Angular momentum conservation in heavy ion collisions at very high energy

F. Becattini*
Dipartimento di Fisica, Università di Firenze, and INFN, Sezione di Firenze, Florence, Italy

F. Piccinini†
INFN, Sezione di Pavia, Pavia, Italy

J. Rizzo‡
Dipartimento di Fisica, Università di Firenze, Florence, Italy
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The effects of angular momentum conservation in peripheral heavy ion collisions at very high energy are investigated. It is shown that the initial angular momentum of the quark-gluon plasma should enhance the azimuthal anisotropy of particle spectra (elliptic flow) with respect to the usual picture where only the initial geometrical eccentricity of the nuclear overlap region is responsible for the anisotropy. In hydrodynamical terms, the initial angular momentum entails a nontrivial dependence of the initial longitudinal flow velocity on the transverse coordinates. This gives rise to a nonvanishing vorticity in the equations of motion, which enhances the expansion rate of the supposedly created fluid compensating for the possible quenching effect of viscosity. A distinctive signature of the vorticity in the plasma is the generation of an average polarization of the emitted hadrons, for which we provide analytical expressions. These phenomena might be better observed at LHC, where the initial angular momentum density will be larger and where we envisage an increase of the elliptic flow coefficient v_2 with respect to RHIC energies.

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I. INTRODUCTION

Nuclei colliding at ultrarelativistic energies have a large initial orbital angular momentum L_0 if their impact parameter is of order of some femtometers; in fact, for symmetric nuclei, $L_0 \simeq A\sqrt{s_{NN}}b/2$ in natural units ($\hbar = 1$). For Au-Au collisions at RHIC energies $\sqrt{s}_{NN} = 200$ GeV and $L_0 \sim 5 \times 10^5$ at an impact parameter $b = 5$ fm. The angular momentum will be almost two orders of magnitude larger in the forthcoming Pb-Pb collisions at LHC, at $\sqrt{s}_{NN} = 5.5$ TeV, with $L_0 \sim 1.4 \times 10^7$. Because of the inhomogeneity of the colliding nuclei in the transverse plane, a significant fraction of L_0 *must* be deposited in the interaction region, in other words should be transferred to the supposedly formed quark-gluon plasma (QGP). Large values of the initial angular momentum of the plasma may give rise, as we will show, to significant observable effects.

According to the to-date generally accepted description of the collision process, a locally equilibrated plasma is formed after a relatively short proper time (of the order of 1 fm/c) followed by a purely ideal-fluid hydrodynamical expansion. This kind of approach proved to be able to reproduce the large observed values of the elliptic flow in peripheral collisions, at a finite impact parameter, and the transverse momentum spectra of particles in the low-p_T region [1]. Usually, in this kind of description, the Bjorken hydrodynamics scaling hypothesis is used either all along the evolution (2+1 hydro) [2] or just at the initial proper time (3+1 hydro [3]). In both cases, the initial longitudinal flow velocity only depends on z, which amounts to making the initial angular momentum vanish unless the energy density has an asymmetric dependence on the transverse coordinates [3]. But even if this is assumed, in which case the initial angular momentum is then nonvanishing, the dynamical evolution would be different from the case of a longitudinal flow velocity depending on transverse coordinates, as we will show later.

In recent papers [4,5] it has been found that amending the ideal-fluid assumption with even a minimal viscosity strongly affects the elliptic flow. Particularly, Song and Heinz pointed out that, to restore the agreement with a hydrodynamical description, one should enforce significant modifications of the initial conditions or the equation of state, thereby raising some doubts about the interpretation of RHIC results. In this paper, we want to show that including the initial angular momentum by a suitable modifications of the initial fluid velocity profile may cure the problem, or at least it may give a contribution in this direction. In fact, a finite angular momentum enhances the elliptic flow coefficient and broadens the transverse momentum spectra, exactly what is needed to counterbalance the quenching effect of viscosity.

The most distinctive signature of an intrinsic angular momentum would be the polarization of the emitted hadrons. This argument has been put forward in Refs. [6,7], where the authors take a QCD perturbative approach. Also, more recently, polarization has been related to the fluid vorticity [8], yet without the development of an explicit mathematical relation. In this paper, we take advantage of a very recent study of the ideal relativistic spinning gas [9] and present a formula relating polarization to the angular velocity of an

*francesco.becattini@fi.infn.it
†fulvio.piccinini@pv.infn.it
‡rizzoj@lns.infn.it

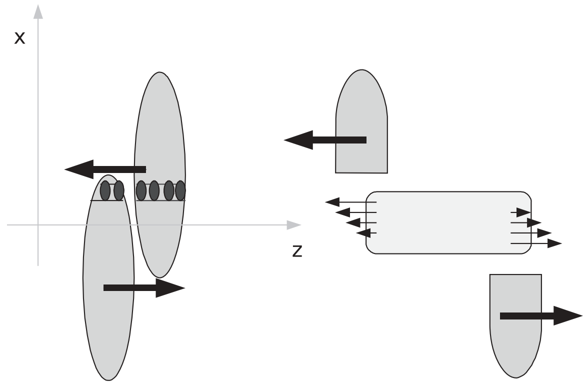

FIG. 1. Sketch of a peripheral heavy ion collision at very high energy in the longitudinal projection. The initial momentum distribution of the interaction region (right) should have a gradient along the axis x orthogonal to the collision axis z stemming from the different transverse densities of the colliding strips (left).

equilibrated (i.e., rigidly rotating) hydrodynamical system. We argue, on the basis of the locality principle, that a such formula should hold for the most general fluid motion where the angular velocity is to be presumably replaced by an expression involving the local acceleration and hence the vorticity of the fluid.

