

Search for strongly deformed structures and observation of multiple nucleon alignments in ^{174}W

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Highly excited states, up to spin $39\hbar$, have been established in ^{174}W , using the Gammasphere array. Ultimate cranked calculations predict the appearance of triaxial, strongly deformed structures above spin $30\hbar$ in ^{174}W . A new approach was developed for a comprehensive search of the data for such structures, similar to those observed in the Lu and Hf isotopes. No evidence was found for strongly deformed bands in the W isotopes populated in this experiment. Existing rotational structures have been considerably extended, allowing for the observation of both neutron and proton alignments in a number of bands. There is evidence for the $i_{13/2}$ neutron and possibly both the $h_{9/2}$ and $h_{11/2}$ proton crossings. The observed neutron and proton crossing frequencies are in good agreement with predictions of Woods-Saxon cranking calculations using an empirical pair-gap energy, and they lead to an improved understanding of the underlying structure of the bands.

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I. INTRODUCTION

The phenomenon of triaxial strongly deformed (TSD) structures in nuclei is unique to the $A \approx 160$ – 170 region of the periodic table. The observation of wobbling bands in ^{163}Lu [1] provided the first conclusive evidence of triaxial nuclear shapes. Though there are a number of instances where triaxial shapes have been inferred in normal deformed (ND) nuclei, these are difficult to establish, given the absence of unambiguous experimental signatures in most cases. It is possible to isolate the nonaxial nature of excited states only when nuclei exist in strongly deformed (SD) triaxial configurations over an extended range of spin. A deformed nucleus with a stable triaxial shape is expected to exhibit a series of rotational (wobbling) bands with very similar moments of inertia and alignment, corresponding to different numbers of wobbling phonons [2–4]. Thus far, only Lu ($Z = 71$) isotopes [1,5–7] have shown unambiguous evidence of wobbling excitations. Although SD structures have also been observed in Tm ($Z = 69$) and Hf ($Z = 72$) isotopes [8–13], it has been determined that most of these arise from configurations which are quite different from the ones in the Lu isotopes, and triaxiality has not been conclusively established. Cranked Nilsson-Strutinsky calculations using the ultimate cranker (UC) code [14] have been used to predict the appearance of TSD energy minima and to explain characteristics of the observed SD structures. While

these calculations account to some extent for the observed TSD structures in the Lu isotopes, the description of the SD bands in Hf nuclei is less accurate [12]. Further, these calculations predict TSD structures to be a feature common to most nuclei in this region, unlike what is actually observed. These calculations also predict TSD structures in W ($Z = 74$) isotopes, near $N = 100$. The investigation of highly excited states in nuclei around ^{174}W is necessary to delineate the region of the periodic chart where TSD structures exist and to test the validity of the UC predictions beyond the Lu isotopes.

Further motivation for this work stems from the need to expand our knowledge of nucleon alignments at very high spins. Though first nucleon alignments have been established and characterized in many deformed nuclei, the knowledge and understanding of subsequent alignments is limited, and discussions addressing the issue can be found for only a small number of nuclei, e.g., Refs. [15–22]. The characterization of nucleon alignments provides insight into the underlying configurations of rotational bands in addition to checking the validity of predictions of cranking calculations at high rotational frequencies.

The use of a thick target in our previous experiment to study high-spin states in W isotopes allowed a clean identification of a number of two- and four-quasiparticle, high- K states in ^{174}W [23], some with half-lives in the 100 ns region. However, this thick target measurement limited the determination of states in the yrast band to a spin of $26\hbar$, beyond the first alignment of $i_{13/2}$ neutrons. A thin target was used in the present experiment, allowing the extension of a number of bands to considerably higher spins. Multiple new alignments have been observed in

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most of these bands, and it has been possible to understand the quasiparticle configurations by comparison with cranking calculations using a Woods-Saxon potential with universal parameters [24]. The use of an empirical value of the pair-gap energy in these calculations, as described in our earlier work [23], provides an accurate description of the observed band crossings.

