Statistical interpretation of multiplicity distributions and forward-backward multiplicity correlations in relativistic heavy ion collisions

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It is shown in this Brief Report that purely statistical considerations are possible to understand both the multiplicity distributions and forward-backward multiplicity correlations measured recently in the highest energy relativistic heavy ion collisions. It is necessary to study carefully whether the observed strong forward-backward correlations come mainly from the overall multiplicity fluctuations or not before we can draw any conclusion on dynamical long-range correlations.

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Recently the PHENIX Collaboration at BNL's Relativistic Heavy Ion Collider (RHIC) measured inclusive charged particle multiplicity distributions as a function of pseudorapidity window size [1]. It was found that the multiplicity distributions in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV are well described by the negative binomial distributions (NBD)

$$P_{k,\bar{n}}(n) = \frac{(n+k-1)!}{n!(k-1)!}\frac{(\bar{n}/k)^n}{(1+\bar{n}/k)^{n+k}}, \tag{1}$$

similar to those in hadron-hadron collisions [2]. Here $\bar{n}$ is the mean of the distribution and the parameter k describes the shape of the distribution. The negative binomial is wider than Poisson as long as k is positive and finite, and it reduces to the Poisson distribution in the limit $k \to \infty$. The dispersion of negative binomial satisfies

$$\frac{\sigma^2(n)}{\bar{n}^2} = \frac{1}{\bar{n}} + \frac{1}{k}. \tag{2}$$

PHENIX measured charged multiplicity distributions and fitted them with negative binomial distributions in pseudorapidity window sizes $\delta\eta$ from 0.066 to 0.7 with a step size of 0.022 in all centrality bins. A typical fitting result for 0–10% most central collisions is reproduced here in Table I.

A notable feature of the fitting results is that the parameter k remains nearly constant when $\delta\eta$ is around 0.3 or larger, which is also true for other centralities [1]. This is very different from what has been observed in $p\bar{p}$ collisions [2], where the parameter k increases almost linearly with the pseudorapidity interval size $\delta\eta$ until the largest measured interval $\delta\eta = 10.0$. The linear evolution of k with $\delta\eta$ has been described by two-particle short-range correlations [1,3]. We suggest in this paper that the observed constant parameter k of the negative binomial fit of multiplicity distributions as a function of pseudorapidity interval sizes can be taken as an indication of independent particle production in Au+Au collisions at RHIC, which will in turn determine the forward-backward multiplicity correlation strength at RHIC.

Suppose that we have a knowledge of the overall multiplicity probability distribution $P(n)$ of a certain pseudorapidity interval. By assuming negative binomial multiplicity distribution and uncorrelated particle emission, it is easy to calculate the parameter k for any subinterval inside the original interval [4]. Let us arbitrarily divide the original interval into two sub-intervals. The populations in the original interval is n and in the two subintervals are n_1 and n_2, respectively, which satisfy $n = n_1 + n_2$. The joint probability distribution of n_1 and n_2 is

$$P(n_1, n_2) = P(n_1|n)P(n), \tag{3}$$

where $P(n_1|n)$ is the conditional probability for finding n_1 given the total is n . Assume that the particles are not correlated. They have fixed probability p to fall into the one subinterval and probability $q = 1 - p$ to fall into the other subinterval. The number of observations in both sub-intervals follow the binomial statistics [5,6]

$$P(n_1|n) = \frac{n!}{n_1!(n-n_1)!}p^{n_1}q^{(n-n_1)}. \tag{4}$$

For easy calculation, following Ref. [4], we redefine the negative binomial distribution Eq. (1) in terms of a Poisson transform

$$P_{k,\bar{n}}(n) = \int_0^\infty dx f(x)\frac{(x\bar{n})^n e^{-x\bar{n}}}{n!}, \tag{5}$$

with $f(x) = \frac{k^k}{\Gamma(k)}x^{k-1}e^{-kx}$. The probability of observing particles in one of the subintervals is