II. ANGULAR MOMENTUM CONSERVATION IN HEAVY ION COLLISIONS

In the usual picture of a peripheral heavy ion collision at ultrarelativistic energy the overlapping region of the two incoming nuclei gives rise to QGP whereas the nonoverlapping fragments fly away almost unaffected. Thereby, only a fraction of the initial angular momentum L_0 is left to the interaction region, while the largest part is carried away by the fragments (see Fig. 1). The angular momentum of the interaction region takes its origin from the inhomogeneity of the density profile in the transverse plane, the so-called thickness function. This is much more clearly seen in a longitudinal projection: The colliding strips of nucleons have, in peripheral collisions, different numbers of nucleons. Although the central strips have the same weight, the strips above it will have a net momentum directed along the negative z axis and the ones below it a net momentum directed along the positive z axis. (see Fig. 1). The net momentum density at each point (x, y) of the overlap region in the transverse plane (see Fig. 2 for the axes definition) for symmetric (equal nuclei) collisions reads

$$\frac{\mathrm{d}P}{\mathrm{d}x\mathrm{d}y} = [T(x - b/2, y) - T(x + b/2, y)]\frac{\sqrt{s}_{NN}}{2}, \quad (1)$$

where $T(x, y)$, the thickness function, that is, the longitudinal integral of the nucleon density, is given by

$$T(x, y) = \int \mathrm{d}z\, n(x, y, z).$$

Only if the two colliding objects were homogeneous in the transverse plane would the angular momentum of the interaction region be vanishing. Yet, the nuclei *are not* homogenous in the transverse plane; for instance, if they are assumed to be homogenous spheres in their rest frame, their thickness function $T(x, y)$ would be proportional to $\sqrt{R^2 - r^2}$, where r is the distance from the center of the nucleus and R is its radius. In this case, Eq. (1) would become

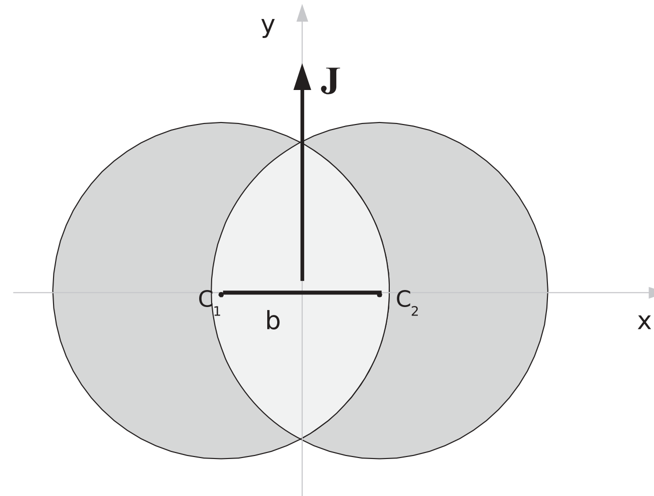

FIG. 2. Sketch of a peripheral heavy ion collision at very high energy in the transverse projection. The overlap almond-shaped region is marked in light gray and has an overall angular momentum directed along the symmetry axis y, orthogonal to the reaction plane.

$$\frac{\mathrm{d}P}{\mathrm{d}x\,\mathrm{d}y} = 2n_0\Big[\sqrt{R^2 - y^2 - (x - b/2)^2} - \sqrt{R^2 - y^2 - (x + b/2)^2}\Big]\frac{\sqrt{s}_{NN}}{2}. \quad (2)$$

From this momentum density, one gets an initial angular momentum J of the interaction region directed along the y axis:

$$\mathbf{J} = 2n_0 \int \mathrm{d}x \int \mathrm{d}y\, x\Big[\sqrt{R^2 - y^2 - (x - b/2)^2} - \sqrt{R^2 - y^2 - (x + b/2)^2}\,\Big]\frac{\sqrt{s}_{NN}}{2}\hat{\mathbf{j}}. \quad (3)$$

In Fig. 3 we show $\mathbf{J}$ for two colliding gold nuclei at $\sqrt{s}_{NN} =$ 200 GeV, in the two cases of hard spheres and a Woods-Saxon distribution. For the former case, it is seen that the angular momentum attains a maximal value at an impact parameter of 2.5 fm and quickly drops thereafter. The maximal value of J is about 7.2×10^4 (i.e., 29% of the initial orbital angular momentum L_0 of the colliding nuclei at that impact parameter). Therefore, J is very large and strongly dependent on the impact parameter b but this effect is usually ignored in the initial conditions assumed for hydrodynamical calculations, as in the commonly used Bjorken model the longitudinal flow velocity only depends on z and it does not thus have any azimuthal anisotropy.

This dependence can be seen again from Fig. 1; since the net momentum of the colliding strips varies monotonically along x, either the proper energy density or the fluid four-velocity or both must have an asymmetric profile in x for the initial angular momentum to be conserved. If we take the

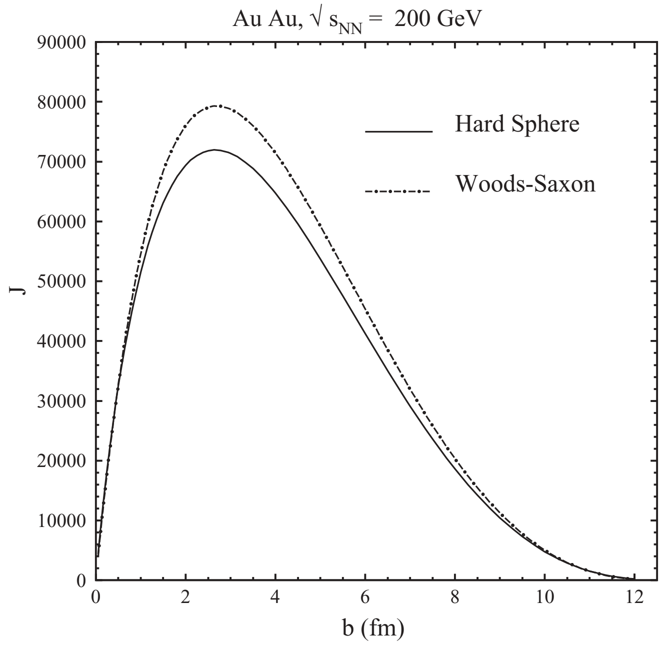

FIG. 3. Angular momentum J of the interaction region as a function of the impact parameter for Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

