Low-momentum interactions in three- and four-nucleon scattering

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Low-momentum two-nucleon interactions obtained with the renormalization group method and the similarity renormalization group method are used to study the cutoff dependence of low-energy $3N$ and $4N$ scattering observables. The residual cutoff dependence arises from omitted short-ranged $3N$ (and higher) forces that are induced by the renormalization group transformations and may help to estimate the sensitivity of various $3N$ and $4N$ scattering observables to short-ranged many-body forces.

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I. INTRODUCTION

Modern few- and many-body calculations of nuclear structure and reactions are based on the picture of pointlike nucleons interacting via two- and three-nucleon potentials. For this purpose, a number of high precision (χ^2/datum $\simeq 1$) but phenomenological meson-exchange models of the two-nucleon ($2N$) force such as the Nijmegen [1], Argonne V18 (AV18) [2], and CD-Bonn [3] potentials have been developed over the past decade. However, with phenomenological models it is not clear how to construct consistent three-nucleon ($3N$) forces and other operators. The lack of a systematic organization or counting scheme results in model-dependent predictions, as there is no way to make controlled comparisons between the different force models. More recently, substantial progress has been made in constructing nuclear interactions from chiral effective field theory (EFT) [4,5], which is based on the most general local Lagrangian with nucleon and pion fields and all possible interactions consistent with the (broken) chiral symmetry of quantum chromodynamics (QCD). In contrast to phenomenological interaction models, the EFT approach is universal and provides a model-independent framework with a systematic organization of consistent $2N$, $3N$, and higher-body forces (and other operators) prescribed by the power counting.

For both phenomenological and EFT potentials, nuclear few- and many-body calculations are complicated by strong short-range repulsion and tensor forces that necessitate highly correlated trial wave functions, nonperturbative resummations, and slowly convergent basis expansions. However, the non-perturbative nature of internucleon interactions is strongly scale dependent and can be radically softened by using the renormalization group (RG) to lower the momentum cutoff that is present in all nuclear interactions. A consequence is that many-body calculations become much more tractable at lower resolutions, resulting in calculations that are amenable to straightforward perturbative methods, simple variational ansätze, and rapidly convergent basis expansions [6–9]. The RG approach has the important advantage of being able to vary the cutoff as a tool to optimize and probe the quality of the many-body solution and to provide estimates of omitted terms in the Hamiltonian.

The above considerations have motivated the construction of low-momentum potentials $V_{\mathrm{low}\,k}$ through the RG method [10,11] and, more recently, by the similarity renormalization group (SRG) method [6,7]. Both methods serve to eliminate the strong coupling between low- and high-momentum modes in the Hamiltonian such that low-energy observables are preserved. In the RG method, one integrates out the problematic high-momentum components of the input interaction above a momentum cutoff Λ, leading to a new energy independent potential $V_{\mathrm{low}\,k}$ that has the same low-energy on-shell transition matrix (t-matrix) as the input potential. In the original approach Λ constitutes a sharp cutoff above which the t-matrix is zero; the method has since been generalized to include a smooth momentum-space regulator to avoid technical difficulties stemming from the sharp cutoff [8]. The SRG method uses a continuous sequence of unitary transformations that weakens off-diagonal matrix elements, driving the Hamiltonian toward a band-diagonal form [6,7]. In contrast to the RG method, SRG preserves both low- and high-energy observables independent of the value of the flow parameter λ that provides a measure of the spread of off-diagonal strength. However, as with the standard RG, the calculation of low-energy observables is decoupled from the high-momentum physics with SRG-evolved potentials (i.e., one can truncate intermediate state summations to low momenta without distorting low-energy observables).