$$\begin{aligned} P(n_1) &= \sum_{n_2=0}^\infty P(n_1, n_2) = \sum_{n_2=0}^\infty P(n_1|n)P(n) \\ &= \sum_{n_2=0}^\infty \int_0^\infty dx f(x)\frac{(x\bar{n})^n e^{-x\bar{n}}}{n!}\frac{n!}{n_1!n_2!}p^{n_1}q^{n_2} \\ &= \int_0^\infty dx f(x)\frac{(xp\bar{n})^{n_1}e^{-xp\bar{n}}}{n_1!}\left(\sum_{n_2=0}^\infty \frac{(xq\bar{n})^{n_2}e^{-xq\bar{n}}}{n_2!}\right) \\ &= \int_0^\infty dx f(x)\frac{(x\bar{n}_1)^{n_1}e^{-x\bar{n}_1}}{n_1!}. \end{aligned} \tag{6}$$

Mean multiplicities of the two sub-intervals $\bar{n}_1 = p\bar{n}$ and $\bar{n}_2 = q\bar{n}$ are used in the derivation. Eq. (6) indicates that the multiplicity distribution in one of the subintervals is unchanged. $P(n_1)$ is also a negative binomial distribution

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TABLE I. NBD fit results in centrality 0–10%. $\langle \bar{n} \rangle$ is the weighted mean of corrected $\bar{n}$ over all window positions. $\langle k \rangle$ is the weighted mean of corrected k over all window positions. $\langle \chi^2/\text{NDF} \rangle$ is the average of reduced χ^2 of NBD fits over all window positions. $\langle \text{NDF} \rangle$ is the average of the degree of freedom of NBD fits over all window positions. $\delta\langle k \rangle$ (total) is total systematic error on $\langle k \rangle$. See Ref. [1] for detail. (Table reproduced from Ref. [1].)

$\delta\eta$	$\langle \bar{n} \rangle$	$\langle k \rangle$	$\langle \chi^2/\text{NDF} \rangle(\langle \text{NDF} \rangle)$	$\delta\langle k \rangle$ (total)
0.700	77.535 ± 0.108	114.28 ± 3.21	0.96 (71.0)	±4.89
0.678	75.445 ± 0.106	113.11 ± 3.29	0.89 (69.7)	±6.78
0.656	72.977 ± 0.103	114.40 ± 3.38	0.95 (68.2)	±6.28
0.634	70.485 ± 0.101	114.02 ± 3.42	0.92 (66.7)	±6.43
0.613	67.998 ± 0.099	114.44 ± 3.52	0.94 (65.3)	±5.66
0.591	65.530 ± 0.096	114.28 ± 3.60	0.95 (63.9)	±5.72
0.569	63.050 ± 0.094	114.62 ± 3.71	0.97 (62.3)	±6.05
0.547	60.569 ± 0.091	114.27 ± 3.80	0.96 (60.8)	±6.03
0.525	58.100 ± 0.089	114.38 ± 3.92	0.95 (59.7)	±6.16
0.503	55.637 ± 0.086	114.36 ± 4.03	0.93 (57.8)	±6.29
0.481	53.164 ± 0.084	114.41 ± 4.17	0.94 (56.3)	±6.85
0.459	50.682 ± 0.081	115.19 ± 4.35	0.98 (54.2)	±6.75
0.438	48.209 ± 0.079	114.89 ± 4.51	0.98 (52.4)	±7.37
0.416	45.743 ± 0.076	115.05 ± 4.71	0.98 (50.3)	±7.57
0.394	43.283 ± 0.074	114.86 ± 4.90	0.97 (48.3)	±7.77
0.372	40.838 ± 0.071	115.20 ± 5.17	1.00 (46.2)	±7.83
0.350	38.424 ± 0.049	115.87 ± 3.88	1.04 (44.5)	±6.88
0.328	36.034 ± 0.047	115.35 ± 4.04	1.04 (42.9)	±7.32
0.306	33.665 ± 0.045	114.46 ± 4.21	1.08 (41.2)	±7.74
0.284	31.288 ± 0.043	113.91 ± 4.39	1.09 (39.9)	±7.65
0.263	28.916 ± 0.041	111.53 ± 4.51	1.10 (37.9)	±7.90
0.241	26.542 ± 0.039	109.53 ± 4.67	1.08 (36.3)	±8.04
0.219	24.155 ± 0.030	107.67 ± 4.03	1.11 (34.4)	±8.07
0.197	21.758 ± 0.028	105.84 ± 4.29	1.15 (32.4)	±8.12
0.175	19.355 ± 0.023	102.63 ± 3.96	1.21 (30.1)	±8.69
0.153	16.948 ± 0.021	97.91 ± 4.19	1.25 (27.6)	±8.96
0.131	14.536 ± 0.017	93.93 ± 4.01	1.34 (24.9)	±8.93
0.109	12.119 ± 0.014	87.92 ± 3.81	1.39 (21.9)	±8.94
0.087	9.695 ± 0.011	78.94 ± 3.35	1.34 (18.7)	±8.79
0.066	7.308 ± 0.008	65.53 ± 2.87	1.09 (15.4)	±8.45