II. EXPERIMENT AND RESULTS

Highly excited states in W isotopes around ^{174}W were populated using a 230 MeV ^{50}Ti beam from the Argonne Tandem-Linac Accelerator System (ATLAS) at Argonne National Laboratory, incident on ^{128}Te targets. The targets had a thickness ranging from 235 to 370 $\mu\text{g}/\text{cm}^2$, with approximately 500 $\mu\text{g}/\text{cm}^2$ of Au on the front and 50 $\mu\text{g}/\text{cm}^2$ on the back. The energy loss of the ^{50}Ti ions in the Au layer on the front was approximately 5 MeV. These targets were mounted on a rotating target wheel to extend the life of the targets. Despite this, some of the targets suffered damage after a few hours of irradiation and had to be replaced. This eventually led to opting for a reduced beam current (<1 pA) for most of the experiment to prevent further loss of targets, which curtailed the statistics considerably. The isotopes ^{174}W and ^{173}W , corresponding to the 4n- and 5n-evaporation channels, had the most population, with lesser cross sections for the 6n- and 3n-channels, ^{172}W and ^{175}W . The Gammasphere array [25], consisting of 100 Compton suppressed Ge detectors for this experiment, was used to record three-fold and higher coincidence data. A total of approximately 800 million triple- γ coincidence events were recorded. The raw data were sorted into three- and four-dimensional symmetric cubes and hypercubes, respectively, and analyzed using the RADWARE [26] suite of programs.

A. Normal deformed bands

The existing level scheme [23] was extended up to spin $39\hbar$ (Fig. 1). Figure 2 is a typical spectrum showing transitions in the yrast sequence of ^{174}W . Bands 1–4, 6, and 7 observed in our previous work [23] were extended from spins of 22, 26, 24, 29, 23, and $26\hbar$ to 32, 38, 30, 39, 35, and $36\hbar$, respectively. The energies of a few of the highest spin transitions observed in the earlier work have been revised, *viz.* $29^- \rightarrow 27^-$ in band 4, $23^- \rightarrow 21^-$ in band 6, and the $26^- \rightarrow 24^-$ transition in band 7. The $21^- \rightarrow 19^-$, 620 keV transition in band 8 was not seen in the present data. At the highest spins, band 2 receives the most population. Bands 1, 3, 4, 6, and 7 are populated with approximately 30, 40, 80, 35, and 60%, respectively, of the intensity of band 2. The intensities of the new transitions in all the bands exhibit the expected smooth decrease with spin. These are not listed here since they do not contribute to the subsequent discussion. Additional nucleon alignments, beyond our earlier observations, were seen in bands 1–4, 6, and 7. Two nucleon alignments were evident in most bands, and possibly three in the yrast structure. The nature of each of these alignments is discussed subsequently. The other bands

observed in the previous work, including bands 5 and 8, in Fig. 1, could not be extended and are, therefore, not discussed.

B. Search for strongly deformed bands

The major motivation for this work was to search for TSD structures in ^{174}W predicted by UC calculations (with standard Nilsson parameters), as shown in Fig. 3. Therefore, an approach was developed which enabled a comprehensive search of all reasonably possible decay sequences in the data which could be associated with strongly deformed bands. First, the characteristics of the known TSD and SD structures in the Lu and Hf isotopes were inspected. Unlike “classic” superdeformed structures, where successive transition energies are very regularly spaced, TSD bands can exhibit a significantly larger stagger in energy differences between successive transitions. Such energy differences in the Lu and Hf bands are plotted in Figs. 4 and 5. It is apparent that almost all energy differences lie between 40 and 70 keV, as indicated by the dotted lines. In general, TSD bands in the Lu isotopes are observed to lower spins and correspondingly smaller rotational frequencies than those in Hf nuclei. The search for possible TSD structures in the W isotopes was based on the premise that these bands would have moments of inertia and, therefore, energy spacings similar to the Lu and Hf bands. The choice of rotational frequency (transition energy) for the band search in W also relied on observations in the Lu and Hf isotopes. The TSD bands in Lu range in rotational frequency from approximately 0.20 to 0.65 MeV, while those in the Hf isotopes extend from 0.35 to 0.70 MeV. Rotational frequencies from 0.40 to 0.60 MeV, corresponding to gating transition energies from 800 to 1200 keV, span the most probable region where TSD bands are expected. Lists were generated of all possible sequences of five transition energies, with the lowest energy extending from 800 to 1200 keV, and energy differences between successive transitions ranging from 40 to 70 keV. The lowest gating energy and energy differences were varied in steps of 2 keV. A smaller energy step was not required, since the peaks were Doppler broadened, with widths of approximately 6 keV around 800 keV. These lists were cataloged as required by the “search band” routine in RADWARE [26]. The total number of lists exceeded 26 million. The optimum number of transitions in each list was determined to be five, since a smaller number of transitions might yield insufficient statistics for weak band structures, while a larger number would have increased the dimension of the search prohibitively.