Observables are scale-independent quantities. It is well-known that RG (SRG) transformations generate short-range many-body forces (in principle, up to A-body) that "run" with the cutoff to maintain exact Λ (λ) independence of A-body observables. If the RG transformation is truncated at the $2N$ level, then the resulting cutoff-dependence in $3N$ observables may provide an estimate of omitted short-range $3N$ forces in the Hamiltonian. Along these lines, low-momentum $2N$ potentials have been recently used in three- and four-nucleon

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($4N$) bound-state calculations [12] as a means to assess the size of omitted higher-body forces by varying the cutoff. There, it was found that the induced $3N$ forces due to the truncation to low momentum are of the same order as the so-called "bare" $3N$ forces attributed to integrating out excitations of nucleons. That is, the cutoff dependence of the $3N$ binding energies was rather weak, varying by only 1 MeV over a large cutoff range, which is comparable to the 0.7–1 MeV binding provided by the missing "bare" $3N$ forces in conventional models and EFT calculations. In this sense, the RG evolution to low momentum does not induce strong short-ranged three-body force contributions to these $3N$ bound state observables. Similar results were obtained in $4N$ bound-state calculations, where the various $2N$ $V_{\mathrm{low}\,k}$ calculations did not differ any more from the phenomenological Tjon-line than did calculations using $2N$ plus adjusted $3N$ forces.

In the current study, we extend the cutoff-dependence study of Ref. [12] to $3N$ and $4N$ scattering observables. In particular, we apply RG- and SRG-evolved $2N$ interactions to study how the neutron-deuteron (n-d) elastic vector analyzing power A_y and the space star cross section in n-d breakup change with the cutoff. These being the two major long-standing failures of realistic interactions in their description of $3N$ data at low energy, one would like to use cutoff dependence as a tool to assess the sensitivity of these observables to omitted short-range $3N$ force effects. Likewise, the same applies to observables in $4N$ scattering that show large deviations to data, namely the total neutron-triton (n-t) cross section σ_t around the resonance region at neutron laboratory energy $E_n = 3.5$ MeV and the p-^{3}He A_y that also misses the data by as much as 25–40%.

In Sec. II we study $3N$ observables and in Sec. III $4N$ observables. Finally in Sec. IV we present the conclusions.

II. THREE-NUCLEON OBSERVABLES

The results shown in this section are obtained from the solution of the symmetrized Alt, Grassberger, and Sandhas (AGS) equations [13] for the $3N$ system using the numerical techniques of Ref. [14]. To relate the present work to the findings of Ref. [12] we repeat in Fig. 1 the cutoff dependence of the triton binding energy ϵ_t for CD-Bonn, AV18, and EFT potential at next-to-next-to-next-to leading order (N3LO) [5] based $V_{\mathrm{low}\,k}$ potentials using RG (left side) and SRG (right side) methodologies. In contrast to the calculations of Ref. [12] with a sharp cutoff Λ, for simpler numerics we use a smooth regulator of the form $\exp[-(k^2/\Lambda^2)^8]$. The results are consistent with the ones of Refs. [6,12]. At first glance, the SRG parameter λ that provides a measure of the spread of off-diagonal strength is not obviously related to the cutoff Λ in the RG. However, in Ref. [7] it was found that the "decoupling scale" for SRG-evolved interactions was of order λ. That is, low-energy phase shifts and binding energies are not distorted if high-momentum modes greater than the decoupling scale are set to zero (or any arbitrary value) by hand. Therefore, it is not surprising that the behavior of ϵ_t in terms of Λ or λ is qualitatively quite similar. We emphasize that the existence of cutoffs where ϵ_t agrees with the experimental value does *not* imply vanishing $3N$ forces, as they will contribute to other observables.

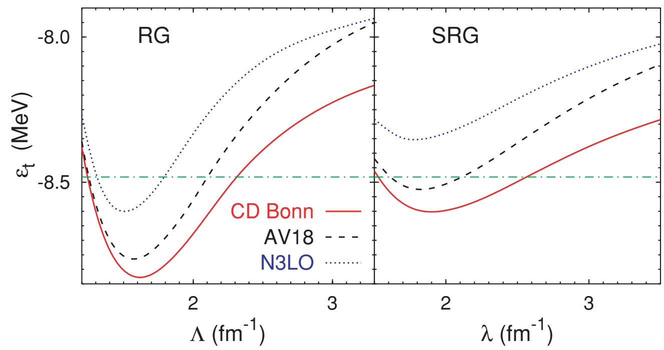

FIG. 1. (Color online) Triton binding energy as function of RG cutoff Λ (left side) and SRG parameter λ (right side). Results derived from CD Bonn (solid curves), AV18 (dashed curves), and N3LO (dotted curves) potentials are shown. The horizontal line at $\epsilon_t = -8.482$ MeV is the experimental value.