with average multiplicity $\bar{n}_1$ and the parameter k in the original interval and the subinterval are the same. Since the subinterval is arbitrarily chosen, the results indicate that, for uncorrelated particle production, the parameter k does not depend on the phase space interval size or position. This suggests that the approximately constant k observed by the PHENIX experiment when pseudorapidity interval size $\delta\eta$ is not too small might be taken as an indication of independent emission, or no strong correlation when the correlation length between particles is dramatically reduced. The analyses in Ref. [1] do suggest smaller correlation length at RHIC energy than those in $p + p$ and low energy $A + A$ collisions. If we take the original pseudorapidity interval as the full pseudorapidity range available for particle production at a certain collision energy, $P(n)$ is simply the overall multiplicity distribution. The property of constant parameter k of multiplicity distributions will persist until the largest $\delta\eta$ interval which is more than 10.0 at RHIC. The PHENIX experiment has a very limited central pseudorapidity coverage $|\eta| \leqslant 0.35$. It is interesting to study experimentally whether k is constant for even larger pseudorapidity intervals. The PHENIX measured increase of k for small $\delta\eta$ intervals might be mainly due to two particle correlations.

If the overall multiplicity distribution is not known or it is not a negative binomial distribution, but the first two moments of the distribution are known, the relations above can be reformulated [7]. Again, let $P(n)$ be the overall multiplicity distribution and δx an arbitrary phase space interval. The probability of a particle falling into δx is p . For uncorrelated particle emission, the variance of observing n_1 particles in δx is

$$\begin{aligned}\sigma^2(n_1) &= \overline{n_1^2} - \bar{n}_1^2 \\ &= \sum_{n=0}^{\infty} P(n) \cdot \left[\sum_{n_1=0}^{n} n_1^2 \binom{n}{n_1} p^{n_1}(1-p)^{n-n_1} \right] - \bar{n}_1^2 \\ &= p^2 \cdot (\overline{n^2} - \bar{n}) + p \cdot \bar{n} - \bar{n}_1^2. \end{aligned} \tag{7}$$

Since $p = \bar{n}_1/\bar{n}$, Eq. (7) can be written as

$$\frac{\sigma^2(n_1)}{\bar{n}_1^2} - \frac{1}{\bar{n}_1} = \frac{\sigma^2(n)}{\bar{n}^2} - \frac{1}{\bar{n}}, \tag{8}$$

indicating that, for uncorrelated particle production, the multiplicity fluctuations in a small phase space interval (local fluctuations) are largely determined by the overall multiplicity fluctuations (global fluctuations). On the other hand, the satisfaction of Eq. (8) might suggest independent particle production. If $P(n)$ is a negative binomial distribution, considering Eq. (2), Eq. (8) is the same as constant parameter k in the overall and the subintervals.

The overall multiplicity fluctuations in nucleus-nucleus collisions, which have broader distributions comparing with the Poisson distributions, originate predominantly from the nuclear geometry, i.e., fluctuations in the number of participating nucleons or the number of binary collisions. The k parameter measured by PHENIX for 0–5% centrality is about three times of that for 0–10% centrality [1], indicating the importance of centrality cut for the shape of the distributions. The multiplicity distribution is much narrower or has fewer fluctuations when a severe centrality cut is used. The overall multiplicity fluctuations will in turn manifest as large fluctuations in restricted regions of phase space even if the particles are produced with little correlations.