The comprehensive search for triple coincidences with and between any five transitions in these lists was performed on a RADWARE cube, using the search band routine. Spectra corresponding to each of the lists of gates described above were generated, and the parameters corresponding to the average number of excess counts per double-gate (above the calculated value) together with the ratio of average number of excess counts to the corresponding standard deviation were recorded for each of the lists. These parameters will henceforth be referred to as mean counts and figure of merit, respectively, following the terminology used in RADWARE. A large value

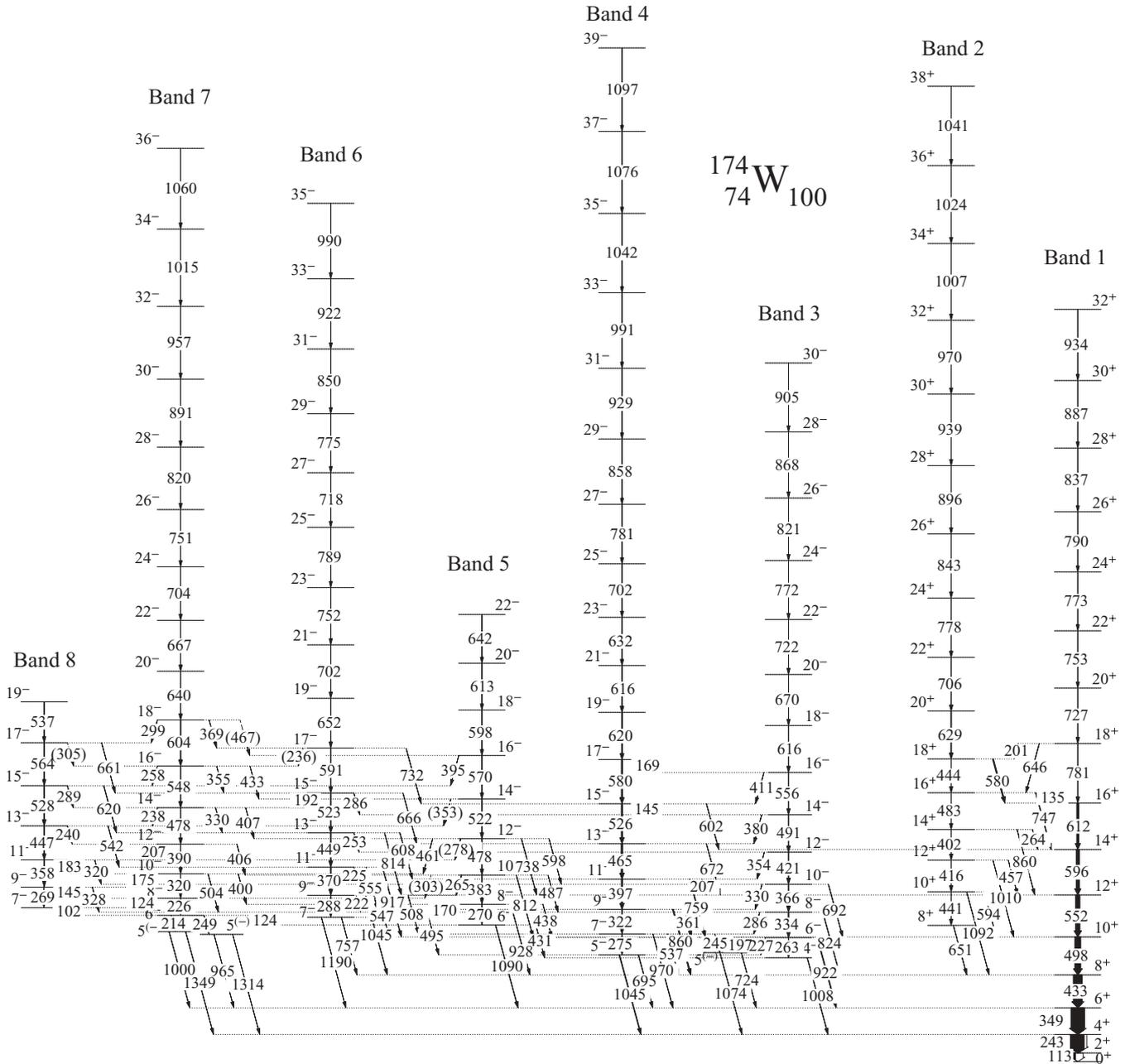


FIG. 1. Extended level scheme for ^{174}W .

