PHYSICAL REVIEW C 77, 014602 (2008)

Separation of two partially coherent beams from their interference pattern

R. da Silveira
Groupe de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France

Ch. Leclercq-Willain*
Physique Nucléaire Théorique et Physique Mathématique, CP229, Université Libre de Bruxelles, B-1050 Brussels, Belgium
(Received 25 July 2007; published 4 January 2008)

Two inequalities are derived which set upper and lower bounds on the intensity of two partially coherent interfering beams. These bounds are obtained from the observed interference pattern. Further, one establishes the connection between these inequalities and the "duality relation" obtained in the context of which-way information in two-path interferometry experiments.

DOI: 10.1103/PhysRevC.77.014602 PACS number(s): 25.70.−z, 03.65.Ta, 25.60.Bx, 42.25.Hz

For a better understanding of the aim of this paper, let us consider an experimental device where an incident beam is split into two beams of intensities σ_1 and σ_2, respectively, which then interfere upon recombining and are counted by a detector. The latter records an interference pattern given by

$$\sigma = \sigma_1 + \sigma_2 + 2\alpha(\sigma_1\sigma_2)^{\frac{1}{2}} \cos\phi, \tag{1}$$

where ϕ is the phase difference between the two beams and α is a parameter which accounts for the possible loss of coherence in one or the other of them (or in both):

(i) fully coherent beams: $\alpha = 1$,
(ii) partially coherent beams: $0 < \alpha < 1$,
(iii) fully incoherent beams: $\alpha = 0$.

Total or partial coherence occurs in a large variety of experiments [1] such as for instance, neutron and atom interferometry or atomic and nuclear collisions. In the case of fully coherent beams, a simple approximate method allowing to separate the interfering components σ_1 and σ_2 from the experimental interference pattern has already been developed [2].

The purpose of the present paper is to extend this method to partially coherent beams. To achieve this we shall derive two inequalities which set upper and lower bounds on the values of the two interfering components σ_1 and σ_2. These bounds can be obtained from the observed interference pattern by using an approximate method [2]. Further we point out the connection between these inequalities and the "duality relation" [3,4], an inequality obtained in the context of which-way information in two-path interferometry experiments. In some sense, this inequality can be regarded as quantifying the notion of wave-particle duality. This will be illustrated by applying the "duality relation" to experimental data of elastic scattering processes.

We first notice that the interference pattern σ in Eq. (1) oscillates between two limiting curves or "envelopes" $E_\pm$ defined by

$$E_\pm = \sigma_1 + \sigma_2 \pm 2\alpha(\sigma_1\sigma_2)^{\frac{1}{2}} \tag{2}$$

with $0 \leqslant \alpha \leqslant 1$. Assuming $\sigma_1 \geqslant \sigma_2$, one readily obtains

$$E_+^{\frac{1}{2}} \leqslant \sigma_1^{\frac{1}{2}} + \sigma_2^{\frac{1}{2}}, \tag{3}$$

$$E_-^{\frac{1}{2}} \geqslant \sigma_1^{\frac{1}{2}} - \sigma_2^{\frac{1}{2}}. \tag{4}$$

This further leads, using the relation

$$\frac{E_+ + E_-}{2} = \sigma_1 + \sigma_2,$$

to the inequalities

$$\sigma_1 \leqslant \bar{\sigma}_1 = \frac{\left(E_+^{\frac{1}{2}} + E_-^{\frac{1}{2}}\right)^2}{4}, \tag{5}$$

$$\sigma_2 \geqslant \bar{\sigma}_2 = \frac{\left(E_+^{\frac{1}{2}} - E_-^{\frac{1}{2}}\right)^2}{4}, \tag{6}$$

where the equal sign holds for fully coherent beams.