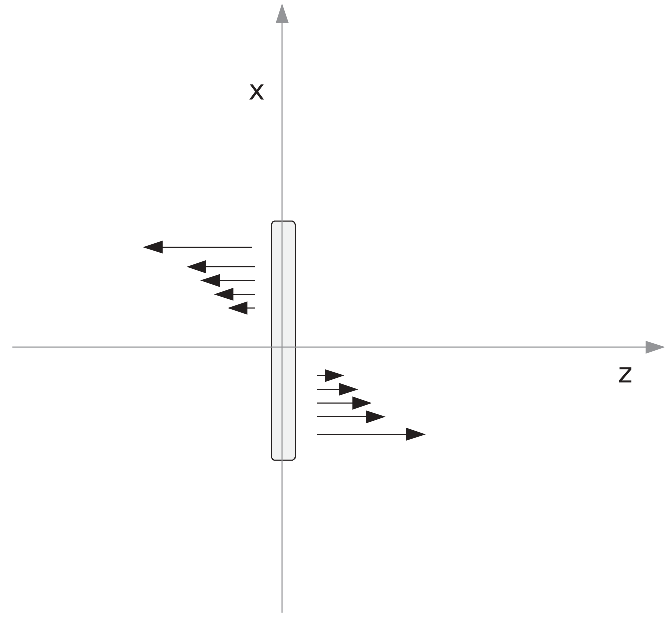

FIG. 4. Initial longitudinal velocity profile for the limiting case of sudden thermalization in the very thin overlap region of the colliding ultrarelativistic nuclei.

reasonable assumption that the proper energy density cannot have such an asymmetric dependence on x because it can only depend on the density of nucleons at each point, the only remaining possibility is to admit that the initial longitudinal flow velocity is asymmetric in x from the very beginning; that is, it is azimuthally anisotropic, in such a way that

$$-\int \mathrm{d}^3x x T^{0z} = -\int \mathrm{d}^3x x(\rho + p)\gamma^2 v_z(x) = J \tag{4}$$

for a perfect fluid and if the initial flow transverse flow velocity is zero; in this equation T is the stress-energy tensor, ρ is the proper energy density, p is the pressure, and $\gamma^2 = (1 - v^2)^{-1}$. The fact that v_z is not azimuthally isotropic implies, in general, a nonvanishing vorticity $\boldsymbol{\omega} = (1/2)\nabla \times \mathbf{v}$ for the fluid motion, and this may have remarkable consequences on the final particle spectra. It should be pointed out that some calculations [3] indeed introduce an x-asymmetric proper energy density function. Still, even if ρ were forced to have such an asymmetric x dependence to fulfill angular momentum conservation (4), the final velocity field would not be the same as when, more reasonably, v_z is asymmetric in x. We will try to illustrate such effects with an oversimplified hydrodynamical scheme in the next section.

III. HYDRODYNAMICAL SCHEME

We are now going to set up a very simple hydrodynamical scheme to show that the azimuthal anisotropy of the longitudinal flow velocity required by the angular momentum conservation must enhance the elliptic flow.

As has been mentioned, the requirement of an initial azimuthal anisotropy of the longitudinal flow velocity breaks the usual Bjorken scheme, where $v_z = z/t$. As a first step, one would like to introduce a minimal change of the Bjorken scheme, which is not an easy task. Thus, to describe the possible effects of an initial dependence of v_z on x in the most transparent way, we will assume an oversimplified scheme in which the two colliding nuclei give rise to a complete thermalization within an infinitesimally thin slab Δz at time $t = 0$ (see Fig. 4). This scheme looks very similar to the Landau hydrodynamical model, were not for the inclusion of an initial flow velocity $v_z(x)$, which ought to vanish at $x = 0$ for an infinitely thin slab, for evident symmetry reasons. Such a picture of the collision should be more realistic at asymptotically large energies, where one expects thermalization to be extremely quick and nuclei are infinitely Lorentz-contracted along their collision axis. Furthermore, we will assume to deal with a perfect fluid and we will focus our attention on the transverse motion only.

First, we can write a relation for the initial momentum density:

$$(\rho_0 + p_0)\gamma_0^2 v_{z0} = \frac{1}{\Delta z}\frac{\mathrm{d}P}{\mathrm{d}x\mathrm{d}y}, \tag{5}$$

where ρ_0, p_0, and v_{z0} are the proper energy density, pressure, and longitudinal flow velocity, respectively, at time $t = 0$, with $\gamma_0^2 = 1/(1 - v_{z0}^2)$ because the initial transverse velocity is vanishing; $\frac{\mathrm{d}P}{\mathrm{d}x\mathrm{d}y}$ is given by Eq. (1). Equation (5) makes it clear that v_{z0} in this approach cannot be zero because of the initial unbalance in momentum and that, in general, it depends on both x and y. The specific functional form of the proper energy density ρ affects the functional dependence of v_{z0} but it should not suppress its dependence on x because it is reasonable to assume that it has a symmetric dependence on x, unlike $\frac{\mathrm{d}P}{\mathrm{d}x\mathrm{d}y}$ as pointed out at the end of Sec. II. From a hydrodynamical point of view, the remarkable consequence of this is that the initial vorticity $\boldsymbol{\omega} = (1/2)\nabla \times \mathbf{v}$ is nonvanishing, unlike in the traditional Bjorken picture where v_z only depends on z. Indeed,

$$\omega_x(t=0) = \frac{1}{2}\frac{\partial v_{z0}}{\partial y}, \quad \omega_y(t=0) = -\frac{1}{2}\frac{\partial v_{z0}}{\partial x}, \tag{6}$$

where the largest component among the two is the one along the y axis, because of the asymmetry of v_{z0} with respect to the y axis. Conversely, v_{z0} is *symmetric* with respect to the x axis [see Eqs. (1) and (2)] and its partial derivative with respect to y ought to vanish at $y = 0$ for any x.

