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Heart-shaped nuclei: Condensation of rotational-aligned octupole phonons

S. Frauendorf
ISP, Forschungszentrum Dresden-Rossendorf, Dresden, Germany,
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA
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The strong octupole correlations in the mass region $A \approx 226$ are interpreted as rotation-induced condensation of octupole phonons having their angular momentum aligned with the rotational axis. Discrete phonon energy and parity conservation generate oscillations of the energy difference between the lowest rotational bands with positive and negative parity. Anharmonicities tend to synchronize the rotation of the condensate and the quadrupole shape of the nucleus forming a rotating heart shape.

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The appearance of rotational bands with alternating parity, such as the one shown in Fig. 1, has been attributed to these nuclei having an intrinsic pear shape, which is generated by combining an axial octupole (Y_{30}) with an axial quadrupole (Y_{20}) shape [1]. The pear shape has the same symmetry as the more general heart shape in Fig. 1. The existence of a reflection plane perpendicular to the rotational axis (z) has the consequence that space inversion $\mathcal{P}$ is equivalent to $\mathcal{R}_z(\pi)$, a rotation by π about the z axis. The invariance with respect to $\mathcal{S} = \mathcal{P}\mathcal{R}_z(\pi)$ implies the existence of the simplex quantum number $\mathcal{S}|\rangle = e^{-i\pi\sigma}|\rangle$ for the deformed intrinsic (mean field) state, which leads to the alternating spin-parity sequence $\pi = (-1)^{I-\sigma}$ of the rotational bands [2,3] (cf. also Ref. [4]). The lowest band in even-even nuclei has $\sigma = 0$, like in Fig. 1. Strong stretched dipole transitions result from combining the octupole distortion of the shape with the quadrupole one, which generates a collective charge dipole (cf. eg. [1]).

For a well-developed pear shape one expects that the negative parity states interleave with the positive parity ones. However, in all alternating bands of even-even nuclei ($\sigma = 0$) the negative parity sequence is up-shifted relative to the positive parity sequence, approaching it with increasing spin. This has been interpreted as a rapid tunneling mode between the two pear shapes related by $\mathcal{P}$, which is progressively suppressed with increasing spin and results in merging of the two sequences [1,5–8]. Figure 2(a) shows the energy difference $\Delta E(I)$ between the $\pi = -$ and $\pi = +$ sequences in $^{220}_{88}\text{Ra}_{132}$. Obviously, the two sequences do not merge but cross. Figure 2(b) displays the angular momentum as a function of the rotational frequency ω, which is the slope of $E(I)$. The $\pi = -$ sequence starts with about $3\hbar$ more angular momentum than the $\pi = +$ sequence but gains less, such that at high ω the $\pi = +$ sequence has more. The conventional interpretation in terms of pear shape and tunneling does not account for these observations in a simple way. In this rapid communication we suggest an alternative interpretation: condensation of rotational-aligned octupole phonons.

To present the concept, we temporally assume that the quadrupole deformed nucleus is a rigid rotor with the moment of inertia $\mathcal{J}$, that the octupole vibration is harmonic with frequency Ω_3, and that there is no interaction between the octupole phonons and the quadrupole deformed potential of nucleus. The modifications caused by anharmonicities, phonon interaction, and nonrigid rotation will be discussed later. The energy of the nucleus in the n-boson state rotating with the angular velocity ω is

$$E_n = \hbar\Omega_3(n + 1/2) + \frac{\omega^2}{2}\mathcal{J}, \tag{1}$$

which is the sum of the boson excitation energy and the rotational energy, respectively. The state with maximal angular momentum for given energy (yrast state) is generated by aligning the angular momenta of all bosons with the axis of rotation. If one boson carries $i \approx 3\hbar$ of angular momentum, the total angular momentum and the energy of the aligned n-boson state are, respectively, given by

