

Strongly paired fermions: Cold atoms and neutron matter

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Experiments with cold Fermi atoms can be tuned to probe strongly-interacting fluids that are very similar to the low-density neutron matter found in the crusts of neutron stars. In contrast to traditional superfluids and superconductors, matter in this regime is very strongly paired, with gaps of the order of the Fermi energy. We compute the $T = 0$ equation of state and pairing gap for cold atoms and low-density neutron matter as a function of the Fermi momentum times the scattering length. Results of quantum Monte Carlo calculations show that the equations of state are very similar. The neutron matter pairing gap at low densities is found to be very large but, except at the smallest densities, significantly suppressed relative to cold atoms because of the finite effective range in the neutron-neutron interaction.

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Strongly-paired fermions are important in many contexts: cold Fermi atom experiments, low-density neutron matter, and QCD at the very high baryon densities potentially found in the center of massive neutron stars. Developing a quantitative understanding of strongly-paired Fermi systems is important since they offer a unique regime for quantum many-body physics, relevant in very different physical settings including the structure and cooling of neutron stars. Constraining neutron matter properties can also be important in understanding the exterior of neutron-rich nuclei by constraining parameters of nuclear density functionals.

Cold-atom experiments can provide direct tests of the equation of state and the pairing gap in the strongly-paired regime, and hence provide a crucial benchmark of many-body theories in these systems. We consider a system of two fermion species and a simple Hamiltonian of the form

$$H = -\frac{\hbar^2}{2m} \sum_i \nabla_i^2 + \sum_{i<j} v(r_{ij}), \quad (1)$$

where i and j represent spin up and down particles, respectively. In cold atoms the interaction $v(r)$ can be tuned through Feshbach resonances to be very attractive, and to produce a specific scattering length. By varying the scattering length one can sweep through various values of Fermi momentum k_F times scattering length a from the BCS side (small $-k_F a$) to unitarity ($-k_F a = \infty$) and beyond. In many experiments ⁶Li atoms are used, and the effective range r_e between the atoms is nearly zero. A variety of beautiful experiments have been performed recently on cold atoms in the strongly paired regime [1–10].

The neutron-neutron interaction, in contrast, is generally quite complicated, with one-pion exchange at large distances, intermediate range spin-dependent attraction dominated by two-pion exchange, and a short-range repulsion. At very low densities, though, as found in neutron star crusts or the exterior of neutron-rich nuclei, the scattering length and effective range are most crucial to the physical properties of the system. The presence of a short-range repulsive core is important primarily in that it prevents collapse to a higher-density state.

A strongly attractive zero-range interaction was proposed as a simple model of neutron matter even before the recent remarkable cold atom experiments [11]. In low-density neutron matter the scattering length is very large, ≈ -18.5 fm, much larger than the typical separation between neutron pairs. The effective range is much smaller than the scattering length, $r_e \approx 2.7$ fm, so $|r_e/a| \approx 0.15$, but only at very low densities is the effective range much smaller than the interparticle spacing.

To the extent that the effects of finite range in the interaction can be neglected, cold atoms and neutron matter are ‘universal’ in the sense that the properties of the system depend only upon the product of the Fermi momentum and the scattering length. Experiments have been performed that probe the sound velocity [1] and collective excitations [2], superfluidity [3] and critical temperature [4], phase separation and phase diagram [5–9] and RF response [10].

We have performed fixed-node quantum Monte Carlo (QMC) calculations for both cold atoms and neutron matter. In each case, the trial wave function is taken to be of the Jastrow-BCS form with fixed particle number and periodic boundary conditions:

$$\Psi_T = \left[\prod_{i<j} f(r_{ij}) \right] \mathcal{A} \left[\prod \phi(r_{ij}) \right]. \quad (2)$$

The BCS pairing function $\phi(r)$ is parametrized with a short- and long-range part as in [12]. Since the ground-state energy in a fixed-node calculation is an upper bound to the true ground state energy, we optimize the parameters to obtain the lowest fixed-node ground-state energy, as in [12].