It is worth pointing out that the evolution equation for the classical vorticity $\boldsymbol{\omega}$ in the relativistic case is more complicated than in nonrelativistic fluid mechanics. However, it is still true that a nonvanishing initial value of the classical vorticity makes the fluid motion a vorticous one in general. This stems from the Carter-Lichnerowicz equation of motion (equivalent to the Euler equations) for a perfect fluid with one conserved charge [10]:

$$u^\mu(\partial_\mu \bar{h} u_\nu - \partial_\nu \bar{h} u_\mu) = -T\partial_\nu \bar{s}, \tag{7}$$

where $\bar{h} = (\rho + p)/n$ and $\bar{s} = s/n$ are the enthalpy and the entropy densities, respectively, normalized to the charge density n, p is the pressure, and T is the temperature. The *vorticity tensor* is usually defined as

$$\Omega_{\mu\nu} = (\partial_\mu \bar{h} u_\nu - \partial_\nu \bar{h} u_\mu) \tag{8}$$

and the *vorticity vector* as [10]

$$\omega^\alpha = \frac{1}{4\bar{h}}\epsilon^{\alpha\mu\nu\sigma}u_\mu \Omega_{\nu\sigma} = \frac{1}{4}\epsilon^{\alpha\mu\nu\sigma}u_\mu(\partial_\nu u_\sigma - \partial_\sigma u_\nu). \tag{9}$$

It is quite straightforward to show from Eq. (9) that the vorticity vector field has the nonrelativistic limit

$$\omega \to \left(0, \tfrac{1}{2}\nabla \times \mathbf{v}\right) \tag{10}$$

so that ω is a proper relativistic generalization of the classical vorticity $\boldsymbol{\omega} = (1/2)\nabla \times \mathbf{v}$. The time component of ω is

$$\omega^0 = \tfrac{1}{2}\gamma^2 \mathbf{v} \cdot \nabla \times \mathbf{v}.$$

Thus, if $\nabla \times \mathbf{v} \neq 0$ then $\omega \neq 0$ and $\Omega \neq 0$. It can be shown, starting from Eq. (7), that the spacial part of the vorticity tensor, that is, $\boldsymbol{\Omega} = \nabla \times \bar{h}\gamma\mathbf{v}$, fulfills the Helmholtz vorticity equation

$$\frac{\partial \boldsymbol{\Omega}}{\partial t} = \nabla \times (\mathbf{v} \times \boldsymbol{\Omega}), \tag{11}$$

provided that the fluid is *isentropic* (i.e., $\nabla \bar{s} = 0$). All classical consequences of the vorticity equations then hold in relativity provided that $\boldsymbol{\omega}$ is replaced by $\boldsymbol{\Omega}$.

Let us now study in more detail the fluid equations of motion at time $t = 0$. We will write the Euler equation, instead of the Carter-Lichnerowicz, for a perfect ultrarelativistic fluid, with equation of state $p = \rho/3$. Accordingly, we have

$$(\rho + p)(u \cdot \partial)u^\mu = g^{\mu\nu}\partial_\nu p - (u \cdot \partial p)u^\mu, \tag{12}$$

and we shall focus on the transverse components at time $t = 0$, when $u_x = u_y = 0$, namely,

$$\rho_0\gamma_0 \left.\frac{\partial u_i}{\partial t}\right|_{t=0} = -\frac{1}{4}\left.\frac{\partial \rho}{\partial x_i}\right|_{t=0} \tag{13}$$

for $i = 1, 2$, where the equation of state has been used. Multiplying both sides by γ_0^2 and manipulating the derivative on the right-hand side gives

$$\rho_0\gamma_0^3 \left.\frac{\partial u_i}{\partial t}\right|_{t=0} = -\frac{1}{4}\left.\frac{\partial \rho\gamma^2}{\partial x_i}\right|_{t=0} + \frac{1}{4}\rho_0 \left.\frac{\partial \gamma^2}{\partial x_i}\right|_{t=0}. \tag{14}$$

Now, since $\gamma_0^2 = 1/(1 - v_{z0}^2)$, we get

$$\left.\frac{\partial \gamma}{\partial x_i}\right|_{t=0} = \gamma_0^3 v_{z0}\frac{\partial v_{z0}}{\partial x_i} \tag{15}$$

and Eq. (14) becomes

$$\rho_0\gamma_0^3 \left.\frac{\partial u_i}{\partial t}\right|_{t=0} = -\frac{1}{4}\left.\frac{\partial \rho\gamma^2}{\partial x_i}\right|_{t=0} + \frac{1}{4}2\rho_0\gamma_0^4 v_{z0} \left.\frac{\partial v_{z0}}{\partial x_i}\right|_{t=0}. \tag{16}$$

Two terms are then responsible for the initial transverse velocity increase: The first is related to the gradient of the energy density in the observer frame; the second depends on the gradient of the initial velocity field [i.e., on the vorticity according to Eq. (6)]. If v_{z0} were independent of x, y the second term would vanish and the transverse expansion would then be driven by the energy density gradient only, as in the usual picture. Because of the eccentricity of the overlap region (see Fig. 2), the system gets an initial kick larger in the x direction than in y and an anisotropy in the final spectra ensues. If, however, the second term is included, the expansion gets an additional contribution because $\partial v_{z0}/\partial x < 0$ (see Fig. 4), and v_{z0} is negative for $x > 0$ and positive for $x < 0$ so that altogether the second term drives an increase of u_x for $x > 0$ and a decrease (starting from zero) for $x < 0$. Moreover, the expansion rate related to this term will be larger in the x direction than in y because, expectedly, $\partial v_{z0}/\partial x > \partial v_{z0}/\partial y$ [see discussion following Eq. (6)], thereby enhancing the elliptic flow. In other words, besides the *geometrical anisotropy*, elliptic flow gets a finite contribution from an initial *kinematical anisotropy* of the longitudinal velocity. This enhancement of elliptic flow can be seen as a centrifugal effect owing to angular momentum conservation: Particles with a momentum orthogonal to $\mathbf{J}$ (i.e., directed along the reaction plane) get an additional momentum kick with respect to those emitted along $\mathbf{J}$.