$$I = ni + \omega\mathcal{J}, \tag{2}$$

$$E_n(I) = \hbar\Omega_3(n + 1/2) + \frac{(I - ni)^2}{2\mathcal{J}}. \tag{3}$$

Figure 3(a) shows the resulting yrast region. At $I_n = \hbar\Omega_3\mathcal{J}/i + i(n + 1/2)$ it becomes energetically favorable to increase I by exciting an aligned phonon instead of further increasing the angular velocity ω. Figure 3(b) shows the energies of the multiphonon states in the frame rotating with the frequency ω,

$$E'(\omega) = E_n(I) - \omega I = \hbar\Omega_3(n + 1/2) - ni\omega - \frac{\omega^2}{2}\mathcal{J}. \tag{4}$$

These 'Routhians' cross at one and the same the critical angular frequency $\omega_c = \Omega_3/i$, which means there is a boson condensation when the intensive variable ω takes the critical value ω_c. It has the characteristic features of a quantum phase transition in a small system. Figure 3(a) illustrates how the transition shows up in the relation $E(I)$ between the extensive variables E and I. It is spread over many quantal states (the yrast line), which are distinguished by the discrete variable I. The energy of these yrast states grows linearly with I *on the average*, $\bar{E}(I) = \text{const} + \omega_c I$, while the individual energies $E(I)$ fluctuate around it. The critical frequency ω_c is the slope of the tangent to the yrast sequence. In a macroscopic system, the fluctuations due to quantization become negligible, and one has the linear relation $dE/dI = \omega_c$ characteristic for a phase transition, which corresponds to a vertical section of

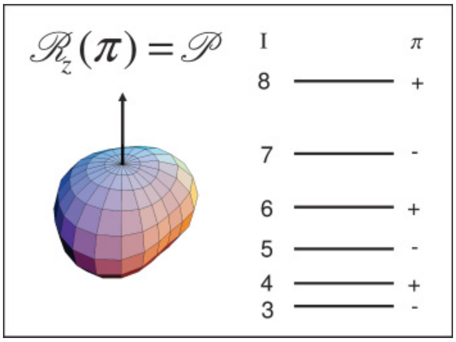

FIG. 1. (Color online) Symmetry of a heart-shaped nucleus and the spin-parity sequence of the rotational ground band (even-even, $\sigma = 0$).

the function $I(\omega)$ at ω_c. In a small system, as the nucleus, the values $\omega(I)$ will fluctuate around ω_c.

The above-presented idealized scenario assumes harmonic, noninteracting bosons. Now we discuss the consequences of the substantial anharmonicities of the octupole mode in real nuclei (cf. Ref. [7]). The energies of n-phonon states grow faster than $n\hbar\Omega_3$, i.e., $E_2 > 2E1, E_3 > 3/2E_2, \ldots$, which signalizes the vicinity of instability to stable octupole deformation. Figure 3(c) shows the consequences: There is no longer a sharp value of ω_c, as discussed for the harmonic case. Rather the average angular frequency ω [average slope of $E(I)$] increases very slowly in the condensation region. In addition, the anharmonicities lead to an interaction between the n-phonon states, which causes a repulsion between crossing bands of the *same parity*, because due to parity conservation only the states with even n or with odd n mix. Figure 3(c) illustrates the case of moderate level repulsion: The order of the $\pi = +$ and $\pi = -$ bands alternates with increasing I. Each change of sign indicates that once more the phonon has entered the condensate. Real nuclei are not rigid rotors, which means the functions $E_n(I)$ grow somewhat slower with increasing I than the parabolas in Fig. 3. Nevertheless, the sequence of the scallop-shaped curves is retained, which represents the boson condensation.