To derive the "duality relation" from Eqs. (5) and (6), let us recall that which-way information from the observed interference pattern in a two-path interferometer is simply defined as the difference between the probabilities $\frac{\sigma_1}{\sigma_1+\sigma_2}$ and $\frac{\sigma_2}{\sigma_1+\sigma_2}$ that a particle is in beam 1 or 2, respectively. The magnitude of this difference

$$P = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} \tag{7}$$

is the predictability of the way followed through the interferometer. Inserting Eqs. (5) and (6) into Eq. (7), on gets the inequality

$$P \leqslant \left[1 - \left(\frac{E_+ - E_-}{E_+ + E_-}\right)^2\right]^{\frac{1}{2}}.$$

On the other side, the fringe visibility or contrast of the interference pattern W is defined as

$$W = \frac{E_+ - E_-}{E_+ + E_-}$$

and one so obtains the "duality relation" [3,4]

$$P^2 + W^2 \leqslant 1. \tag{8}$$

*cwillain@ulb.ac.be

Let us now consider the application of inequalities (5) and (6) to the experimental data. An interesting case of two-path interference pattern is provided by the angular distributions in nucleus-nucleus collisions for which a large amount of experimental data is available. Furthermore it happens that it is precisely the domain best know by the authors. We shall consider collisions involving two identical nuclei either in the entrance or in the exit channel (or in both) for which the angular dependence of the differential cross section is given in Eq. (1). In this case σ_1 and σ_2 become respectively the differential cross sections for particles scattered through the angles θ and $\pi - \theta$. The expression (1) then applies to various collision processes [2]: i) elastic scattering of identical spinless particles or of nonzero spin particles when spin dependent forces can be neglected; ii) inelastic and transfer reactions involving identical nuclei under particular conditions.

The upper part of Fig. 1 shows the angular distribution obtained in the elastic scattering of ^{12}C on ^{12}C at $E_{\mathrm{c.m.}} = 5$ MeV [5]. As the ^{12}C nucleus has spin 0, the symmetrization procedure gives $\alpha = 1$ in Eq. (1) and therefore one expects that the interference pattern results from full coherent beams. In this case the equal sign holds in Eqs. (5) and (6) and the $\sigma_{1,2}$ are simply given by $\sigma_{1,2} = \bar{\sigma}_{1,2}$.

To obtain $\bar{\sigma}_{1,2}$, one should be able to draw the limiting curves or envelopes $E_\pm$ from the experimental data. Obviously, there is no unique and rigorous way to do so. We adopt as a reasonable criterion for drawing such curves, to assume that the envelope $E_+(E_-)$ should be the "simplest" smooth curve passing though the maxima (minima) of the oscillatory angular distribution. This procedure has already proved accurate in decomposing angular distributions into interfering components [2]. The interpolated envelopes $E_\pm$ are shown in the upper part of the figure. The resulting values for $\bar{\sigma}_{1,2}$ are shown in the lower part of the figure, by squares ($\bar{\sigma}_1$) and triangles ($\bar{\sigma}_2$). This result can be compared with almost exact theoretical predictions. In fact, since the collision occurs under the Coulomb barrier (≈8.5 MeV) only the Coulomb field makes non-negligible contributions to the elastic scattering [7]. In this case $\sigma_{1,2}$ are given by the familiar Rutherford formula

$$\sigma_1 = \sigma_R(\theta) = \frac{\left(\frac{\eta}{2k}\right)^2}{\sin^4\left(\frac{\theta}{2}\right)}, \tag{9}$$

$$\sigma_2 = \sigma_R(\pi - \theta), \tag{10}$$

where η is the Sommerfeld parameter. The resulting expression for σ is the well-known Mott differential cross section. The calculated curves $\sigma_R(\theta)$ and $\sigma_R(\pi - \theta)$ are shown in the lower part of the figure. As seen, these curves and the values obtained