The neutron analyzing power A_y in n-d elastic scattering at neutron laboratory energy $E_n = 3$ MeV has a maximum at the center of mass (c.m.) scattering angle $\theta_{\mathrm{c.m.}} = 104$ deg, where the predictions based on realistic interaction models underestimate the experimental value by about 20%. In Fig. 2 we plot the maximum value of A_y as a function of RG cutoff Λ and SRG parameter λ. The cutoff dependence is quite weak, indicating that this observable is not a sensitive probe of short-range force effects. The net variation of A_y over the range of cutoffs is smaller than the discrepancy from experiment of the initial interactions, which implies that short-range $3N$ forces are not likely to solve the A_y problem.

The cutoff dependence is even weaker for the n-d breakup differential cross section in the space star configuration. We demonstrate that in Fig. 3 for the differential cross section close to the center of the space star configuration at $E_n = 13$ MeV; the values measured in two different experiments are shown as a reference. These flat curves are again an indication that space star cross section is not sensitive to short-range physics as already found in conventional calculations with different $2N$ interactions or by adding a $3N$ force [16,17].

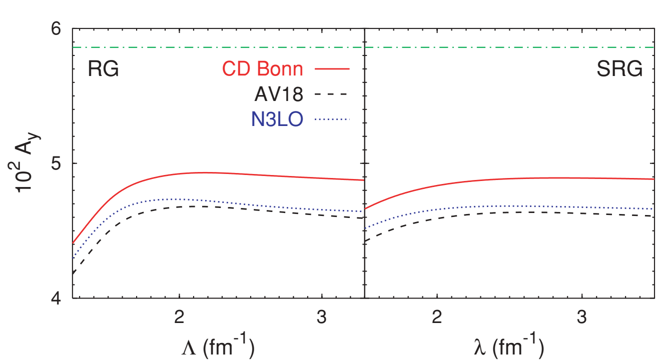

FIG. 2. (Color online) Neutron analyzing power A_y for n-d scattering at $E_n = 3$ MeV and $\theta_{\mathrm{c.m.}} = 104$ deg as function of RG cutoff Λ (left side) and SRG parameter λ (right side). The horizontal line at $10^2 A_y = 5.86$ is the experimental value from Ref. [15].

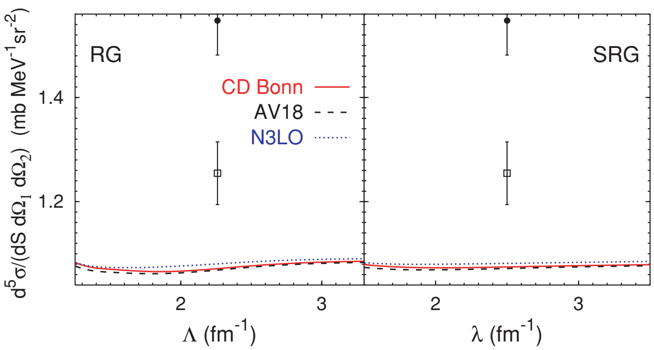

FIG. 3. (Color online) Differential cross section for n-d breakup at $E_n = 13$ MeV in the space star configuration (50.5°, 50.5°, 120°) at arclength $S = 6.25$ MeV as function of RG cutoff Λ (left side) and SRG parameter λ (right side). The experimental data are from Ref. [18] (square) and [19] (circle).