The interaction for cold atoms is taken as $v(r) = -v_0 \frac{2\hbar^2}{m} \frac{\mu^2}{\cosh^2(\mu r)}$, with $\mu = 24/r_0$, or an effective range of $r_0/12$, with $1/\rho = (4/3)\pi r_0^3$. The interaction range is small enough not to significantly affect the energy or pairing gap from the BCS regime to unitarity. For the neutron-neutron interaction, we take the s-wave part of the AV18 [13] interaction. This interaction fits nucleon-nucleon scattering very well at both low- and high-energies. For our purposes the important thing is that the scattering length and effective range are correctly

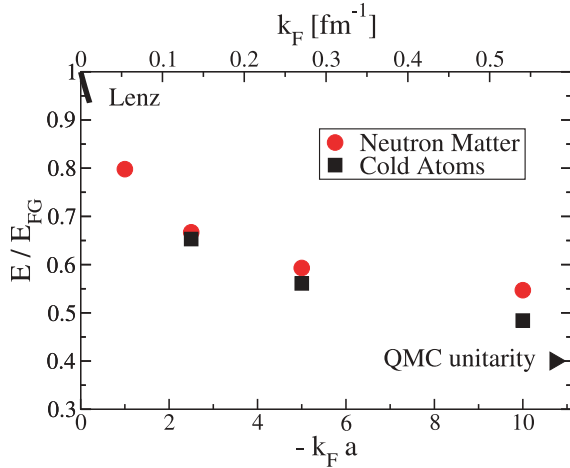


FIG. 1. (Color online) Zero-temperature equation of state for cold atoms and neutron matter. Near zero density we show the analytical expansion of the ground-state energy of a normal fluid, and at high density we show the cold atom result at unitarity ($k_F a = \infty$, arrow). QMC calculations are shown as circles and squares for neutron matter and cold atoms, respectively.

described. We use this interaction only between spin up-down pairs, which sets the interaction in the $L = 1$, $M = \pm 1$ pairs to zero. We correct for the artificial attraction in the $L = 1$, $M = 0$ pairs perturbatively. This correction is 10% for the ground-state energy at the largest densities considered, and typically much smaller. The correction to the pairing gap is always smaller than the statistical error in the calculation.

The $T = 0$ equations of state for cold atoms and neutron matter are compared in Fig. 1. The horizontal axis is $k_F a$, with the equivalent Fermi momentum k_F for neutron matter shown along the top. The vertical axis is the ratio of the ground-state energy to the free Fermi gas energy (E_{FG}) at the same density; it must go to one at very low densities and decrease as the density increases and the interactions become important. The curve at lower densities shows the analytical result [14] for normal matter: $E/E_{FG} = 1 + \frac{10}{9\pi} a k_F + \frac{4}{21\pi^2} (11 - 2 \ln 2) (a k_F)^2$. This curve should be valid at very low densities. While it ignores the contributions of superfluidity, these are exponentially small in $(1/k_F a)$.

The neutron matter and cold atom equations of state are very similar even for densities where the effective range is comparable to the interparticle spacing. Hence cold atom experiments can tell us something rather directly about the neutron matter equation of state. Near $k_F a = -10$ the energy per particle is not too far from QMC calculations [12,15] and measurements [16] of the ratio ξ of the unitary gas energy to E_{FG} ; previous calculations give $\xi = 0.42(1)$. Extrapolations of recent QMC calculations to $r_e = 0$ and also AFMC calculations suggest that $\xi = 0.40(1)$ [17] (arrow in Fig. 1).

The results near $k_F a = -10$ for neutron matter are compatible with previous calculations of the neutron matter equation of state at somewhat higher densities ($k_F \geq 1 \text{ fm}^{-1}$) [18–20]. Results shown are for 66 particles in periodic boundary conditions; calculations have also been performed near $N = 20, 44$, and 90. Based on these results, finite-size effects for $N = 66$ and beyond are expected to be quite small, of the order

of a couple percent. Calculations of the cold atom equation of state are very similar to those reported previously in [15] and [21]; the energies reported here are slightly lower (up to $\approx 10\%$ in some cases) because of larger system sizes and better optimizations.

Realistic microscopic calculations that incorporate strong pairing thus provide important constraints on the neutron matter equation of state in the subnuclear saturation density regime. Skyrme models or more generally density functionals are used, for example, to determine the structure of neutron star crusts [22] and the neutron skin thickness of nuclei [23]. A realistic treatment of these problems should incorporate the physics of the rapid transition of neutron matter from nearly free particles to a strongly-paired system at very low densities.

The pairing gap is the other fundamental zero-temperature property of superfluid systems. Calculations of the s -wave pairing gap in neutron matter have varied enormously over the past 20 years [24,25]. The difficulties in accurately calculating corrections to the BCS pairing gaps in the strongly-paired regime are significant, and hence calculations of the pairing gap [24–30] can differ by large factors (from 4 to 10) in the low-density regime. Cold atom experiments can provide a critical test of theories of the pairing gap in this regime. We first compare our calculations of the pairing gap in cold atoms and neutron matter, and then compare with previous results.