Since the multiplicity distributions and the forward-backward multiplicity correlations are closely related, it is interesting to see how the forward-backward multiplicity correlations will be according to this picture of uncorrelated particle production in heavy ion collisions.

Correlations among particles emitted at various values of rapidity are important probes of the mechanisms of particle production in high-energy reactions. Many experiments show strong short-range correlations, indicating final state particles to be grouped in clusters over a range of about one unit in rapidity [2,8]. Besides these short-range correlations, long-range correlations have been studied with forward-backward multiplicity correlations in e^+e^- annihilation [9] and hadron-hadron collisions [10]. More recently, forward-backward multiplicity correlations in heavy ion collisions have been studied by the STAR collaboration at RHIC in Au+Au collisions at center of mass energy 200 GeV [11]. The strength of the correlations between the event multiplicity in the forward hemisphere n_F and that in the backward hemisphere n_B can be defined as [10]

$$b = \frac{\sigma(n_F, n_B)}{\sigma(n_F)\sigma(n_B)} = \frac{\langle (n_F - \langle n_F\rangle)(n_B - \langle n_B\rangle)\rangle}{[\langle (n_F - \langle n_F\rangle)^2\rangle\langle (n_B - \langle n_B\rangle)^2\rangle]^{1/2}}. \quad (9)$$

The forward-backward intervals are generally two symmetric, nonoverlapping intervals around midrapidity. In the STAR measurements, the interval width $\delta\eta$ is chosen as 0.2 and the distance between the forward and backward bin centers $\Delta\eta$ ranging from 0.2 to 1.8. The STAR results for 0–10% most central collisions are reproduced here in Fig. 1. STAR observed that the correlation strength b in central collisions is larger than that in pp collisions and it is essentially constant as a function of $\Delta\eta$ in all measured centralities [11]. The results suggest that the correlations between the forward and backward hemisphere are strong, and the correlations do not decrease when the distances between the forward and backward bins are increased. The measured correlation strength b is divided into contributions from both the short- and long-range correlations, see Fig. 1. The short-range correlations are expected to

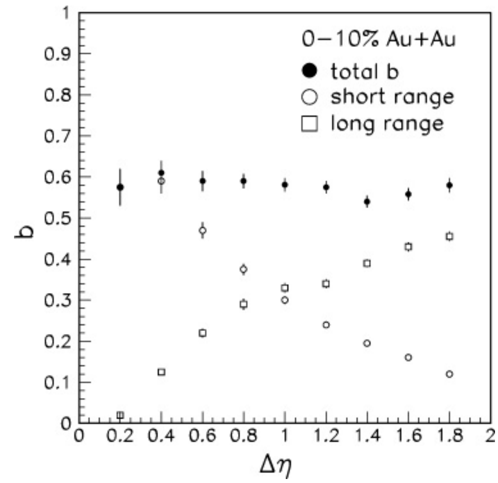

FIG. 1. Forward-backward correlation strength b as a function of the distance between the forward and backward pseudorapidity bin centers $\Delta\eta$ for 0–10% most central collisions. The total correlation strength is divided into the short- and long-range correlations. Data and analysis are from Ref. [11].

decrease exponentially with increasing $\Delta\eta$. The long-range correlation part is obtained by subtracting the short range component from the measured correlation strength. Since b is flat, the long-range correlations increase with $\Delta\eta$.