It is now interesting to make an estimate of how large this contribution is in our simple scheme. Assuming that the energy density is proportional to the total energy of nucleons in the overlap region so that

$$(\rho_0 + p_0)\gamma_0^2 - p_0 = \frac{1}{\Delta z}\frac{\mathrm{d}E}{\mathrm{d}x\mathrm{d}y} = \frac{1}{\Delta z}[T(x - b/2, y) + T(x + b/2, y)]\frac{\sqrt{s_{NN}}}{2}, \tag{17}$$

and by using Eq. (5) and the equation of state $p = \rho/3$, we can obtain the expressions of the initial proper energy density ρ:

$$\rho_0 = \frac{1}{\Delta z}\sqrt{4\left(\frac{\mathrm{d}E}{\mathrm{d}x\mathrm{d}y}\right)^2 - 3\left(\frac{\mathrm{d}P}{\mathrm{d}x\mathrm{d}y}\right)^2} - \frac{1}{\Delta z}\frac{\mathrm{d}E}{\mathrm{d}x\mathrm{d}y} \tag{18}$$

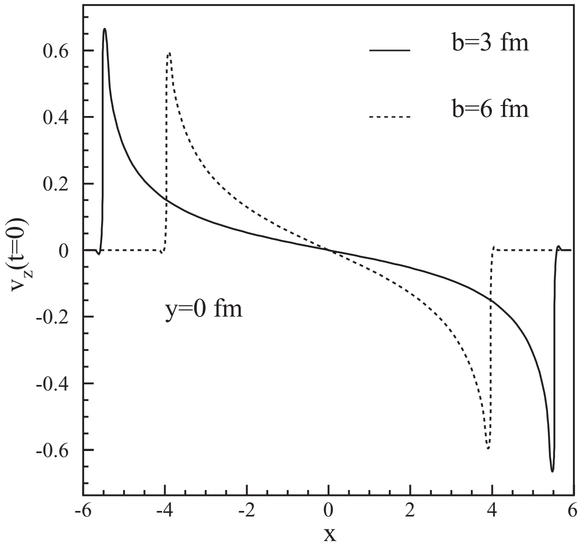

FIG. 5. Initial longitudinal velocity profile along the reaction plane $y = 0$ for two different impact parameters for the collision of two hard-sphere nuclei with 7-fm radius.

and the flow velocity v_{z0}:

$$v_{z0} = \frac{3\frac{\mathrm{d}P}{\mathrm{d}x\mathrm{d}y}}{\sqrt{4\left(\frac{\mathrm{d}E}{\mathrm{d}x\mathrm{d}y}\right)^2 - 3\left(\frac{\mathrm{d}P}{\mathrm{d}x\mathrm{d}y}\right)^2} + 2\frac{\mathrm{d}E}{\mathrm{d}x\mathrm{d}y}}, \tag{19}$$

which is shown in Fig. 5 for the case of hard-sphere nuclei with 7-fm radius. According to Eq. (18), the proper energy density is an even function of x, as was expected with the assumption (17), whereas v_{z0} is an odd function of x. Also, it can be seen from Fig. 5 that v_{z0} has a singular derivative at the edge of the overlap region, a consequence of the hard-sphere assumption; such singularities disappear with smooth density profiles. By using Eqs. (19), (18), (5), and (17) we can compute the ratio of the second to the first term in Eq. (16) for the x axis:

$$-\frac{2\rho_0\gamma_0^4 v_{z0} \left.\frac{\partial v_{z0}}{\partial x}\right|_{t=0}}{\left.\frac{\partial\rho\gamma^2}{\partial x}\right|_{t=0}} \tag{20}$$

and thereby evaluate the importance of the vorticity term for the expansion rate. This ratio is shown in Fig. 6 for the case of hard-sphere nuclei for two different y values at an impact parameter $b = 6$ fm. It is seen that the second term is a consistent fraction of the first term even near the collision center $x = 0$ (about 20%) whereas it steeply increases at larger x values; at the boundary of the x interval the ratio shows spikes owing to the hard-sphere assumption and it is not shown. Of course, these numbers refer to an oversimplified example and just for the initial expansion kick, but the conclusion that the longitudinal velocity gradient cannot be neglected in more realistic hydrodynamical calculations should hold.

As has been mentioned, in some hydrodynamical calculations [3,11], a nonvanishing angular momentum of the plasma is tacitly introduced by enforcing an asymmetric x dependence for the proper energy density in peripheral collisions keeping the Bjorken longitudinal scaling (i.e., the independence of v_z on the coordinates x, y). Thereby, longitudinal momentum density [Eq. (5)] conservation is fulfilled even though v_z is independent of x and the angular momentum conservation [Eq. (4)] is also fulfilled. We think that this assumption is quite unnatural. First, it cannot hold in our specific example of instantaneous thermalization at infinitely large energy (with the infinitesimally thin fluid in Fig. 4) because the only velocity that is compatible with symmetry and independent of x is 0, thus making both momentum and angular momentum density vanishing. However, even in the more realistic and more general case of finite thermalization time, it does not lead to the same flow velocity field as in the case of angular momentum conserved through Bjorken scaling breaking because of the *absence of the vorticity term*. This can be shown by enforcing the equality of angular momentum densities in the two approaches:

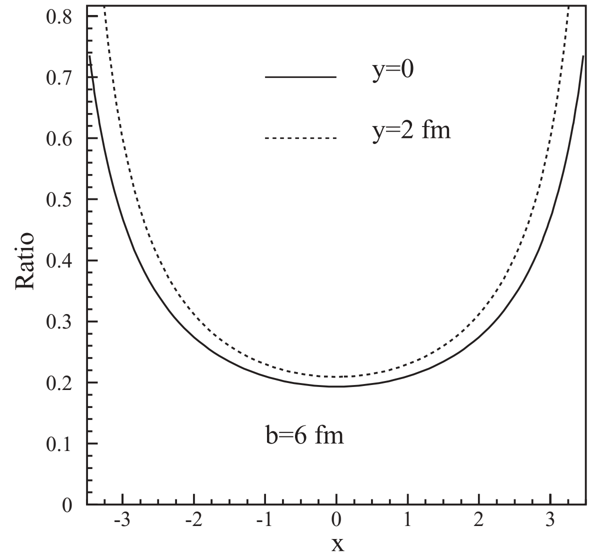

FIG. 6. Ratio of the term proportional to the vorticity and the term proportional to energy density gradient along x in Eq. (16) as a function of x for $y = 0$ and $y = 2$ fm for the collision of two hard-sphere nuclei with 7-fm radius at an impact parameter $b = 6$ fm.