for one or both of the parameters typically indicates that the transitions in the list are in coincidence with each other and also with other γ rays, as expected for a band structure. A relatively small number of such potentially promising lists of transitions could therefore be distinguished from the initial, extremely large number of lists. The procedure was tested on high-statistics (2.6×10^9 four-fold coincidences) Gammasphere data for the nucleus ^{174}Hf [12], for which a number of SD structures have been established. Figures 6 and 7 present two-dimensional plots with mean counts and figures of merit for each of the lists, with the search performed on the ^{174}Hf and the present (^{174}W) data, respectively. Coincidence spectra corresponding to a few thousand of the most

promising lists of transitions, i.e., those with the highest values of mean counts and figures of merit, were visually inspected to determine whether a new band structure had been isolated. The efficacy of this technique could be determined from the ^{174}Hf data where a number of SD bands are known. It can be seen from Fig. 6 that most of the reported bands in ^{174}Hf could be easily isolated using this approach. A similar search was performed with the data obtained from the present experiment (Fig. 7); however, no new bands were found in any of the W isotopes populated in the experiment. It should be noted that the sensitivity of the current experiment is considerably less than that of the ^{174}Hf experiment, given the large difference in statistics. This difference is reflected in the mean counts

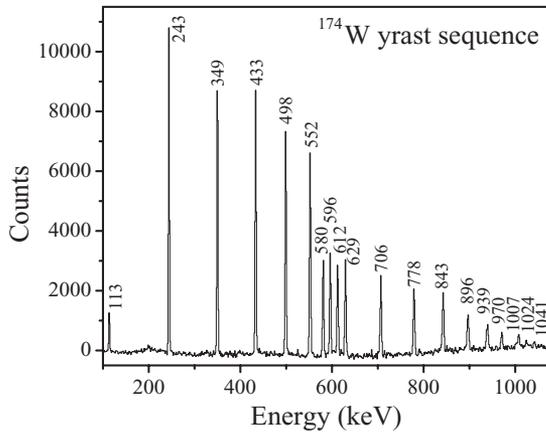


FIG. 2. Summed coincidence spectrum obtained from a RADWARE hypercube [26] by gating on transitions in the yrast sequence above the region of crossing of the *g* and *s* bands, showing all the transitions in the yrast structure of ^{174}W .

and figures of merit obtained (Figs. 6 and 7) in the two cases and may be a contributing factor in the lack of observation of SD structures in ^{174}W . In addition to the search technique described above, a visual inspection of all coincidence spectra using lists of two and three transitions spanning the same energy range and spacings as described earlier was performed on both the coincidence cubes and hypercubes. This search also

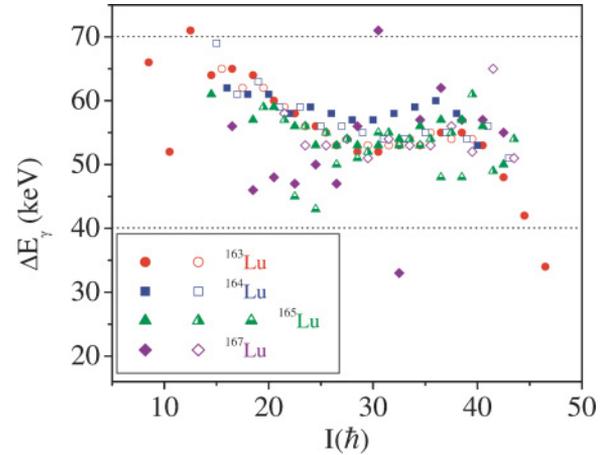


FIG. 4. (Color online) Energy spacings between successive transitions in the TSD bands observed in the Lu isotopes.

did not reveal the presence of new band structures. Based on the results of these searches, we conclude that it is unlikely that strongly populated TSD structures are present in ^{174}W . Weakly populated, nonyrast TSD bands might be present. However, more sensitive experiments will be required to isolate these structures. The new approach described above appears robust and can, therefore, be adopted for a band search in any nucleus.