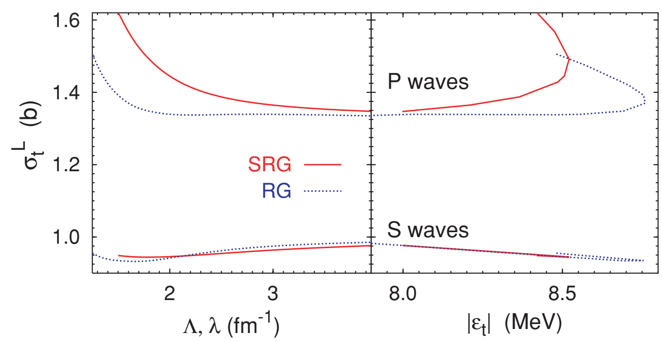

FIG. 5. (Color online) S- and P-wave contributions to the total cross section for n-^{3}H scattering at $E_n = 3.5$ MeV. On the left side they are shown as functions of RG cutoff Λ or SRG parameter λ, whereas on the right side their correlation with the ^{3}H binding energy is shown. The SRG and RG interactions are derived from the AV18 potential.

III. FOUR-NUCLEON OBSERVABLES

The results shown in this section are based on the solution of the AGS equations [20] in a symmetrized form following the technical developments expressed in Refs. [21–23] for all elastic and transfer $4N$ reactions below three-body breakup threshold.

As discussed in Ref. [21], one of the simplest observables in $4N$ scattering is the total n-^{3}H cross section σ_t that exhibits a resonance around $E_n \simeq 3.5$ MeV. This peak of the total cross section results from a complicated interference between ^3P_J n-^{3}H partial waves whose relative strength is sensitive to the realistic $2N$ force one uses. Although at threshold we find the usual scaling between σ_t and ϵ_t (σ_t decreases as $|\epsilon_t|$ increases), at $E_n \simeq 3.5$ MeV we observe a breakdown of scaling when we use N3LO [21], which is a low-momentum potential when compared with the meson-exchange potentials. There N3LO yields the largest cross section while not having the lowest $|\epsilon_t|$.

Furthermore, in Ref. [24] it was found that adding the Urbana IX $3N$ force to AV18 slightly reduces σ_t at the peak while more significantly lowering the cross section at threshold toward the data as expected through scaling.

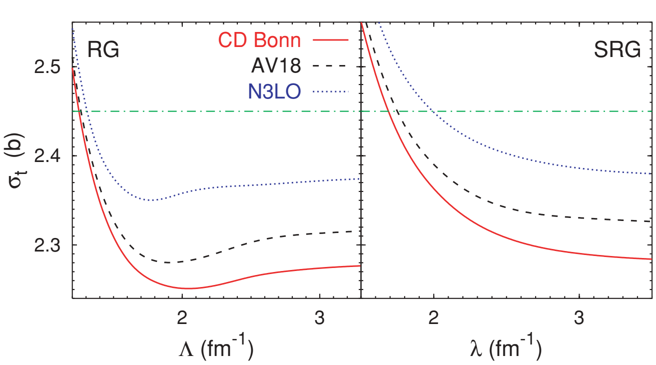

FIG. 4. (Color online) Total cross section for n-^{3}H scattering at $E_n = 3.5$ MeV as function of RG cutoff Λ (left side) and SRG parameter λ (right side). The horizontal line at $\sigma_t = 2.45$ b is the experimental value from Ref. [25].

Therefore, to investigate the effect of low-momentum potentials on σ_t we plot in Fig. 4 the total cross section at the peak versus $\Lambda(\lambda)$. In contrast to studied $3N$ observables, σ_t shows stronger dependence on Λ or λ, which is not surprising because the ratio of triples to pairs increases.

In Fig. 5 we split up the total cross section into n-^{3}H relative S- and P-wave contributions using AV18-based $V_{\mathrm{low}\,k}$. The S-wave contribution scales well with the ^{3}H binding energy; that scaling is slightly violated for RG approach at $\Lambda < 1.5$ fm^{-1}. In contrast, P-waves show no correlation with ϵ_t and are responsible for an increase of the total cross section at small $\Lambda(\lambda)$ values. This is consistent with the findings of Ref. [21]. In Fig. 6 we use the AV18 potential to show σ_t versus