Instead of taking it as a sum of short- and long-range components, the constant b as a function of $\Delta\eta$ can also be understood by assuming uncorrelated emission of charged particles. For independent emission, one has [10]

$$b = \frac{\sigma^2(n_{FB}) - \bar{n}_{FB}}{\sigma^2(n_{FB}) + \bar{n}_{FB}}, \quad (10)$$

where $n_{FB} = n_F + n_B$ is the combined multiplicity in the forward and backward intervals, and $n_F \approx n_B \approx \frac{1}{2}n_{FB}$ is assumed. If the distribution of n_{FB} is a negative binomial distribution, Eq. (10) becomes [12]

$$b = \frac{\bar{n}_{FB}}{\bar{n}_{FB} + 2k}, \quad (11)$$

with k the negative binomial parameter. Equations (10) and (11) indicate, for uncorrelated emission, the forward-backward correlation strength b depends only on the average multiplicity and the fluctuations of multiplicity in the combined forward and backward intervals. For the central rapidity region $|\eta| < 1$ measured in STAR, the average multiplicity is approximately constant when the pseudorapidity interval size $\delta\eta$ is the same [13]. The parameter k is the same for any phase space interval for independent particle production. Thus, from Eq. (11), b does not depend on the position of the forward and backward phase space intervals in central rapidity region, i.e., it is flat as a function of $\Delta\eta$. This conclusion can be got similarly from Eq. (10) by considering Eq. (8). In this case, the forward-backward multiplicity correlation strength b is mainly determined by the overall multiplicity fluctuations which have large contributions from fluctuations in nuclear geometry. This idea was used originally in explaining the much stronger forward-backward multiplicity correlations in hadron-hadron collisions than that in e^+e^- annihilation [5,6,14].

From the PHENIX results of multiplicity distributions we can make a crude estimate of how large the correlation strength b will be by assuming uncorrelated particle production. Take 0–10% centrality as an example. From Table I, for $\delta\eta = 0.2$, $\bar{n}_F \approx 22$ and k as a function of $\delta\eta$ has a saturation value of about 114. Since charged tracks in the PHENIX paper [1] are only from $\Delta\phi < \pi/2$, while the STAR experiment has full azimuthal coverage [11], we estimate the combined average multiplicity in forward and backward intervals for STAR as $\bar{n}_{FB} \approx 22 \times 4 \times 2$. The k parameter assumes to be the same as the PHENIX value because it does not depend on the phase space intervals according to our assumptions. With the above values of $\bar{n}_{FB}$ and k, we get $b \simeq 0.44$ from Eq. (11). It is slightly lower than that from the STAR measurements. Purely statistical considerations do result in strong forward-backward correlations and they are flat as a function of $\Delta\eta$ in central rapidity region. Since different centrality selection criterions are used in STAR and PHENIX [1,11], which might cause different overall multiplicity distributions for a given centrality. Collision centrality is determined by the off-line cuts on the TPC charged particle multiplicity in STAR, while by the correlations between BBC charges versus ZDC energy in PHENIX. It is better to measure the multiplicity distributions and the forward-backward multiplicity correlations both in the same experiment and find out whether those strong forward-backward correlations are mainly from the overall multiplicity fluctuations or not before we can draw any conclusion on dynamical long-range correlations [15,16].

A interesting test is to measure the forward-backward correlations for both 0–10% centrality and 0–5% centrality. In the uncorrelated emission picture, correlation strength b would be less for 0–5% centrality because the overall multiplicity fluctuations are much reduced by narrowing the centrality window. For instance, for 0–5% centrality, PHENIX measured $n_{FB} \approx 24 \times 4 \times 2$ and $k \approx 360$ [1], which leads to $b \approx 0.25$. It is much less than that for 0–10% centrality. However, if the correlations are of dynamical origin, the forward-backward correlation strength might increase from 0–10% to 0–5% because the collisions become more violent.

In summary, we argued in this paper that the large and flat forward-backward multiplicity correlation strength as a function of the forward and backward pseudorapidity interval gap size in central rapidity region can be understood naturally from simple statistical considerations of uncorrelated production of charged particles in the highest energy Au+Au collisions at RHIC. This idea is supported by the constant negative binomial parameter k of multiplicity distributions as a function of pseudorapidity interval size measured in the same reactions. The observed large local multiplicity fluctuations in small phase space intervals are to a large extent determined by the overall multiplicity fluctuations. In nucleus-nucleus collisions, the overall multiplicity fluctuations are closely related to fluctuations in nuclear geometry. It is better to measure both the multiplicity distributions and the forward-backward multiplicity correlations in the same experiment to decide whether the observed strong correlations are mainly from the overall multiplicity fluctuations or not.

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