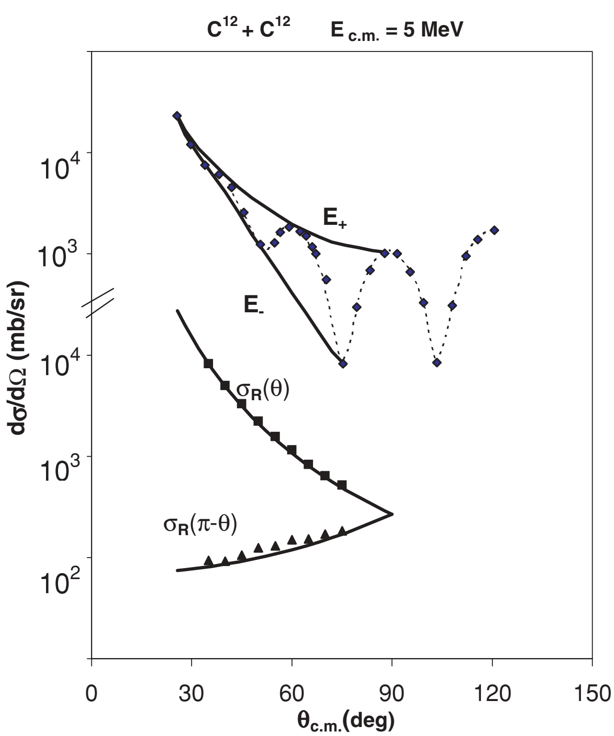

FIG. 1. Upper part: experimental data (diamonds) of the differential elastic scattering cross section of ^{12}C on ^{12}C for $E_{\mathrm{c.m.}} = 5$ MeV (from [5]) The dotted curve through the data is to guide the eye. The full curves $E_\pm$ are the "envelopes"of the oscillatory pattern. Lower part: the squares and triangles are the values of $\bar{\sigma}_1$ and $\bar{\sigma}_2$, respectively, obtained from the relations (5) and (6) on using the "envelopes" $E_\pm$(see main text). The curves are the Rutherford cross sections [$\sigma_R(\theta)$] and [$\sigma_R(\pi - \theta)$] for particles scattered through the angles θ and $(\pi - \theta)$, respectively.

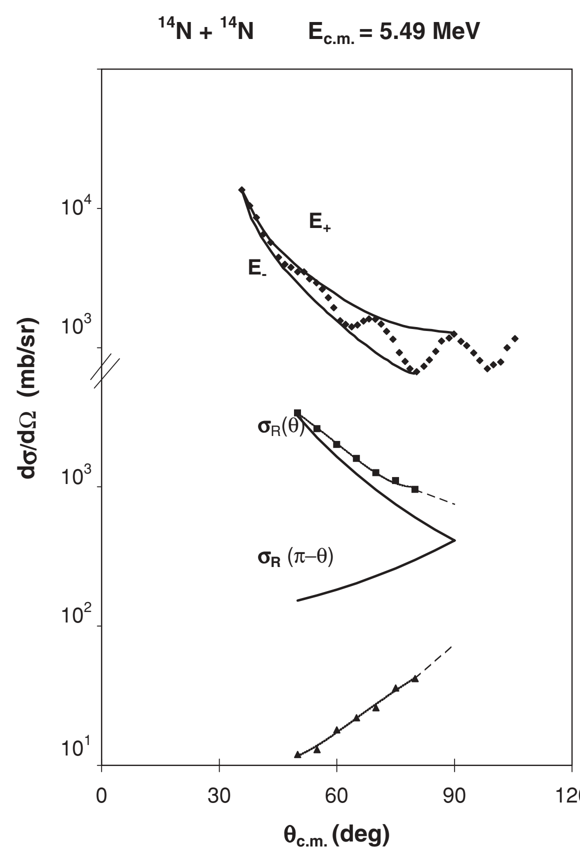

FIG. 2. Upper part: same as for Fig. 1. for elastic scattering of ^{14}N on ^{14}N at $E_{\mathrm{c.m.}} = 5.49$ MeV, from [6]. Lower part: same as for Fig. 1. The broken lines are to guide the eye and have been extrapolated to $\frac{\pi}{2}$ (see main text).

from Eqs. (5) and (6) remain very close. We think that this example also provides a test of the accuracy of the "envelope method."