$$\tfrac{4}{3}\tilde{\rho}_0\tilde{\gamma}_0^2\tilde{v}_{z0} = \tfrac{4}{3}\rho_0\gamma_0^2 v_{z0}, \tag{21}$$

where quantities with a tilde on the left-hand side are such that only $\tilde{\rho}$ depends on x whereas on the right-hand side we have the standard ones in our approach. From this equation it follows that

$$\left.\frac{\partial\tilde{\rho}}{\partial x}\right|_{t=0}\tilde{\gamma}_0^2\tilde{v}_{z0} = \left.\frac{\partial\rho}{\partial x}\right|_{t=0}\gamma_0^2 v_{z0} + \rho_0\left.\frac{\partial\gamma_0^2 v_{z0}}{\partial x}\right|_{t=0}. \tag{22}$$

Using Eqs. (22) and (21) to obtain $\partial\rho/\partial x$ in the equation of motion at time $t = 0$ [Eq. (13)], we get, after some manipulations,

$$\left.\frac{\partial u_x}{\partial t}\right|_{t=0} = -\frac{1}{4\gamma_0\rho_0}\left.\frac{\partial\rho}{\partial x}\right|_{t=0} = -\frac{1}{4\tilde{\rho}_0\tilde{\gamma}_0}\left.\frac{\partial\tilde{\rho}}{\partial x}\right|_{t=0}\frac{\tilde{\gamma}_0}{\gamma_0} + \frac{1}{4\gamma_0^3 v_{z0}}\left.\frac{\partial\gamma^2 v_{z0}}{\partial x}\right|_{t=0}. \tag{23}$$

Conversely, in an approach where velocity is uniform with the same angular momentum density, one would have

$$\left.\frac{\partial u_x}{\partial t}\right|_{t=0} = -\frac{1}{4\tilde{\rho}_0\tilde{\gamma}_0}\left.\frac{\partial \tilde{\rho}}{\partial x}\right|_{t=0}. \tag{24}$$

Therefore, even though the same angular momentum density was enforced by modifying the energy density profile, the expansion rate could be consistently different from the one with nonvanishing vorticity because of the factor $\tilde{\gamma}/\gamma$ and, chiefly, the additional term proportional to the derivative of longitudinal velocity, which in general speeds up expansion, as we have seen.

The ratio of the expansion-driving terms in Eq. (20) depends only on geometry and not on the center-of-mass energy, because so do both ρ and v_{z0} according to Eqs. (18) and (19) and the expressions (5) and (17). This apparent energy independence is just a specific feature of our simple scheme where the longitudinal dimension was shrunk to a very thin slab. In fact, this cannot be the case in a more realistic situation, where thermalization is not instantaneous and, therefore, the gradient of v_z is distributed on a larger volume. In other words, the relative vorticity contribution will not be as large as it turned out to be by enforcing instantaneous thermalization in an infinitesimal slab Δz. This can be better seen by rewriting the angular momentum, for a perfect fluid, as

$$\begin{aligned}
\mathbf{J} &= \int \mathrm{d}^3x\,\mathbf{x}\times\boldsymbol{\pi} = \int \mathrm{d}^3x\,\nabla\frac{x^2}{2}\times h\gamma^2\mathbf{v} \\
&= \int \mathrm{d}^3x\,\nabla\times\frac{x^2}{2}h\gamma^2\mathbf{v} - \int \mathrm{d}^3x\,\frac{x^2}{2}\nabla\times h\gamma^2\mathbf{v} \\
&= -\int \mathrm{d}^3x\,\frac{x^2}{2}\nabla h\gamma^2\times\mathbf{v} - \int \mathrm{d}^3x\,x^2h\gamma^2\boldsymbol{\omega},
\end{aligned} \tag{25}$$

where $\pi_i = T^{0i} = h\gamma^2 v_i$ is the momentum density, $h = \rho + p$ is the enthalpy density, and $\boldsymbol{\omega} = (1/2)\nabla\times\mathbf{v}$; in this equation we assumed that the enthalpy density vanishes outside a compact region. In general, the sum of the two terms in Eq. (25) is constrained by angular momentum conservation but their relative contribution to it can, and will, vary with the center-of-mass energy. Most likely, at lower energy, the vorticity term will be less important whereas, at higher energy, its relative contribution should approach the limiting one calculated in our simple scheme because thermalization is expected to be faster, with an initial denser plasma and a higher angular momentum density. If this is the case, at the LHC, a further increase of the elliptic flow with respect to RHIC ought to be observed.

IV. ELLIPTIC FLOW FOR A SPINNING SYSTEM

Taking viscosity into account implies a modification of the hydrodynamical equations but this should not affect our conclusions. This can be understood with a simple argument: Angular momentum has to be conserved anyway and dissipative effects such as viscosity will speed up entropy increase. Thus, the system will tend to the maximal entropy configuration, which, for a system with finite angular momentum and finite volume, is a rigidly spinning fluid, with velocity field $\mathbf{v} = \boldsymbol{\omega}\times\mathbf{x}$, $\boldsymbol{\omega}$ being a constant vector related to the total angular momentum $\mathbf{J}$ [9,12]. Of course, the quick expansion will prevent the system from reaching global equilibrium before decoupling, but still the system will evolve toward that configuration.

Hence, although viscosity, along with other dissipative forces, will not be able to create a fully equilibrated rigidly spinning plasma fireball, it is interesting to show that in this ideal case the effect of a finite angular momentum on elliptic flow and other observables is remarkable.

