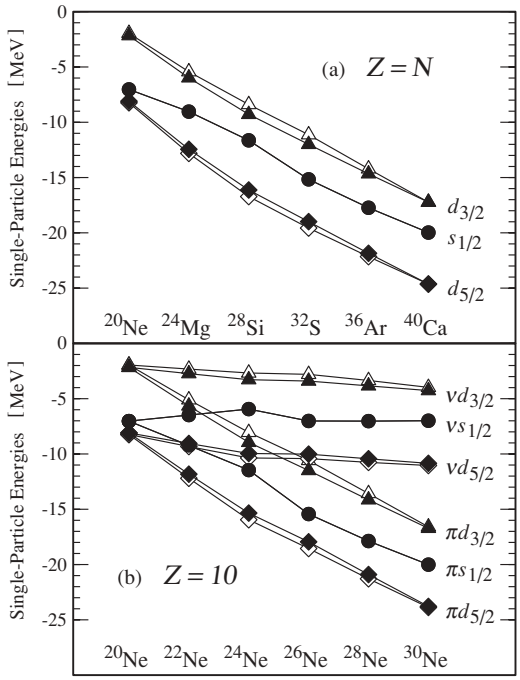

FIG. 5. Single-particle energies of the three orbits in sd-shell for ground states of (a) $Z = N$ nuclei and (b) Ne isotopes, calculated with (filled) and without (open) the tensor interaction.

Eq. (50), which can be rewritten as

$$\begin{aligned} 6 \cdot 4\, \Delta\varepsilon_{d_{5/2}d_{3/2}}(\mathrm{TN}) + 6^2\, \Delta\varepsilon_{d_{5/2}d_{5/2}}(\mathrm{TN}) = 0, \\ 6 \cdot 4\, \Delta\varepsilon_{d_{5/2}d_{3/2}}(\mathrm{TN}) + 4^2\, \Delta\varepsilon_{d_{3/2}d_{3/2}}(\mathrm{TN}) = 0, \end{aligned}$$

where effects of the tensor force are completely canceled out. A similar trend can be seen also in Ne isotopes, as shown in the lower panel of Fig. 5. In ^{30}Ne, the tensor force has no effect on the proton orbits because of the closure of the neutron shell and the complete cancellation shown above, whereas a small effect remains in the neutron orbits since the nucleus has only two protons on the ^{16}O core.

E. Quadrupole interaction

Now, we discuss contributions of higher multipoles, which are displayed by black bars in Fig. 2. Though the contribution shares a small fraction of the total binding energy, it is important for open-shell nuclei, such as ^{20}Ne and ^{24}Mg, since it is a result of configuration mixing, which is not obtained by the monopole interaction.

We have decomposed them into multipoles with $k > 0$ for the proton-neutron interaction, while for the proton-proton and neutron-neutron interactions into $\langle j_1 j_2 | V_{pp/nn} | j_1' j_2' \rangle_J$ with different J-values. It is found that the magnitudes of higher-multipole contributions are correlated with the quadrupole component $(k = 2)$ of the proton-neutron interaction. Figure 6 shows the calculated result for even-even nuclei in sd-shell. There is a tendency that the energy gain is large in the $Z = N$ nuclei, especially in the first half of sd-shell, and becomes smaller as the neutron number becomes closer to the neutron magic number $N = 20$. In each nucleus,

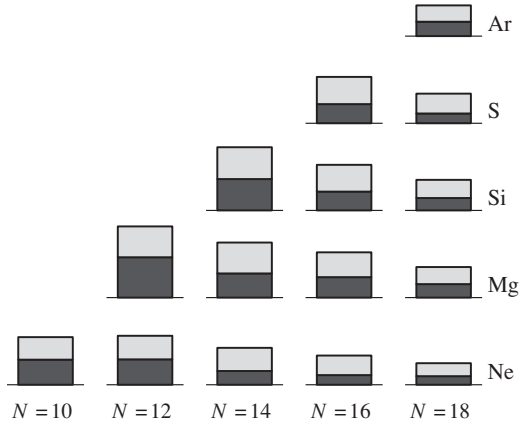

FIG. 6. The energy gain due to the higher multipoles ($k \neq 0$), calculated for 0^+ ground states of even-even nuclei with $10 \leqslant Z \leqslant N \leqslant 18$ in the *sd* shell. The largest energy gain is 30.4 MeV for ^{24}Mg. The black bar corresponds to the quadrupole ($k = 2$) component of the proton-neutron interaction.

approximately a half of the energy gain is caused by the quadrupole interaction of the proton-neutron interaction, that is, there are large effects of the quadrupole interaction in the symmetric nuclei.