The first steps of the condensation are seen in ^{220}Ra, which correspond to the realistic case Fig. 3(c). Figure 2(a) shows the energy difference $\Delta E = E_- - E_+$ between the lowest rotational sequence of each parity. The one-phonon band crosses the zero-phonon band before it feels much of the two-phonon band. At the crossing, ΔE changes sign. When the two-phonon band encounters the zero-phonon one, the two states mix and exchange character (avoided crossing). The level repulsion attenuates the growth of $-\Delta E$, which starts decreasing when the $\pi = +$ band has become predominantly the two-phonon state. When the two-phonon band crosses the one-phonon band, ΔE changes sign again. Its growth is attenuated and reversed when the avoided crossing between the one- and three-phonon bands is encountered, the beginning of which is still visible. Figure 2(b) illustrates how the phonon condensation shows up in the functions $I(\omega)$. The $\pi = -$ one-phonon band starts with $3\hbar$ more than the $\pi = +$ zero-phonon band at the same ω, which is the expected angular momentum carried by an aligned octupole phonon. The difference decreases, when the $\pi = +$ two-phonon band, which carries additional $6\hbar$, starts mixing into the zero-phonon band. The (interpolated) $\pi = -$ and $\pi = +$ bands have equal angular momentum at the frequency of maximal mixing ($0.5 \times 0\hbar + 0.5 \times 6\hbar = 3\hbar$). Near the one-two-phonon band crossing at $I = 24$, where the mixing is small, the angular momentum difference is $-3\hbar$ (at $\hbar\omega = 0.26$ MeV). The other indication for condensation is the slow growth of ω above 0.25 MeV, in particular for $\pi = +$. If the phonon spectrum was harmonic the two functions $I_\pm(\omega)$ would be about vertical, oscillating around the critical frequency ω_c. It would be interesting to see if the oscillations of ΔE and $I(\omega)$, which are the hallmarks of the condensation, continue in experiment as shown in Fig. 3(c).

Classically, the n-phonon states correspond to an octupole wave running over the surface of the deformed nucleus with the angular velocity $\omega_3 = \Omega_3/3$ [see Fig. 4(a)]. The factor 1/3 accounts for the fact that the octupole wave reaches an identical position after turning 120°. The quadrupole distortion rotates with the angular velocity ω. If $\omega = \omega_3$, the quadrupole and octupole distortions combine to a heart shape rotating with ω. In general, $\omega \neq \omega_3$, which means the octupole distortion travels with angular velocity $\omega_3 - \omega$ relative to the quadrupole-deformed shape [cf. Figs. 4(b) and 4(c)]. At the yrast line, the frequencies ω, which is the slope of $E_n(I)$ in Fig. 3(c), and $\omega_3 = \omega_c$, which is the slope of the envelope, tend to be equal. However, they cannot completely synchronize because the phonon number is discrete. The frequency difference is reflected by the fluctuations of the yrast line above the envelope. They become negligible for a macroscopic system, for which the boson number n can be considered as a continuous variable measuring the amplitude of the oscillation. The frequency difference suppresses the electric dipole transitions of the

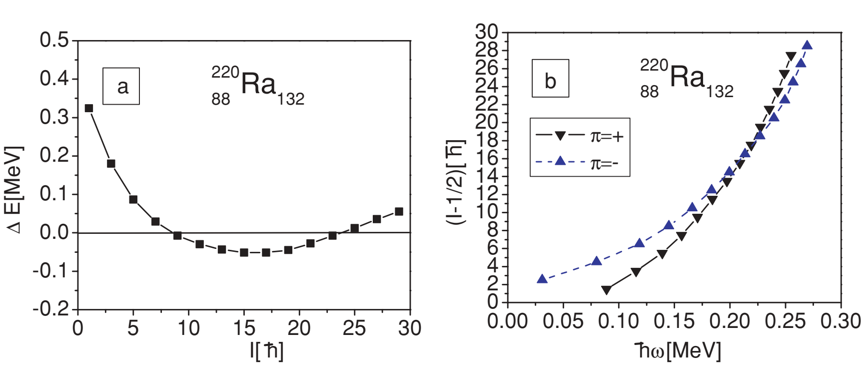

FIG. 2. (Color online) (a) Energy difference $\Delta E(I) = E_-(I) - (E_+(I+1) + E_+(I-1))/2$ between the positive and negative parity yrast sequences in ^{220}Ra. (b) Angular momentum $I(\omega)$ as a function of the angular frequency $\hbar\omega(I) = (E(I) - E(I-2))/2$ of the two sequences. Data are from the ENSDF base.

