Addendum to triton and hypertriton binding energies calculated from SU_6 quark-model baryon-baryon interactions

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Previously we calculated the binding energies of the triton and hypertriton, using an SU_6 quark-model interaction obtained by a resonating-group method of two baryon clusters. In contrast to the previous calculations employing the energy-dependent interaction kernel, we present new results using a renormalized interaction that is energy-independent and still preserves all the two-baryon data. The new binding energies are slightly smaller than the previous values. In particular the triton binding energy turns out to be 8.14 MeV with a charge-dependence correction of the two-nucleon force, 190 keV, being included. This indicates that the energy to be accounted for by three-body forces is about 350 keV.

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The QCD-inspired spin-flavor SU_6 quark model (QM) for the baryon-baryon interaction, developed by the Kyoto-Niigata group, has achieved accurate descriptions of available NN and YN experimental data [1]. In particular, the most recent model fss2 produces accuracy comparable to modern realistic meson-exchange NN potentials. Since the QM description of the short-range part is quite different from that of meson-exchange potentials, it is interesting to apply the QM interaction to calculate some properties of three-baryon systems, for example, the binding energies of the triton (^{3}H) and the hypertriton ($^3_\Lambda$H). For this purpose, we developed in Refs. [2,3] a three-cluster equation, called an εK prescription, which employs energy-*dependent* two-cluster exchange kernels of the resonating-group method (RGM). Applying the εK formulation to the case of the quark-exchange kernels, we obtained the following results for fss2: The triton binding energy is $B_t = 8.518$ MeV [4,5], and the Λ separation energy of $^3_\Lambda$H is $B_\Lambda = 289$ keV [6]. The calculation for ^{3}H assumes charge independence of the NN interaction. Correcting for the well-known charge dependence of about 190 keV, we find that this result indicates a value of approximately 8.3 MeV, which lies in between standard results from meson-exchange models (up to approximately 8.0 MeV) and the experimental value of 8.482 MeV. For the $^3_\Lambda$H result, it should be noted that most of the Nijmegen models fail to reproduce enough separation energy of Λ, except for the NSC89 and more recent soft-core models NSC97 (e) and (f) [7,8], whereas our value is larger than the experimental value of 130 ± 50 keV.

Recently, important progress has been made in applying an energy-*independent* RGM kernel to the 3α system [9] and other three-cluster systems [10], through a standard procedure [11] of eliminating the energy dependence of the RGM kernel. This renormalized kernel will naturally give results different from the εK prescription when it is applied to the few-body systems interacting via the QM baryon-baryon interactions. We will report these new results in this paper.

The energy-independent renormalized RGM kernel $V^{\rm RGM}$ for a two-cluster system reads

$$V^{\rm RGM} = V_D + G + W, \tag{1}$$

where V_D is the direct potential and G is the sum of the exchange kinetic-energy and interaction kernels. It is the kernel W that appears through the elimination of the energy dependence, and it is given by

$$W = \Lambda \frac{1}{\sqrt{1-K}} h \frac{1}{\sqrt{1-K}} \Lambda - h. \tag{2}$$

Here K is the exchange normalization kernel, h denotes $h_0 + V_D + G$ with h_0 being the kinetic energy for the two-cluster relative motion, and $\Lambda = 1 - |u\rangle\langle u|$ is a two-cluster Pauli projection operator, where $|u\rangle$ is a Pauli-forbidden state satisfying $K|u\rangle = |u\rangle$. An advantage of using the $V^{\rm RGM}$ is that the two-cluster RGM equation takes the form of the usual Schrödinger equation in the Pauli-allowed model space, and the relative wave function is properly normalized [11]. This Schrödinger-type equation for the relative wave function gives the same asymptotic behavior as the original RGM equation, thus preserving the phase shifts and physical observables for

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the two-cluster system. The difference between the previous energy-dependent RGM kernel, $V^{\rm RGM}(\varepsilon) = V_D + G + \varepsilon K$, and $V^{\rm RGM}$ in Eq. (1) is essentially a replacement of $\Lambda(\varepsilon K)\Lambda$ with W. Here ε is the two-cluster relative energy measured from its threshold. The value is however not defined in the three-cluster system and it was determined in a self-consistent procedure in the εK treatment.