A schematic quadrupole interaction is the well known $Q \cdot Q$ force,

$$V^{(2)}_{pn,\mathrm{sch}} = Q_p \cdot Q_n, \tag{51}$$

where Q_p and Q_n are quadrupole operators acting on proton and neutron systems, respectively, and it enhances deformation of the nucleus.

Wave functions of 0^+ states can be expressed, symbolically, as

$$|0^+\rangle = \sum_J \beta_J |J^+_p \otimes J^+_n\rangle, \tag{52}$$

where $|J^+_p \otimes J^+_n\rangle$ represents wave-function components in which both proton system and neutron system have the same value of angular momentum J, and β_J corresponds to their amplitudes. Figure 7 shows the decomposition of 0^+ ground-state wave functions, where probabilities of $|0^+_p \otimes 0^+_n\rangle$ and $|2^+_p \otimes 2^+_n\rangle$ shell-model basis states are separately plotted. A strong correlation is observed, by comparing Figs. 6 and 7, that the nuclei with a large energy gain in Fig. 6 has a large $|2^+_p \otimes 2^+_n\rangle$ fraction. This is well understood by the matrix element of the schematic $Q \cdot Q$ force,

$$\begin{aligned}&\langle 2^+_p \otimes 2^+_n | Q_p \cdot Q_n | 0^+_p \otimes 0^+_n\rangle \\ &\quad = \frac{1}{\sqrt{5}} \langle 2^+_p \| Q_p \| 0^+_p\rangle \langle 2^+_n \| Q_n \| 0^+_n\rangle.\end{aligned} \tag{53}$$

First, a large mixture of the $|2^+_p \otimes 2^+_n\rangle$ component induces a large energy gain by configuration mixing. Secondly, the magnitude of the matrix element is determined by quadrupole collectivity. The latter is most enhanced when the proton and neutron systems collaborate to deform, namely, in symmetric $Z = N$ nuclei. As the neutron number tends to the closed shell, $N = 20$, the neutron system becomes spherical and prevents the proton system from deforming.

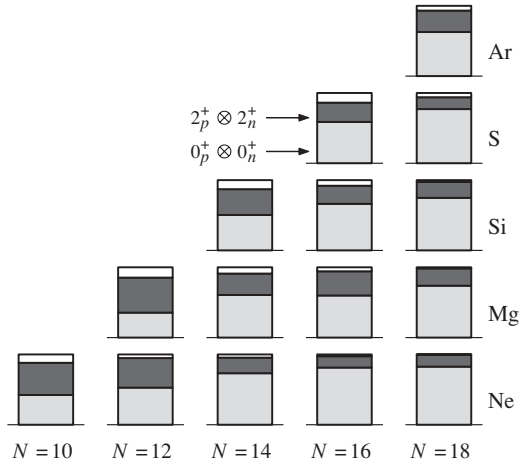

FIG. 7. Decomposition of the wave functions of 0^+ ground states of even-even nuclei with $10 \leqslant Z \leqslant N \leqslant 18$.

The pairing interaction, i.e., the like-nucleon interaction with $J = 0$, gives always a positive gain of binding energy. But, we cannot say that it dominates the rest of higher multipole contributions, after subtracting the quadrupole proton-neutron interaction. Higher J-value components of the like-nucleon interaction sometimes appear in negative sign, and then there is considerable cancellation between $J = 0$ and higher-J contributions.

V. CONCLUSION

We have calculated binding energies of *sd*-shell nuclei with the USD interaction, which is known to reproduce experimental binding energies very well, by employing three types of decompositions of two-body interaction.

In the first decomposition, the present calculation demonstrates numerically the well known fact that, on average, the strongly attractive proton-neutron interaction is much more important for binding energies than like-nucleon interactions which are weakly attractive or sometimes repulsive.

The multipole expansion clearly defines the monopole interaction as the $k = 0$ component, which has been discussed in a number of previous papers, and it is shown that the monopole strengths $\Delta\varepsilon_{jj'}$ are exactly the same quantity which appears in the definition of single-particle energies which arises interaction between valence nucleons. The binding energies are dominated by the monopole interaction. The ratio R, defined by Eq. (47), is found usually to be very close to unity, and this means that the monopole interaction can be represented by one-body single-particle energies, and higher multipoles are then residual interactions which cause configuration mixing. Among the higher multipoles, the quadrupole component of the proton-neutron interaction is responsible for a considerable gain of binding energy in some nuclei, by coherent mixing of various configurations which describes deformation.

The spin-tensor decomposition has shown the importance of the attraction of the central interaction in the triplet-even

channel with isospin $T = 0$. In the $T = 1$ channels, the singlet-even attraction is canceled to a large extent by the triplet-odd repulsion, leading to a much smaller contribution to binding energies. As part of the monopole interaction, effects of the tensor force are found in the behaviors of single-particle energies, which is recently pointed out as the second mechanism of shell structure evolution.

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