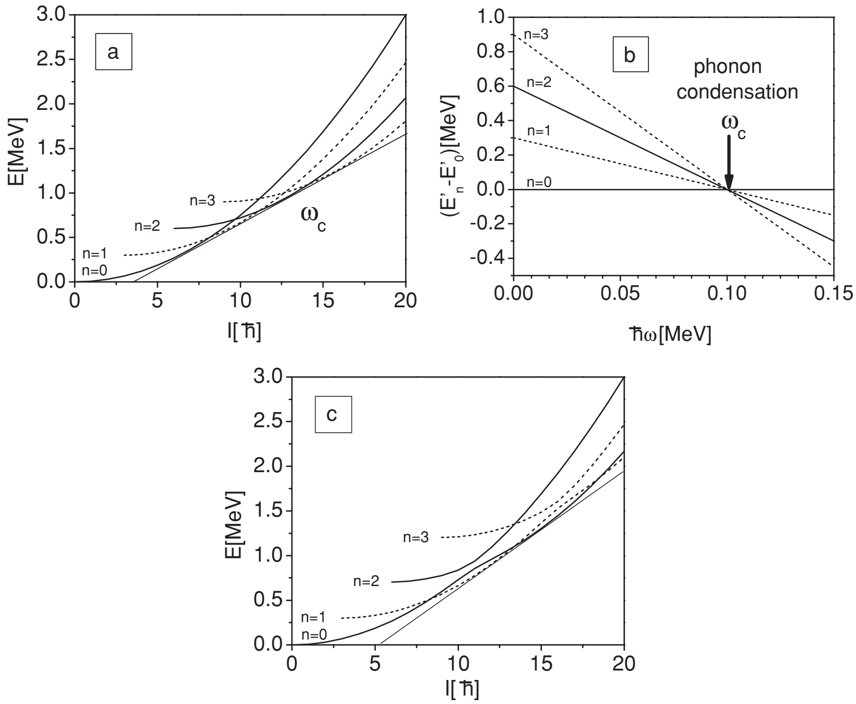

FIG. 3. (a) Energies of aligned octupole multiphonon bands. Full lines show $\pi = +$ states and dashed lines show $\pi = -$ states. The straight line is the common tangent of the curves with the slope ω_c. (b) Energy in the rotating frame (Routhian), relative to the zero-phonon state. (c) Same as Panel (a), assuming an interaction between the phonons and anharmonicities such that $E_3 > 3/2E_2 > 2E_1$. The slope of the tangent, ω, increases slowly during condensation.

type $I^+ \rightarrow (I-1)^-$. The difference $\omega(I^+) - \omega((I-1)^-) = [I - ((I-1)-3)]/\mathcal{J}$ corresponds to a decrease of the angular momentum of the quadrupole rotor by $4\hbar$. The dipole moment is proportional to the product of the quadrupole moment Q_2 and the octupole moment Q_3 (cf. Refs. [7] and [1]). The transition is suppressed because Q_2 can only transfer $\pm 2\hbar$ to the rotor. The difference $\omega(I^-) - \omega((I-1)^+) = [I - 3 - (I-1)]/\mathcal{J}$ corresponds to an increase of $2\hbar$, which can be transferred by Q_2. The transitions $I^- \rightarrow (I-1)^+$ are allowed.