The three-cluster equation to be solved reads

$$P\left[E - H_0 - V_\alpha^{\rm RGM} - V_\beta^{\rm RGM} - V_\gamma^{\rm RGM}\right]P\Psi = 0, \qquad (3)$$

where α, β, and γ denote three independent pairs of two-cluster subsystems; H_0 is the free three-body kinetic-energy operator; and $V_\alpha^{\rm RGM}$ stands for the RGM kernel in Eq. (1) for the α pair, etc. The three-body operator P projects on the Pauli-allowed space with a proper symmetry of clusters, and it is constructed from the orthogonality constraint that each pair of two-cluster subsystems is free from any Pauli-forbidden states [12–14]. This definition of the three-cluster Pauli-allowed space may not be exactly equivalent to the standard definition given by the three-cluster normalization kernel. We however employed this orthogonality condition in the εK prescription. See Refs. [2,3] for details. We use the same definition of P in this paper as well.

Table I lists the three-nucleon bound-state properties predicted by the Faddeev calculations with fss2. The np interaction is employed in the isospin basis. The momentum discretization points for solving the Faddeev equation are the same as in Ref. [4]. The charge root mean square (rms) radii of ^{3}H and ^{3}He are calculated with the finite-size corrections of the proton and neutron through [17]

$$\begin{aligned}\langle r^2\rangle_{^3\rm H} &= [R_C(^3\rm H)]^2 + (0.8750)^2 + 2(-0.1161),\\ \langle r^2\rangle_{^3\rm He} &= [R_C(^3\rm He)]^2 + (0.8750)^2 + \tfrac{1}{2}(-0.1161),\end{aligned} \qquad (4)$$

TABLE I. The energy and charge rms radius of the three-nucleon ground state calculated by using the energy-independent renormalized RGM kernel derived from the QM fss2 potential. The column n gives the number of three-nucleon channels, including the two-nucleon systems up to the total angular momentum J. The proton and neutron size corrections for the rms radii are made by using Eq. (4). The column $E^{\varepsilon K}(^3\rm H)$ denotes the triton energy obtained by the εK prescription [4,5]. The experimental values are $E^{\rm exp}(^3\rm H) = -8.482$ MeV and $\sqrt{\langle r^2\rangle_{^3\rm H}} = 1.755 \pm 0.086$ fm [15] and $\sqrt{\langle r^2\rangle_{^3\rm He}} = 1.959 \pm 0.030$ fm [15], 1.9642 ± 0.0011 fm [16].

n	$E(^3\rm H)$ (MeV)	$E^{\varepsilon K}(^3\rm H)$ (MeV)	$\sqrt{\langle r^2\rangle_{^3\rm H}}$ (fm)	$\sqrt{\langle r^2\rangle_{^3\rm He}}$ (fm)
$2(S)$	-7.952	-7.807	1.80	1.95
$5(S, D)$	-8.261	-8.189	1.76	1.92
$10(J \leqslant 1)$	-7.962	-8.017	1.77	1.95
$18(J \leqslant 2)$	-8.228	-8.439	1.75	1.93
$26(J \leqslant 3)$	-8.313	-8.503	1.75	1.92
$34(J \leqslant 4)$	-8.322	-8.514	1.75	1.92
$42(J \leqslant 5)$	-8.326	-8.517	1.75	1.92
$50(J \leqslant 6)$	-8.326	-8.518	1.75	1.92

where ${R_C}^2$, given in units of square femtometers, stands for the squared charge radius for each isospin state of the point nucleons.

To calculate ${R_C}^2$ from the Faddeev components, previously we first calculated the charge form factor and then derived the ${R_C}^2$ value by differentiating it with respect to the squared momentum transfer at zero momentum. This time, we improved the method by taking the second-order differentiation of the momentum-space Faddeev components using the fifth-order spline interpolation formula. This approach leads to a stable rms radius within four digits, whereas in the previous method even the third digit fluctuates. In the present calculation, the Coulomb force and the relativistic correction terms [18] of the charge current operator are entirely neglected.

The final fss2 prediction for the triton energy is -8.326 MeV, which is higher by 192 keV than the $E^{\varepsilon K}$ value -8.518 MeV. Compared to the experimental value $E^{\rm exp}(^3\rm H) = -8.482$ MeV, the calculated value is too high by 156 keV. In fact, we have to take into account the effect of the charge dependence of the two-nucleon force, which is estimated to result in an energy loss of about 190 keV [19]. Therefore our calculation using the QM potential misses the triton binding energy by 346 keV ($\approx$350 keV). Comparing the energy convergence between the present and $E^{\varepsilon K}$ methods in Table I, we note that, in the present treatment, the 5-channel calculation already gives energy close to the converged value. Namely, the energy gain from the 5-channel to 50-channel calculation is only 65 keV, whereas it is 329 keV in the εK prescription. We will see that this is also the case in the hypertriton calculation.