We calculate the pairing gap from the odd-even energy staggering $\Delta = E(N+1) - (E(N) + E(N+2))/2$, where N is an even number of particles. Finite-size effects for the pairing gap are considerably larger than for the ground state energy. In order to estimate the convergence of the gap to the continuum value with increasing N we have solved the BCS equations:

$$\Delta(\mathbf{k}) = - \sum_{\mathbf{k}'} \langle \mathbf{k} | V | \mathbf{k}' \rangle \frac{\Delta(\mathbf{k}')}{2\sqrt{\epsilon(\mathbf{k}')^2 + \Delta(\mathbf{k}')^2}}, \quad (3)$$

$$\langle N \rangle = \sum_{\mathbf{k}} \left[1 - \frac{\epsilon(\mathbf{k})}{\sqrt{\epsilon(\mathbf{k})^2 + \Delta(\mathbf{k})^2}} \right]$$

in periodic boundary conditions for different $\langle N \rangle$.

The line in Fig. 2 is the continuum BCS result for $k_F a = -10$, and the open symbols are the solutions of the BCS equations for different $\langle N \rangle$. The continuum results are nearly identical for the AV18 interaction and the simple cosh potential adjusted to yield the same scattering length and effective range. For the finite systems BCS results are shown for the cosh potential. Unlike the case of cold atoms near unitarity, where $-k_F a \gg 1$ and $r_e \approx 0$, the BCS gap shows sizable oscillations for small numbers of particles. The BCS value approaches the continuum limit (straight line) near $N = 66$, and oscillations from that point on are fairly small, comparable in size to the statistical error in the QMC calculations. We also show as solid points the gaps obtained from particle-projected BCS wave functions in variational Monte Carlo calculations and the odd-even staggering formula. The projection to definite particle number is a small effect.

The lower points in Fig. 2 are QMC results for $k_F a = -10$. At very small values of N the gap is quite large, as is also seen

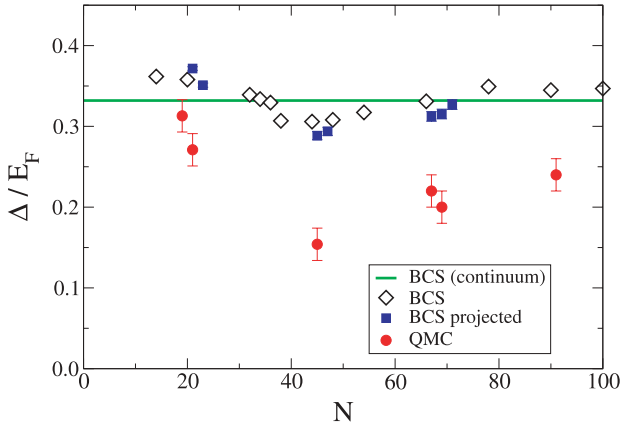


FIG. 2. (Color online) Neutron matter pairing gap at $k_F a = -10$ versus particle number in periodic boundary conditions, BCS and QMC calculations.

in the BCS calculations. This is due to the coarse description of the Fermi surface in such small systems; the momentum grid spacing in occupied states is similar in magnitude to the Fermi momentum. When the pairing is very strong, as in cold atoms in the unitary regime, this coarse description is not too critical. However for weaker coupling or the larger effective range in neutron matter this becomes more important. The gap in both BCS and QMC calculations reaches a minimum near 44 particles (near the midpoint between closed shells at 38 and 54), and then increases to values near the continuum limit. Pairing gap results for $N = 66-92$ are consistent within the statistical errors.

For all values of N the gap is considerably smaller than the BCS results. For comparison, at unitarity in cold atoms BCS calculations give a gap of $0.69E_F$ while the QMC result is $0.50(3)E_F$. [31] These calculations are in good agreement with recent polarized cold atom experiments [9,32]. For cold atoms the BCS equations will produce the exact gap in the

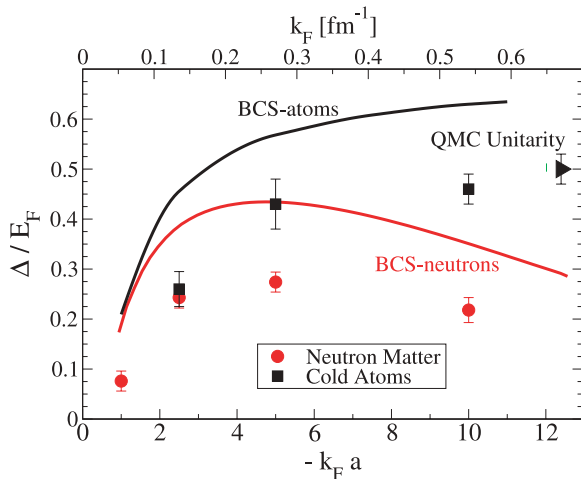


FIG. 3. (Color online) Superfluid pairing gap versus $k_F a$ for cold atoms ($r_e \approx 0$) and neutron matter ($|r_e/a| \approx 0.15$). BCS (solid lines) and QMC results (points) are shown.