The anharmonicities, which cause the repulsion between the bands with n and $n+2$ phonons shown in Fig. 3(c), attenuate the frequency difference and as a consequence the suppression of the $I^+ \rightarrow (I-1)^-$ transitions. For sufficiently strong interaction, the $\pi = +$ and $\pi = -$ sequences eventually merge into a smooth sequence of good simplex $\pi = (-1)^{I-\sigma}$, which characterizes a static heart shape. In this case the transitions $I^+ \rightarrow (I-1)^-$ and $I^- \rightarrow (I-1)^+$ have equal

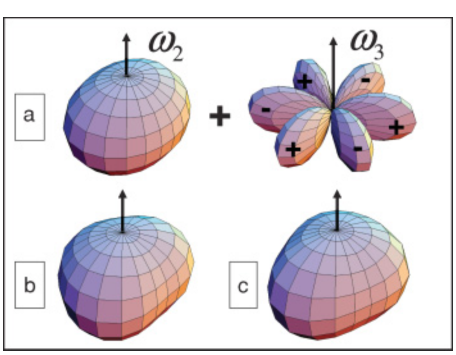

FIG. 4. (Color online) An octupole wave traveling over the surface of a quadrupole-deformed nucleus. In Panel (c), the octupole wave has turned by 90° as compared to that in Panel (b).

strength, as expected for a rotating static dipole. (It is noted that the discussion above does not assume rigid rotation. The $\mathcal{J}$ in the estimates is to be understood as the dynamical moment of inertia $\mathcal{J}^{(2)}$.)

Figure 5 shows the Th isotopes. For $N = 130, 132$, the $\pi = -$ sequence starts with the extra $\approx 3\hbar$ of the rotational-aligned octupole phonon. Its distance to the $\pi = +$ sequence, $\Delta E(I)$, changes sign at $I_1 = \hbar\Omega_3\mathcal{J}/i + i/2$. The mixing and repulsion of the zero- and two-phonon bands increases with N, which subsequently delays and attenuates the cross-over phenomenon. For $N = 134$, the $\pi = -$ sequence starts with only $\approx 2\hbar$ of extra angular momentum, because the $\pi = +$ sequence contains a substantial two-phonon component right from the start, which reduces the angular momentum difference with the $\pi = -$ sequence. The tendency is continued for $N = 136$. The two sequences start with an angular momentum difference of only $\approx 1.5\hbar$ and rather merge than cross. The progressive synchronization of the angular velocities of the $\pi = +$ and $-$ sequences with increasing N signalizes the appearance of a static heart shape. The traditional view is that the nucleus attains an octupole deformation and there is strong inversion tunneling, which is suppressed by rotation [1,5,6]. The two interpretation do not contradict each other. From the energies and the $B(E3)$ values of the 3^- states one can estimate [7] that the average elongation of the two-phonon state, $\sqrt{\langle 2|\beta_3^2|2\rangle} \sim 0.1$, is comparable with the static deformation given by mean field calculations around $N = 134$ [1]. Such a deformation is generated by a strong admixture of the two-phonon state and some admixture of the four-phonon state to the zero-phonon state. However, we argue that the heart shape should be energetically preferred over the pear shape studied so far, which needs to be confirmed by appropriate mean field calculations. Such studies are on the way [11].

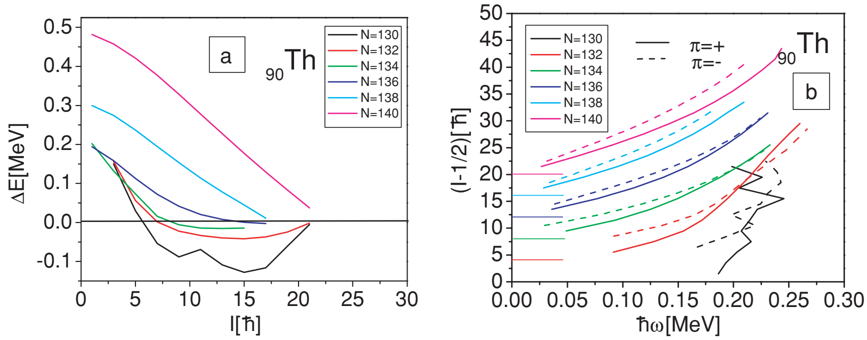

FIG. 5. (Color online) Same as Fig. 2 except for the Th isotopes. Data are from the ENSDF base and Ref. [9]. For more expanded versions of Panel (b) see Fig. 15 of Ref. [10].