For a realistic calculation of the ^{3}H binding energy, it is important to use an NN interaction that reproduces various deuteron properties and some typical NN scattering data, such as the ε_1 parameter, which are strongly correlated to the D-state probability, P_D, of the deuteron [20]. The effective range parameters of $^1\rm S_0$ scattering also play an essential role. To demonstrate the correlation between B_t and P_D, we display in Fig. 1 the updated energies of fss2 and FSS together with the values calculated by various realistic NN potentials. Here the FSS [27], an earlier version of fss2, less accurately reproduces the NN phase shifts. We find that fss2 gives a larger binding energy than the modern realistic meson-exchange potentials such as Bonn-C and AV18, whereas the result of FSS is not very far from that of Bonn-C. It is interesting to note that our QM points are apparently off the line on which the data points of the modern meson-exchange potentials fall. The five-channel calculation of the model QCM-A by Takeuchi *et al.* [28] gives almost the same result as Bonn-C.

The results of the hypertriton Faddeev calculations are listed in Table II for the model fss2. The Λ separation energy of the hypertriton is $B_\Lambda = 262$ keV, which is 27 keV less than the εK value, 289 keV. So far all the Faddeev calculations, using the energy-independent renormalized RGM kernels, yield less binding than the εK prescription, as long as the full model space with enough angular momenta is taken into account. Compared with the experimental value, ${B_\Lambda}^{\rm exp} = 130 \pm 50$ keV, the model fss2 gives an overbinding of 82–182 keV. In spite of the large experimental error bar, we may conclude that the ΛN interaction of fss2 is probably slightly

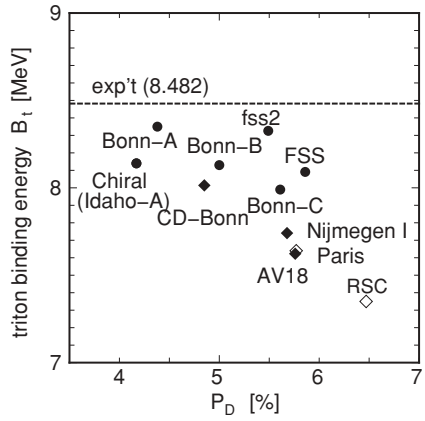

FIG. 1. Calculated ^{3}H binding energies B_t as a function of the deuteron D-state probability P_D. Calculations are made in the isospin basis, by using the np interaction, for fss2, FSS, Bonn-A, Bonn-B, Boon-C, and Chiral (denoted by black circles). The group including CD-Bonn, Nijmegen I, and AV18 (denoted by black diamonds) takes into account the effect of charge dependence of the interaction. In the Paris and RSC results (denoted by the open diamonds), the $^1S_0$ interaction is determined from the pp scattering data. Those energies denoted by black circles go down by about 190 keV when the charge dependence of the NN force is taken into account. The calculated values are taken from Refs. [19,20] 6 (RSC, Paris, Bonn-A, Bonn-B, and Bonn-C), [21,22] (AV18), [21,23] (CD-Bonn), [24,25] (Nijmegen), and [26] (Chiral). The experimental value, $B_t = 8.482$ MeV, is shown by the dashed line.

too attractive. The accuracy of the present-day YN interaction makes it impossible for us to discuss the effect of the ΛNN three-body force and the charge-symmetry breaking effect of the Λp and Λn interactions.

TABLE II. Hypertriton ($^3_\Lambda$H) properties calculated by using the energy-independent renormalized RGM kernels derived from the QM fss2 potential. The momentum discretization points used in the Faddeev calculations are the same as in Ref. [6]. The deuteron binding energy given by fss2 is $\varepsilon_d = 2.2247$ MeV ($\varepsilon_d^{\rm exp} = 2.2246$ MeV). The column n stands for the number of three-baryon channels, including the two-baryon systems up to the total angular momentum J, the column E lists the $^3_\Lambda$H energy measured from the $N+N+\Lambda$ threshold, and the column B_Λ is the Λ separation energy. The column $B_\Lambda^{\varepsilon K}$ is the energy obtained in the εK prescription. The column P_Σ denotes the probability, given in percent, of the ΣNN admixture. The experimental value is $B_\Lambda^{\rm exp} = 130 \pm 50$ keV.

n	E (MeV)	B_Λ (keV)	$B_\Lambda^{\varepsilon K}$ (keV)	P_Σ (%)
6(S)	−2.392	167	137	0.566
15(SD)	−2.451	226	198	0.775
30($J \leqslant 1$)	−2.404	179	178	0.679
54($J \leqslant 2$)	−2.467	243	273	0.792
78($J \leqslant 3$)	−2.483	259	285	0.824
102($J \leqslant 4$)	−2.486	261	288	0.828
126($J \leqslant 5$)	−2.487	262	289	0.830
150($J \leqslant 6$)	−2.487	262	289	0.830