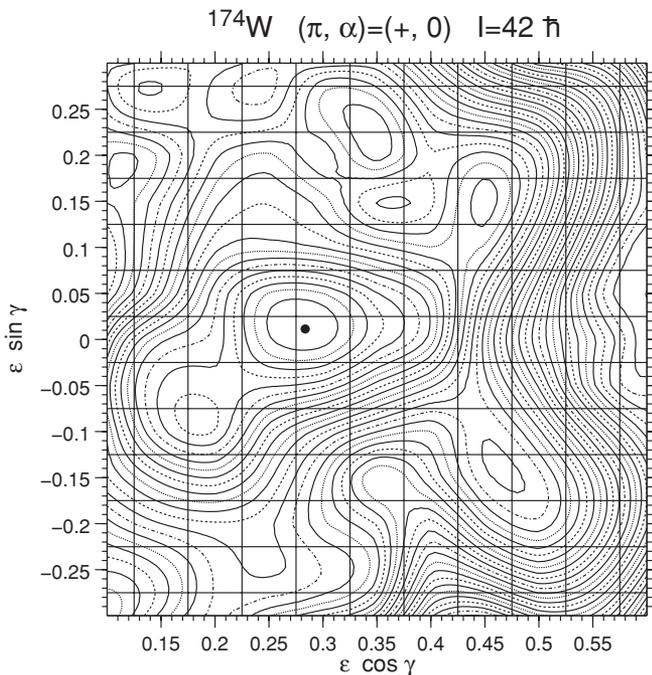


FIG. 3. UC calculation of the potential energy surface for the $(\pi, \alpha) = (+, 0)$ configuration at $I = 42\hbar$ in ^{174}W , with standard Nilsson parameters. The lowest energy minimum, indicated by the black dot, is visible for an axially symmetric prolate shape. Other minima are evident for strongly deformed triaxial shapes at $(\epsilon, \gamma) = (0.47, +18)$ and $(0.48, -17)$, the first of which is approximately 800 keV lower in energy than the second. The spacing between adjacent energy contours is 200 keV.

III. DISCUSSION

A. TSD structures in the $A \approx 160\text{--}170$ region

Considerable experimental effort has recently been focused on attempting to establish TSD structures in the Lu-Hf region; however, confirmation of wobbling excitations has been possible only in the odd-*A* Lu isotopes, $^{161\text{--}167}\text{Lu}$. It is therefore apparent that there is a confluence of conditions favorable to the realization of wobbling excitations in these

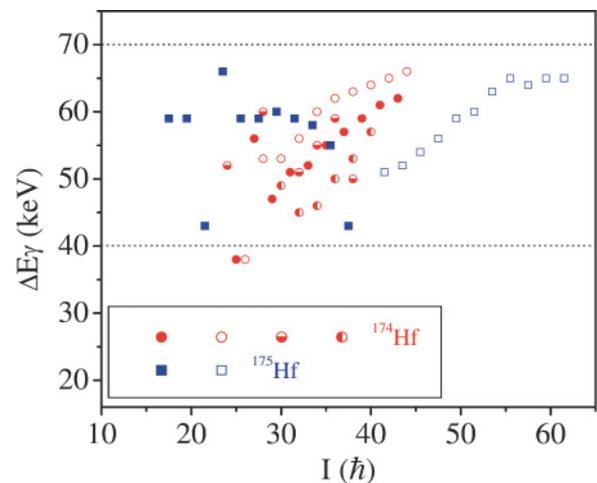


FIG. 5. (Color online) Energy differences between consecutive transitions in the strongly deformed bands in the Hf isotopes.

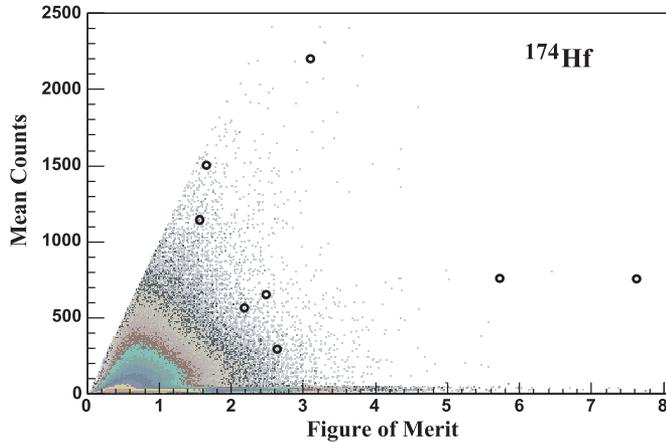


FIG. 6. (Color online) Results of the band search for ^{174}Hf . Each point represents a unique sequence of five transitions which were considered as potential candidates for SD bands, with the various colors representing different densities of such points. An overwhelming number ($>99.9\%$) of the sequences considered could be rejected based on the small values of both parameters. The open circles indicate the values determined for the known SD bands in ^{174}Hf . It is evident that almost all of these bands can be clearly distinguished from the rest of the sequences due to their high figures of merit and/or mean counts.

nuclei. Some of the considerations which distinguish the Lu isotopes from other nuclei are outlined below.