That a globally spinning interaction region brings about an anisotropy in the particle azimuthal spectra was argued many years ago by Hagedorn and Wambach [13] and recently rediscussed in Ref. [14]. Assuming statistical hadronization for a fully equilibrated subsystem of the plasma, one can calculate the elliptic flow coefficient as that of a rigidly spinning ideal hadron-resonance gas with angular momentum $\mathbf{J}_\omega$ such that $J_\omega < J$ and fixed angular velocity $\boldsymbol{\omega} = (1/2)\nabla\times\mathbf{v}$ parallel to it and linked to $\mathbf{J}_\omega$ by a thermodynamic relation that is linear for small ω/T values [9]. In the Boltzmann approximation for primary hadrons, this reads [9]

$$v_2^{(J)} = \frac{\int \mathrm{d}^3x\,\frac{\mathrm{K}_1\left(m_T\sqrt{1-|\omega\times\mathbf{x}|_\parallel^2}/T\right)}{\sqrt{1-|\omega\times\mathbf{x}|_\parallel^2}}\mathrm{I}_2\left(\frac{p_T z\omega}{T}\right)}{\int \mathrm{d}^3x\,\frac{\mathrm{K}_1\left(m_T\sqrt{1-|\omega\times\mathbf{x}|_\parallel^2}/T\right)}{\sqrt{1-|\omega\times\mathbf{x}|_\parallel^2}}\mathrm{I}_0\left(\frac{p_T z\omega}{T}\right)}, \tag{26}$$

where K_1 and I_n are modified Bessel functions and T is the global temperature. It should be stressed that the global temperature T in a spinning relativistic gas is related to the *local* proper temperature T_0, measured by a comoving thermometer, by the relation [9]

$$T_0(r) = \frac{T}{\sqrt{1-\omega^2r^2}}, \tag{27}$$

where r is the distance from the rotation axis $\boldsymbol{\omega}$. Since it is the local, and not the global, temperature that determines the phase of the system, the decoupling should occur when the highest local temperature reaches the critical value T_c for the quark-hadron transition, that is, when

$$\frac{T}{\sqrt{1-\omega^2R^2}} = T_c, \tag{28}$$

where R is the maximal distance from the rotation axis.

The behavior of the "rotational" $v_2^{(J)}$ (which would simply vanish if $J = 0$) as a function of p_T for primary hadrons is very similar to that driven by pressure gradients in usual hydrodynamical calculations, and it turns out to be almost independent of the particle mass. This behavior is shown in Fig. 7 for $\omega/T = 0.03$ at the chemical freeze-out temperature $T_c = 165$ MeV, for a spherical source with radius $R = 10.1$ fm and a total angular momentum $J_\omega \simeq 10^4$ (i.e., of the same order of the J of the interaction region at RHIC energies). The $v_2^{(J)}$ of primary hadrons from a globally spinning region would be therefore very large, although resonance decays should lower the final one consistently.

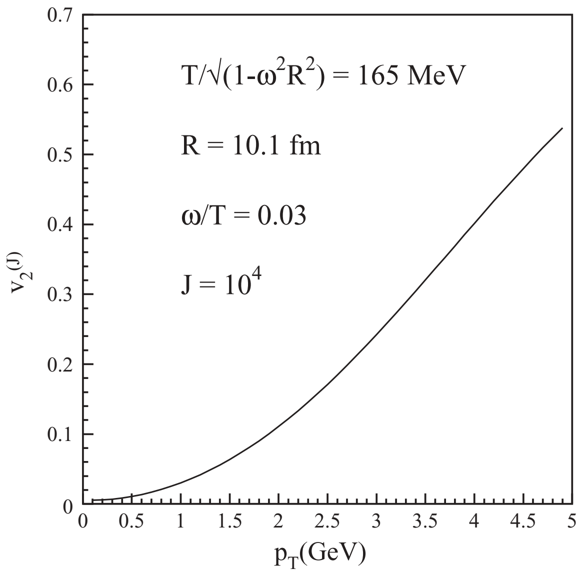

FIG. 7. Elliptic flow coefficient $v_2^{(J)}$ as a function of p_T for hadrons originated from a spherical spinning plasma at a chemical freeze-out $T = 165$ MeV and a radius of 10.1 fm for $\omega/T = 0.03$. The elliptic flow would simply vanish if $J = 0$.

V. POLARIZATION

Elliptic flow is not a unique consequence of an intrinsic rotation. There is, however, a distinctive signature thereof: a polarization of the emitted hadrons along the angular momentum direction (in the observer frame). That a large angular momentum in peripheral heavy ion collisions could give rise to polarization of the final hadrons was first proposed in Ref. [6], where a quantitative assessment was performed within a perturbative QCD framework, with the polarization of quarks assumed to be effectively transferred to final hadrons. Recently, it has been pointed out that a plasma with polarized quarks could be probed by observing the polarization of direct photons [15]. We take a different approach here and determine the polarization of particles by invoking local thermodynamical equilibrium and the statistical hadronization dogma, which is successful in describing hadronic multiplicities: Every multihadronic state compatible with conservation laws is equally likely. Therefore, since the total angular momentum is not vanishing, when the plasma hadronizes, available spin states will not be evenly populated and a net polarization of the produced hadrons will show up. In this approach, there is no need to invoke any special dynamical mechanism for the polarization of quarks to be transferred to hadrons, as it should happen as a consequence of the statistical nature of this process.

The proper polarization vector $\mathbf{\Pi}_0$ of particles in a relativistic rotating ideal gas has been calculated by the authors [9] as

$$\mathbf{\Pi}_0 = \frac{1}{2}\tanh\frac{\omega}{2T}\left[\frac{\varepsilon}{m}\hat{\boldsymbol{\omega}} - \frac{\hat{\boldsymbol{\omega}}\cdot\mathbf{p}\mathbf{p}}{m(\varepsilon+m)}\right] \quad (29)$$

for spin 1/2 particles and

$$\mathbf{\Pi}_0 = \frac{\sum_{n=-S}^{S} n e^{n\omega/T}}{\sum_{n=-S}^{S} e^{n\omega/T}}\left[\frac{\varepsilon}{m}\hat{\boldsymbol{\omega}} - \frac{\hat{\boldsymbol{\omega}}\cdot\mathbf{p}\mathbf{p}}{m(\varepsilon+m)}\right] \quad (30)$$

for generic spin particles, where ε is the energy and $\mathbf{p}$ the momentum of the particle. The polarization along $\hat{\boldsymbol{\omega}}$ (i.e., $\mathbf{\Pi}_0\cdot\hat{\boldsymbol{\omega}}$) turns out to be maximal for particles emitted orthogonally to $\boldsymbol{\omega}$ (i.e., along the reaction plane for an equilibrated spinning system) and increases for increasing p_T up to momenta of the order of $2mT/\omega$, where the rotational grand-canonical ensemble scheme fails and more complicated expressions arise [9]. Also, the vector mesons show spin alignment in that the 00 component of the spin density matrix turns out to be different from 1/3, specifically [9],

$$\rho_{\omega\,00}(p) = \frac{1}{2\cosh(\omega/T)+1}\left\{\cosh(\omega/T) + \frac{(\mathbf{p}\cdot\boldsymbol{\omega})^2}{\mathrm{p}^2\omega^2}\times[1-\cosh(\omega/T)]\right\}, \quad (31)$$

which, for small ω/T, reduces to

$$\rho_{\omega\,00}(p) \simeq \frac{1}{3} + \frac{1-3(\hat{\mathbf{p}}\cdot\hat{\boldsymbol{\omega}})^2}{18}\frac{\omega^2}{T^2}. \quad (32)$$