We again confirm in Table II that the 15-channel calculation with S and D states only is a good approximation to the full calculation. The value of B_Λ is 226 keV in the 15-channel calculation, and the energy gain extended to the full calculations is only 36 keV. The corresponding energy gain in the previous εK prescription is as large as 91 keV. The NN (ε_{NN}) and ΛN($\varepsilon_{\Lambda N}$) expectation values defined by $\varepsilon_{BN} = \langle P\Psi|h_{0(BN)} + V_{BN}^{\rm RGM}|P\Psi\rangle/\langle P\Psi|P\Psi\rangle$ and the admixture of the ΣNN component (P_Σ) are not much different from the previous values in the εK prescription. The converged value of P_Σ is 0.83% for fss2 versus the previous value of 0.80%. The decomposition of the ε_{NN} value into the kinetic-energy and potential-energy contributions is 19.034−20.723 = −1.689 MeV, which was previously 19.376–21.032 = −1.657 MeV.

As to the overbinding of the model fss2, we have discussed in Ref. [6] that a slight increase of the κ-meson mass will improve the fit to the experimental value, without changing good reproduction of the low-energy ΛN cross-section data. If we modify the κ-meson mass from the value used in fss2, $m_\kappa = 936$ MeV, to 995 MeV, we would obtain $B_\Lambda = 134$ keV with $P_\Sigma = 0.56\%$, which is very close to the NSC89 prediction $B_\Lambda = 143$ keV with $P_\Sigma = 0.5\%$ [7,8]. The effective range parameters calculated with this modified interaction are $a_s = -2.18$ fm, $r_s = 3.03$ fm, $a_t = -1.78$ fm, and $r_t = 2.88$ fm. The phase-shift difference is only 2.2° at $p_{\rm lab} = 200$ MeV/c.

Rather moderate modification of the present results from the previous εK prescription is related to a simple structure of the quark-exchange normalization kernel $\Lambda K \Lambda$ in the Pauli-allowed space. For the NN interaction, $\Lambda = 1$ since there is no Pauli-forbidden state. For the positive-parity states, the largest eigenvalue of K in absolute magnitude is 1/9 for the (0s) harmonic-oscillator state. Although almost Pauli-forbidden states appear in the P states, such partial waves give minor contributions to the binding energy of the triton. For the ΛN-ΣN interaction, we have a Pauli-forbidden state classified by the SU_3 label $(11)_s$. Once this component is properly eliminated, the eigenvalues of $\Lambda K \Lambda$ also become very small. These are the main reasons why the present treatment by the energy-independent renormalized RGM kernel gives results that are not very much different from the previous energy-dependent εK prescription. However, the difference between $\Lambda(\varepsilon K)\Lambda$ and W sometimes becomes large in nuclear cluster systems, because the number of Pauli-forbidden states in general increases and they play more complicated roles in determining the structure of nuclear systems. For example, the ground-state energy of ^{9}Be, calculated in an $n\alpha\alpha$ microscopic three-cluster model, is −2.61 MeV, whereas it turns out to be −2.16 MeV in the renormalized RGM approach [10] and −3.86 MeV in the εK prescription [29].

In summary, we have recalculated the triton and hypertriton binding energies in a new semimicroscopic three-cluster equation, using the energy-independent renormalized RGM kernels of the quark-model baryon-baryon interactions. This formulation produces slightly less attractive effect on the three-baryon systems, compared to the previous energy-dependent treatment of the two-cluster RGM kernels. For the triton binding energy, the fss2 quark model gives 8.326 MeV, which is 156 keV smaller than the experimental value of

8.482 MeV. By taking into account the charge-dependence correction of 190 keV, we conclude that the quark-model potential underbinds the triton by approximately 350 keV. Thus the energy to be accounted for by three-nucleon forces may not be as large as 0.5–1 MeV, which most standard meson-exchange potentials [21] predict. These different predictions for the contributions of the three-body force prompt us to examine what results the present quark-model NN interaction produce in other three-nucleon observables, especially in the nd and pd scatterings. This will be examined soon and reported elsewhere. For the Λ separation energy of the hypertriton, the overbinding of the model fss2 is slightly reduced. We still have large ambiguities in ^{1}S and ^{3}S ΛN interactions, before further details, such as the charge-symmetry breaking of Λp, Λn interactions, and the ΛNN three-body force, come into play. The comparison of the fss2 value, 262 keV, with the experimental one, $B_\Lambda^{\rm exp} = 130 \pm 50$ keV, indicates that the ^{1}S interaction of fss2 is still too attractive, which could be corrected by choosing a slightly heavier κ-meson mass.

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