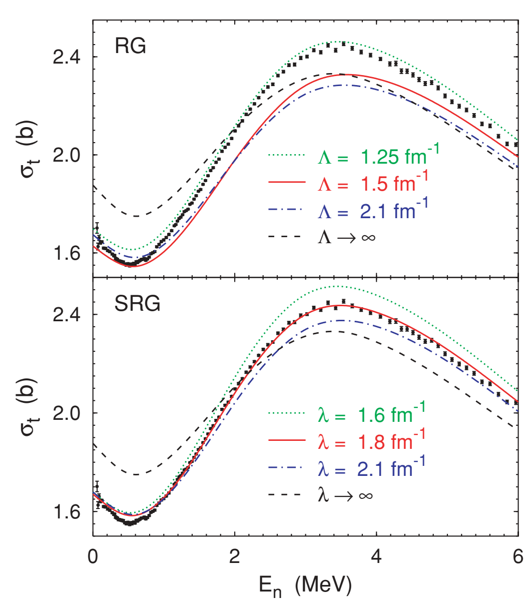

FIG. 6. (Color online) Total cross section for n-^{3}H scattering as function of neutron lab energy for different values of RG cutoff Λ (top) and SRG parameter λ (bottom). All results are derived from the AV18 potential. The predictions of the original AV18 potential (dashed curves) are also shown. The experimental data are from Ref. [25].

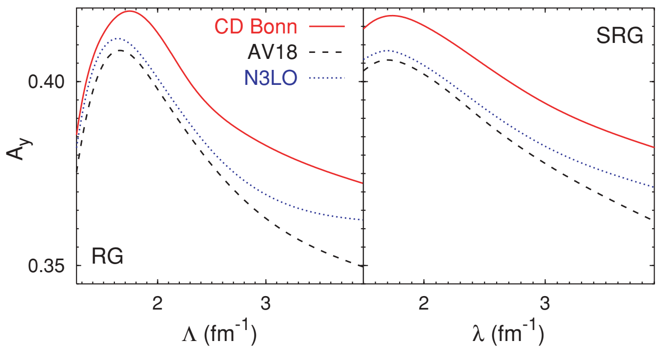

FIG. 7. (Color online) Maximum of the neutron analyzing power A_y for n-^{3}H scattering at $E_n = 3.5$ MeV as function of RG cutoff Λ (left side) and SRG parameter λ (right side).

E_n for the values of $\Lambda(\lambda)$ that fit the experimental triton binding energy and for the one that yields deepest binding. Although one finds that one may describe the total neutron cross section over a wide energy range by using $\Lambda \approx 1.25$ fm^{-1} in RG method or $\lambda \approx 1.8$ fm^{-1} in the SRG approach that also yield reasonable values for ϵ_t, we emphasize once again that these particular values of $\Lambda(\lambda)$ do not imply vanishing $3N$ and $4N$ forces, as they will contribute to other few- and many-nucleon observables, for example, to the ground-state energies of light nuclei that do not match experiment with those "special" choices of $\Lambda(\lambda)$ [9].

There is a clear correlation between maximum values of nucleon analyzing power A_y in p-^{3}He and n-^{3}H scattering [21,22]; we therefore study only the latter case. Though A_y in n-d and n-^{3}H scattering are also correlated to some extent, their dependence on cutoff is different as shown in Fig. 7; it is considerably stronger for n-^{3}H. The largest increase of A_y value at the maximum, by a factor 1.13 (N3LO) to 1.21 (AV18), is observed around cutoff values that yield experimental or deepest binding. However, according to Ref. [22], the experimental A_y value at the maximum for p-^{3}He scattering in the same energy region is larger than theoretical predictions by a factor 1.45 (CD Bonn) to 1.55 (AV18). In Fig. 8