It is interesting to note that the polarization (more generally the spin density matrix) depends on the ratio of angular velocity to global temperature, that is, on using Eq. (27) on $\gamma\omega/T_0$, where T_0 is the local temperature. Therefore, it can be conjectured, by invoking the locality principle, that a polarization should appear in a generic accelerated hydrodynamical cell at local equilibrium fully determined by local quantities. Hence, $\boldsymbol{\omega}$ is to be plausibly replaced by the vector

$$\boldsymbol{\omega} \to \frac{1}{v^2}\mathbf{a}\times\mathbf{v}, \quad (33)$$

which is the local angular velocity for a general trajectory, according to the Frenet formulas. If this conjecture is true, every hadronizing hydrodynamical cell will produce hadrons with polarization vector (30) with $\boldsymbol{\omega}$ equal to the right-hand side of expression (33).

The expected polarization values are of the order of ω/T, which is reasonbly some percent or less (see previous section) but they should increase with particle momenta up to momenta of a few GeVs and hopefully become observable.[1] However, the expressions (30) and (31) refer to primary hadrons (i.e., those emitted from the source at decoupling) and resonance decays can further dilute the polarization, so that a more detailed study is needed.

It is difficult to predict the evolution of polarization values as a function of center-of-mass energy. However, it can be argued that they should increase by considering that angular momentum *density* at freeze-out increases as a function of $\sqrt{s}_{NN}$. This happens because the size of the system at freeze-out increases approximately logarithmically whereas the angular momentum of the interaction region increases linearly with $\sqrt{s}_{NN}$ [see Eq. (3)]. Since the angular momentum density must be somehow related to the final local angular velocity (33), a fair conclusion follows that polarization effects should increase with the collision energy.

[1]Recent measurements by RHIC experiments set a limit on average Λ polarization to be ~ 0.02 [16].

VI. SUMMARY AND CONCLUSIONS

We have pointed out that angular momentum conservation in peripheral ultrarelativistic heavy ion collisions at very high energy should give an additional contribution to the azimuthal momentum anisotropy, thereby enhancing the elliptic flow coefficient v_2. By using a very simple hydrodynamical scheme, we have shown that taking angular momentum conservation properly into account implies, most likely, a nonuniform longitudinal flow velocity in the transverse plane, breaking the usual assumption of Bjorken scaling. This in turn generates a nonvanishing initial vorticity term in the equations of motion that enhances the transverse expansion rate and may be able to balance the elliptic flow deficit observed by Song and Heinz [5] in minimally viscous hydrodynamical calculations. Angular momentum conservation is also implemented in current hydrodynamical calculations by keeping the Bjorken scaling hypothesis, but the resulting expansion rates are different. We expect this effect to be more visible at the very large energies, where the vorticity contribution to the angular momentum density tends to an upper geometrical limit, that we have analyzed in our simplified scheme. Hence, we predict that v_2 should increase from RHIC to LHC energy, although we cannot give a definite quantitative estimate.

The most characteristic signature of the vorticity induced by angular momentum conservation would be a polarization of the emitted particles, which is predicted to be, in the observer frame, for a globally spinning system, orthogonal to the reaction plane and maximal for particles with momentum parallel to the reaction plane in case of a globally spinning plasma. A quantitative assessment of these effects for the actual hydrodyamical evolution is very difficult to make, but we argued that the polarization should be there for a general accelerated fluid motion. Also this effect should be better observed at the LHC, where the angular momentum density should be larger.

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[1] P. F. Kolb and U. W. Heinz, in *Quark Gluon Plasma 3*, edited by R. C. Hwa and X.-N. Wang (World Scientific, Singapore, 2004).
[2] P. F. Kolb, J. Sollfrank, and U. W. Heinz, Phys. Rev. C **62**, 054909 (2000).
[3] T. Hirano and K. Tsuda, Phys. Rev. C **66**, 054905 (2002).
[4] P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99**, 172301 (2008).
[5] H. Song and U. W. Heinz, Phys. Lett. **B658**, 279 (2008).
[6] Z. T. Liang and X. N. Wang, Phys. Rev. Lett. **94**, 102301 (2005).
[7] J. H. Gao, S. W. Chen, W. T. Deng, Z. T. Liang, Q. Wang, and X. N. Wang, LBNL-63515, arXiv:0710.2943.
[8] B. Betz, M. Gyulassy, and G. Torrieri, Phys. Rev. C **76**, 044901 (2007).
[9] F. Becattini and F. Piccinini, to appear in Ann. Phys.
[10] E. Gourgoulhon, in *Stellar Fluid Dynamics and Numerical Simulations: From the Sun to Neutron Stars,* EAS Ser. **21**, 43 (2006).
[11] U. W. Heinz and P. F. Kolb, J. Phys. G **30**, S1229 (2004).
[12] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon Press, New York, 1980).
[13] R. Hagedorn and U. Wambach, Nucl. Phys. **B123**, 382 (1977).
[14] P. Castorina, D. Kharzeev, and H. Satz, Eur. Phys. J. C **52**, 187 (2007).
[15] A. Ipp, A. Di Piazza, J. Evers, and C. H. Keitel, arXiv:0710.5700.
[16] B. I. Abelev *et al.* [STAR Collaboration], Phys. Rev. C **76**, 024915 (2007).