The TSD bands observed in the Lu isotopes have an intrinsic structure involving the $i_{13/2}$ proton orbital. The presence of the aligned $i_{13/2}$ proton appears crucial for the realization of TSD structures and allows the comparison of the interband transition rates between a family of wobbling bands to those predicted by particle-rotor calculations, to ascertain triaxiality [4]. The strongly downsloping $i_{13/2}$, [660]1/2 Nilsson orbital is expected to approach the Fermi surface with increasing rotational frequency and become the lowest positive parity orbital at the highest frequencies, leading to states with

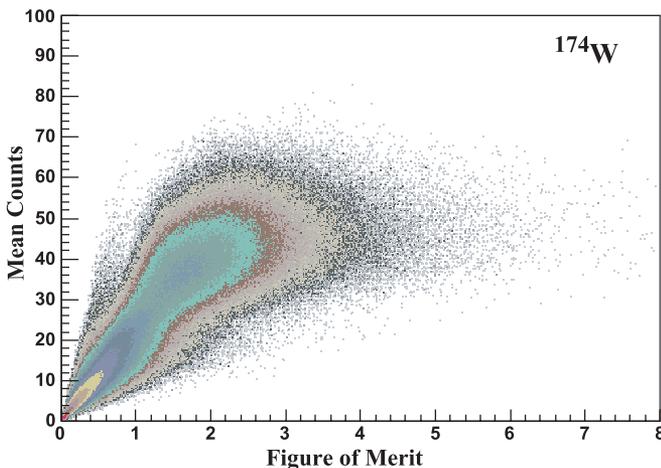


FIG. 7. (Color online) Results of the SD band search in W isotopes from the present data. None of the sequences with high mean counts and/or figures of merit could be associated with SD bands.

deformations considerably larger than those arising from the occupation of other proton orbitals. With increasing proton number above $Z = 71$, the [660]1/2 orbital should be in the vicinity of the Fermi surface at lower rotational frequencies, which should apparently create conditions favorable for the observation of SD structures at lower excitation in nuclei with higher Z . The observation of rotational bands built upon the [660]1/2 orbital in $^{169,171}\text{Ta}$ ($Z = 73$) [21,22] and $^{171,173}\text{Re}$ ($Z = 75$) [27,28], similar to the Lu isotopes, is consistent with this expectation. An additional factor which has to be taken into account is described in a recent work in which TSD bands resulting from particle-hole (p-h) excitations were observed in ^{163}Tm [8]. Wobbling excitations are expected to be favored over p-h excitations only when there is a large energy gap between signature partners of the $i_{13/2}$ proton orbital [8]. This situation is realized at $Z = 71$ in the Lu isotopes. However, the opposite may be true for other odd- Z isotopes, in which case the wobbling bands would be nonyrast. It should be noted that the $N = 94$ gap is also responsible for the stabilization of TSD structures, which means that the even- N Lu and Ta isotopes around $N = 94$ are particularly well placed for the observation of TSD bands. Since ^{163}Lu ($N = 92$) and ^{165}Lu ($N = 94$) have provided the best examples of wobbling excitations so far, the corresponding isotones ^{165}Ta and ^{167}Ta are probably well suited for the observation of TSD bands. Whether wobbling bands can be observed will depend on the relative energies of the wobbling and p-h excitation modes. Excited states have been established in these Ta nuclei; however, the proton $i_{13/2}$ band has not been observed, primarily because these nuclei are difficult to populate with large cross sections in fusion-evaporation reactions.

Recent calculations [29] suggest that there is a substantial gap around $N = 100$ for triaxial deformation. While evidence for this gap is not as conclusive as the $N = 94$ gap, it is thought that this gap is one of the factors responsible for the SD structures in the heavier Hf isotopes. For $N = 100$, as compared with $N = 94$, an effect of the increased neutron number is that the neutron Fermi surface moves closer to the deformation-driving $j_{15/2}$ orbital. This creates a situation appropriate for the realization of SD states with configurations involving both $i_{13/2}$ protons and $j_{15/2}$ neutrons. This is consistent with the large difference in quadrupole moments, excitation energies, and spins between the Lu and Hf isotopes, all of which are considerably higher for the bands in the Hf isotopes, indicating a larger number of quasiparticles in the underlying configurations, as described in earlier works [12,13]. The deformations inferred from the measured transition quadrupole moments for the SD bands in the Lu and Hf isotopes are not consistent with UC predictions, with a greater discrepancy evident for Hf nuclei. Furthermore, there appears to be no evidence of the predicted TSD structures in ^{174}W . This indicates that refinements to the available models are probably necessary.