The decrease of 3^- energy and the increase of the anharmonicities and phonon coupling with N reflect the neutron Fermi level moving into the region where $g_{9/2}$ and low-$\Omega j_{15/2}$ orbitals are close together. As seen in the Nilsson diagram in Figs. 5–3 of Ref. [7], for $\delta = 0.2$ the orbitals [633]5/2 ($g_{9/2}$) and [761]3/2 ($j_{15/2}$) are both at the Fermi surface for $N = 136$ and the [770]1/2 orbital is the next below. The coupling of these orbitals generates increasingly soft, anharmonic octupole phonons that easily align with the rotational axis. For $N > 136$, the 3^- energy increases strongly with N. The angular momentum difference of $i \approx 3$ near $\Delta E = 0$ indicates that the coupling between the zero- and two-phonon bands must be reduced. This can be attributed to the Fermi level moving from the $g_{9/2}$ to the $d_{5/2}$–$i_{11/2}$ orbitals ([622]5/2 and [631]1/2 in Figs. 5–3 of Ref. [7]), which couple much weaker with the $j_{15/2}$ orbitals via the octupole field. The larger quadrupole deformation in the heavier isotopes increases the quadrupole-octupole coupling, which favors the $K = 0$ octupole phonon. As a consequence, the full alignment of the octupole phonon is only reached for $\hbar\omega > 0.15$ MeV, where the Coriolis force overcomes the coupling of the phonons to the quadrupole-deformed potential. In the light isotopes the phonons align immediately, because the deformation is small.

As seen in Figs. 5–3 of Ref. [7], the orbitals [622]5/2 and [642]7/2, which have a strong $g_{9/2}$ component at the larger deformation of $\delta \approx 0.3$, come close to the Fermi surface around $N = 148$. Their coupling with the nearby $j_{15/2}$ orbitals [743]7/2 and [734]9/2 may lead to rotation-induced condensation of octupole phonons again. An enhancement of octupole correlations around $N = 146$ has been reported in Refs. [12] and [13].

For $N = 130$ the quadrupole deformation is no longer stable. The yrast line is formed by a combined condensation of quadrupole and octupole phonons. As suggested in Ref. [9], the rotating condensate forms a heart-shaped wave running over the nuclear surface. The condensation is seen in Fig. 5(b) as the roughly vertical $\pi = +$ and $\pi = -$ sequences fluctuating around the critical frequency $\hbar\omega_c \approx 0.21$ MeV. The $Z = 88$ and 86 isotones behave similarly [10].

In summary, the strong octupole correlations of rotational bands in the light actinides may be interpreted as the condensation of rotational-aligned octupole phonons. Condensation sets in when the nucleus reaches the angular velocity of the condensate. During the condensation of harmonic phonons the energy of the yrast states increases on the average linearly with angular momentum. The discreteness of the phonon energy combined with parity conservation causes oscillations of the lowest positive and negative parity rotational bands around this classical mean value, which are in antiphase. The mismatch of their angular velocities causes a preference of electromagnetic dipole transitions $I^- \rightarrow (I-1)^+$ over $I^+ \rightarrow (I-1)^-$. The first oscillations of this quantum phase transition are clearly seen in the $N = 132$ isotones up to the encounter of the three phonon state. The anharmonicity and interaction of the phonons increase with N, which softens the phase transition and attenuates the oscillations. As a result, the angular velocities of the octupole condensate and the quadrupole shape of the nucleus are progressively synchronized approaching a rotating static heart shape. The $N = 136$ isotones are closest to this limit, which shows up as rotational band of levels with alternating parity that interleave and equal strength of dipole transitions in both directions. For larger N the phonons become again more harmonic. The strong octupole correlations of rotational bands observed in other mass regions can also be interpreted as phonon condensation.

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