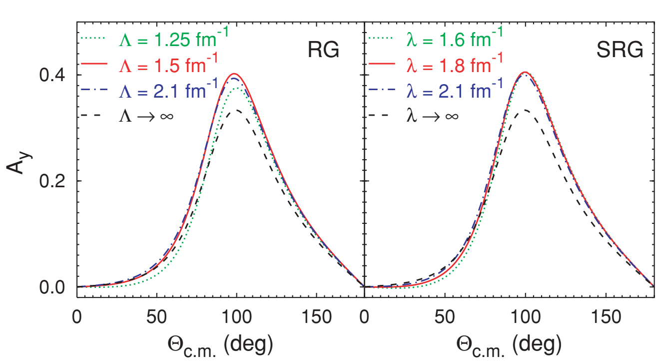

FIG. 8. (Color online) Neutron analyzing power A_y for n-^{3}H scattering at $E_n = 3.5$ MeV as function of center-of-mass scattering angle for different values of RG cutoff Λ (left side) and SRG parameter λ (right side). All results are derived from the AV18 potential. The predictions of the original AV18 potential (dashed curves) are also shown.

we use the AV18 potential to show A_y versus $\theta_{\text{c.m.}}$ for the values of $\Lambda(\lambda)$ that fit the experimental triton binding energy and for the one that yields deepest binding.

IV. CONCLUSIONS

To probe the sensitivity of $3N$ and $4N$ scattering observables to short-range physics, we used AV18, CD Bonn, and N3LO based $V_{\text{low}\,k}$ potentials that are generated through the RG (SRG) method to study their evolution with the cutoff $\Lambda(\lambda)$. Truncating the RG (SRG) equations to the two-body level amounts to neglecting short-ranged $3N$ (and higher) forces that are generated to preserve exact cutoff independence. Therefore, one expects to find residual cutoff dependence in few-body observables when only $2N$ low-momentum interactions are used. That cutoff dependence may provide a measure of the sensitivity of a given observable to omitted short-ranged $3N$ (and higher) forces because the RG evolution does not distort the long-ranged forces arising from pion exchange, provided Λ (or λ) is well above the pion mass, and it is only short-ranged operators that "run" to maintain cutoff independence.

Comparing the results shown in Figs. 2 and 3 with those in Figs. 4 and 7 one cannot help noticing that the cutoff dependence of $3N$ observables is much weaker than the one observed for $4N$ observables. Clearly for the $3N$ observables, the cutoff dependence is rather weak, which seems to imply that short-ranged $3N$ forces are not likely to fix the two long-standing discrepancies with data mentioned above. This is indeed what has been found when the leading missing "bare" $3N$ force, which contains both long- and short-ranged operators, is added. Nucleon-deuteron A_y in elastic scattering and the space star differential cross section for breakup barely change by adding a two-π-exchange $3N$ force [16,26,27], an effective $3N$ force due to the explicit Δ-isobar excitation [17,28], or the more recent leading $3N$ force from chiral EFT [29]. However, there is hope that the subleading long-range $3N$ forces from chiral EFT might be important for the resolution of these problems due to their novel space, spin, and isospin structures.

On the contrary, $4N$ scattering observables seem to be more sensitive to omitted short-ranged many-body forces as demonstrated by the more pronounced dependence on the cutoff. In n-^{3}H scattering at low energy the total cross section σ_t is dominated by S and P waves in the relative n-^{3}H motion. The S waves ($^1S_0$, $^3S_1$) are Pauli repulsive and therefore simply scale with ϵ_t over the whole energy region shown in Fig. 6. Therefore sensitivity to $2N$ forces comes through the P waves ($^3P_0, {}^3P_1 - {}^1P_1, {}^3P_2$), which, in the resonance region, have a very complex behavior with the cutoff parameter, leading to breaking of scaling with ϵ_t. This is consistent with the previous findings [21] obtained with various $2N$ potentials. It also indicates that the σ_t discrepancy may be sensitive to missing short or intermediate range $3N$ forces, in contrast to the p-^{3}He A_y puzzle [22].

From these studies one may conclude that $4N$ scattering observables are more sensitive to short-range physics than the $3N$ observables where, at low energy, they seem to be constrained, to a large extent, by on-shell $2N$ scattering and three-particle unitarity, as was expressed long ago by Brayshaw [30]. Recent developments [26,27] indicate that one needs to fit triton binding energy or neutron-deuteron doublet scattering length to constrain some other $3N$ observables that, unlike A_y, are sensitive to scaling. Nevertheless, this is already fine-tuning on top of results that are already very close to those of the experimental data. This is not the case for low-energy $4N$ observables.

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