B. Neutron and proton alignments in ^{174}W

The first alignment in the yrast band of ^{174}W is the $i_{13/2}$ neutron AB crossing at $\hbar\omega = 0.29$ MeV, which was established in our earlier work [23]. The gain in alignment as a result of this

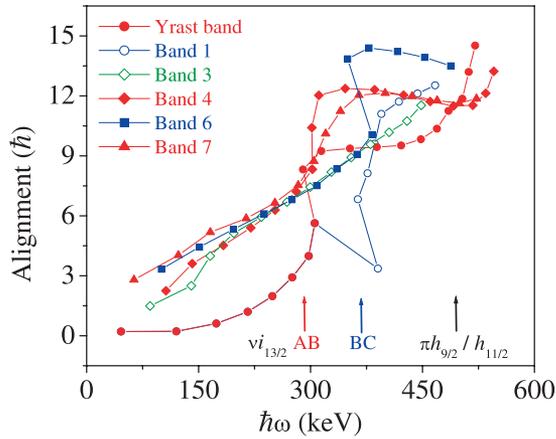


FIG. 8. (Color online) Alignments as a function of rotational frequency for bands in ^{174}W illustrating various neutron and proton crossings.

crossing (Fig. 8) is $9\hbar$, typical of an $i_{13/2}$ neutron AB crossing. The orbitals corresponding to the various quasiparticle labels (for both neutrons and protons) are listed in Table I. Table II summarizes the expected crossing frequencies from our calculations of proton and neutron quasiparticle levels in ^{174}W [23]. The observed frequencies are listed alongside for comparison. Two alignments appear to be visible in the yrast structure following the AB crossing: the first is a gradual upbend around $\hbar\omega = 0.48$ MeV, while the second appears as a more pronounced upbend at $\hbar\omega = 0.51$ MeV (Figs. 8 and 9), possibly due to the effect of the almost simultaneous alignments. Figure 9, which shows the variation of dynamic moment of inertia $J^{(2)}$ with rotational frequency, highlights these two almost simultaneous alignments. No neutron alignments are predicted at these rotational frequencies [23]. The $h_{9/2}$ ($A_p B_p$) and $h_{11/2}$ ($C_p D_p$) proton crossings are, however, expected; therefore, we associate the observed alignments with these crossings. Three nucleon alignments have seldom been observed in a rotational band, one example being the yrast band of ^{162}Hf [15], where there is evidence for crossings in

TABLE I. Neutron and proton Nilsson orbitals in the vicinity of the Fermi surface in ^{174}W and their respective quasiparticle labels. The proton $i_{13/2}$, [660]1/2 state crosses the $d_{5/2}$, [402]5/2 orbital and is the lowest positive parity, proton orbital for rotational frequencies beyond 0.3 MeV.

Orbital	Configuration	$\alpha = +1/2$	$\alpha = -1/2$
Neutrons			
$i_{13/2}$	[633]7/2 ⁺	A	B
$i_{13/2}$	[642]5/2 ⁺	C	D
$p_{3/2}$	[521]1/2 ⁻	E	F
$h_{9/2}$	[512]5/2 ⁻	G	H
Protons			
$h_{9/2}$	[541]1/2 ⁻	A_p	B_p
$h_{11/2}$	[514]9/2 ⁻	C_p	D_p
$d_{5/2}$	[402]5/2 ⁺	E_p	F_p
$g_{7/2}$	[404]7/2 ⁺	G_p	H_p

TABLE II. Predicted and observed neutron and proton crossing frequencies (MeV) in ^{174}W .

Crossing	Predicted	Observed	Band
Neutrons			
AB	0.28	0.29	Yrast
AB	0.28	0.30	Band 4
AB	0.28	0.31	Band 7
BC	0.36	0.38	Band 1
BC	0.36	0.37	Band 6
Protons			
$A_p B_p$	0.47	0.48	Yrast
$C_p D_p$	0.53	0.51	Yrast
$A_p B_p$	0.47	0.45	Band 3
$C_p D_p$	0.53	0.52	Band 4
$C_p D_p$	0.53	0.52	Band 7

the yrast structure involving all three of the above orbitals. It should be noted that the data for the ^{174}W yrast sequence could be consistent with one instead of two almost simultaneous alignments around $\hbar\omega \approx 0.5$ MeV.