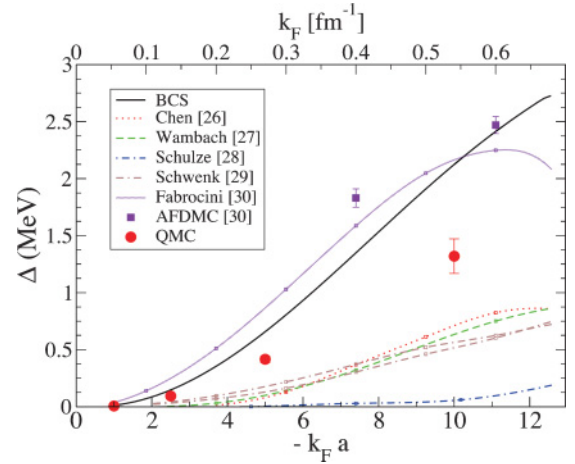


FIG. 4. (Color online) Superfluid pairing gap versus $k_F a$ for neutron matter compared to previous results.

BEC limit where the pairs are strongly bound. No such limit is relevant for a finite-range interaction.

In Fig. 3 we plot the pairing gap as a function of $k_F a$ for both cold atoms and neutron matter. BCS calculations are shown as solid lines, and QMC results are shown as points with error bars. QMC pairing gaps are shown from calculations of $N = 66-68$ particles. For cold atoms away from unitarity the pairing gaps are smaller than calculated previously [21] due to more complete optimizations and because these larger simulations reduce the finite-size effects. For very weak coupling, $-k_F a \ll 1$, the pairing gap is expected to be reduced from the BCS value by the polarization corrections calculated by Gorkov [33] $\Delta/\Delta_{\text{BCS}} = (1/4e)^{1/3}$. Because of finite-size effects, it is difficult to calculate pairing gaps using QMC in the weak coupling regime. The QMC calculations at the lowest density, $k_F a = -1$, are roughly consistent with this reduction from the BCS value. At slightly larger yet still small densities, where $-k_F a = \mathcal{O}(1)$ but $k_F r_e \ll 1$ for neutron matter, one would expect the pairing gap to be similar for cold atoms and neutron matter. The results at $k_F a = -2.5$, where $k_F r_e \approx 0.35$, support this expectation. Beyond that density the effective range becomes important and the QMC results are significantly reduced in relation to the cold atoms where $r_e \approx 0$.

These results for the pairing gap are compared to selected previous results in Fig. 4. The results of our calculations are much larger than the diagrammatic [26-28] and renormalization group [29] approaches. As these approaches assume a well-defined Fermi surface or calculate polarization corrections based upon single-particle excitations it is not clear how well they can describe neutron matter in the strongly-paired regime, or the similar pairing found in cold atoms.

The results here are significantly smaller than the AFDMC results of Fabrocini *et al.* [30]. These calculations are somewhat similar to those reported here. The disadvantage of the AFDMC approach is that it does not provide a variational bound to the energy, and hence the wave functions are chosen from another approach. In the calculation of Ref. [30] the wave function was taken from a correlated basis function approach that included a BCS initial state. The pairing in that variational

state is unusually large, and in fact increases as a fraction of E_F when the density is lowered. The advantage of the AFDMC approach is that it includes the full interaction rather than the simple s-wave interaction used here. AFDMC calculations with larger particle numbers are underway. [34]

In summary, we have calculated the $T = 0$ equations of state and pairing gaps for cold atoms and neutron matter. These systems are quite similar in that both are very strongly paired, and both have pairing gaps of the order of the Fermi energy. Experiments on the cold-atom equation of state would be very valuable in constraining the neutron matter equation of state. Pairing gaps in neutron matter are found to be suppressed compared to cold atoms and BCS theory, but much larger than in most other approaches. Again, cold-atom experiments could provide very valuable tests of many-body theories. It could be very important to explore finite-range effects experimentally using other atomic systems.

Note added in proof. We recently became aware of new calculations of the equation of state and pairing gap for cold atoms using auxiliary field Quantum Monte Carlo techniques [35]. Their results are similar to, but slightly different than, those presented here.

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