Bands 3–8 (Fig. 1) are considerably mixed at lower spins as evidenced by the large number of interband transitions. These bands have contributions to varying degrees from different configurations, as described in our earlier work [23]. At higher spins, each of these bands presumably evolves to a predominantly single, unique underlying structure, since the interband transitions are no longer seen. The associated configurations could be determined by characterizing the newly observed alignments in these bands. Band 3 was assigned an AE configuration in our earlier work. The AB crossing is blocked as a result. The BC crossing is possible; however, it is not clearly evident. The gradual alignment gain for band 3 in the vicinity of the BC crossing frequency could be consistent with a crossing with large interaction strength. The onset of an alignment is seen around 0.45 MeV (Fig. 9) near the predicted $A_p B_p$ crossing frequency. Band 3 can therefore be assigned an $AE A_p B_p$ or an $AE B C A_p B_p$ (if the BC crossing is present) configuration at the highest

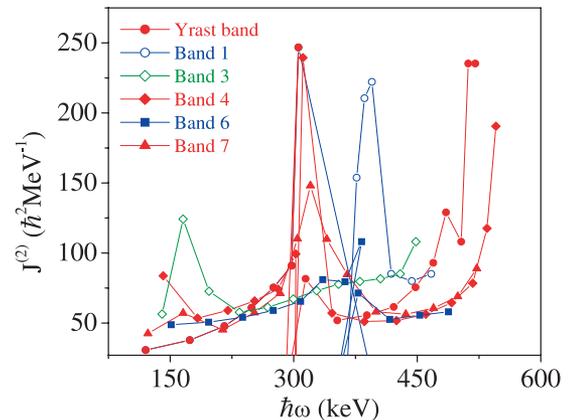


FIG. 9. (Color online) Dynamic moments of inertia $J^{(2)}$ for bands in ^{174}W . The proton crossings are more evident than in the alignment plots of Fig. 8.

spins. Bands 4 and 7 exhibit two upbends, at $\hbar\omega \approx 0.30$ and 0.52 MeV (Figs. 8 and 9). The first one can be associated with the AB crossing, while the second is probably due to the $h_{11/2}$ ($C_p D_p$) crossing, expected at this frequency (Table II). Band 4 should have a predominantly $A_p G_p$ configuration in the region of the AB crossing, in accordance with our earlier speculation [23]. The contribution from the $h_{9/2}$ proton (A_p) accounts for the absence of the $A_p B_p$ crossing in band 4, which is seen in the yrast structure and band 3 at around 0.45 MeV. Band 7 exhibits an alignment behavior similar to band 4. Bands 7 and 8, which are signature partners at low spins, appear to have a two-quasineutron configuration near the bandhead, as reported in our earlier work. However, the two bands differ in their behavior at higher excitation in terms of interband transitions and alignments, and band 7 is observed to considerably higher spins. Therefore, band 7 probably evolves to a $A_p H_p ABC_p D_p$ configuration, suggesting that it is a signature partner of band 4 at the highest spins.

Band 1 is the yrast structure (g band) up to $16\hbar$, beyond which band 2 (s band) becomes favored in energy. Band 1 exhibits an alignment at $\hbar\omega \approx 0.37$ MeV, which is identified with the BC crossing. It should be noted that the observation of the g band and the BC alignment in this structure considerably above the region of the g - and s -band crossing is quite unusual. Band 6, with an AG configuration as described in our earlier work, also shows evidence of the BC crossing at the expected rotational frequency. Therefore, band 6 is assigned an AGBC configuration at high spins.

It can be seen from Table II that the predicted and observed neutron and proton crossing frequencies are in good agreement. The predicted values are from Woods-Saxon (with universal parameters) cranking calculations which have been performed using an empirical value of the pair-gap energy extracted from a five-point formula of odd-even mass

differences [30], as described in our earlier work [23]. Similar agreement is obtained for first nucleon alignments in a large number of rare earth nuclei using this prescription. This highlights the predictive power of cranking calculations and validates the choice of an empirical pair-gap energy.

IV. SUMMARY

A comprehensive search was performed for TSD structures predicted by UC calculations in ^{174}W , by populating excited states at very high spin. A new approach was developed to effectively examine an extremely large number of coincidence spectra which is required for a thorough band search, using the RADWARE suite of programs. No evidence was found for SD bands, suggesting that UC predictions of TSD structures in the $A \approx 170$ region need to be reexamined in some instances or that more sensitive experiments are required to isolate bands that might be very weakly populated. A number of the bands were considerably extended, and nucleon alignments, including the $i_{13/2}$ neutron AB and BC and the $h_{9/2}$ and $h_{11/2}$ proton crossings, were observed. Three nucleon alignments are probably evident in the yrast structure. Woods-Saxon cranking calculations, with an empirical value of the pair-gap energy, provide a good description of all crossing frequencies.

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