Let us now consider, in Fig. 2, the angular distribution for elastic scattering of ^{14}N on ^{14}N at $E_{\mathrm{c.m.}} = 5.49$ MeV [6]. The ^{14}N nucleus has spin 1 which implies $\alpha = \frac{1}{3}$ in Eq. (1). This is an example where the interference pattern results from partial coherency. To evaluate $\bar{\sigma}_{1,2}$ as given in Eqs. (5) and (6) we proceed as for the preceding example by drawing the envelopes $E_{\pm}$. These are shown in the upper part of Fig. 2. The resulting values for $\bar{\sigma}_{1,2}$ are shown on the lower part of the figure, by squares ($\bar{\sigma}_1$) and triangles ($\bar{\sigma}_2$). The broken lines serve to guide the eye. Extrapolating these lines, one observes that they do not cross at $\theta = \frac{\pi}{2}$ where the amplitudes of the two interfering beams should coincide. This means that σ_1 and σ_2 should lie inside the two border lines $\bar{\sigma}_{1,2}$ defined in Eqs. (5) and (6). This can also be confirmed by an "exact" theoretical calculation. In fact $\sigma_{1,2}$ can again be identified with the Rutherford cross section as the collision takes place under the Coulomb barrier (≈11 MeV). The curves $\sigma_R(\theta)$ and $\sigma_R(\pi - \theta)$ are shown in the lower part of Fig. 2. It is apparent that they satisfy the inequalities (5) and (6).

Nucleus nucleus scattering is a good alternative to conventional two-way interferometers for discussing and illustrating the interplay between particle and wave information. For a given W, P is limited by inequality (8). From the angular distributions of Figs. 1 and 2, one easily observes the interplay between P and W when the detector moves from $\theta = 0$ to $\theta = \frac{\pi}{2}$. In the near forward direction, where the Rutherford cross section (9) grows as θ^{-4}, the contrast becomes negligibly small, $W \approx 0$, and therefore one can say that almost all the particles detected are those scattered through the angle θ. When the latter increases from $\theta = 0$ to $\theta = \frac{\pi}{2}$ the fringe visibility also increases until the two interfering amplitudes become equal. Thus, at $\theta = \frac{\pi}{2}$, for both ^{12}C and ^{14}N scattering, one has $\sigma_1 = \sigma_2$ and $P = 0$. For ^{12}C, $P = 0$ implies $W = 1$, but for ^{14}N (partially coherent case) the fringe visibility W is less than 1.

ACKNOWLEDGEMENT

The authors would like to thank Dr. S. Klarsfeld for a critical reading of the paper.

[1] A. Bramon, G. Garbarino, and B. C. Hiesmayr, Phys. Rev. A **69**, 022112 (2004).
[2] R. da Silveira and Ch. Leclercq-Willain, Phys. Rev. C **38**, 543 (1988); Z. Phys. A **34**, 63 (1983); for other applications of the "envelope method" see M. Lassaut, Nucl. Phys. **A442**, 1 (1985); H. M. Khalid, K. W. Mc Voy, and M. M. Shalaby, *ibid.* **A455**, 100 (1985); J. Y. Hostachy *et al.*, *ibid.* **A490**, 441 (1988).
[3] D. M. Greenberger and A. Yasin, Phys. Lett. **A128**, 391 (1988).
[4] B. G. Englert, Phys. Rev. Lett. **77**, 2154 (1996).
[5] D. A. Bromley, J. A. Kuehner, and E. Almquist, Proceedings of the 2nd Conference on Reactions Between Complex Nuclei, Gatlinburg, 1960, p. 151.
[6] L. A. Jacobson, Phys. Rev. **188**, 1509 (1969).
[7] D. M. Brink, Semiclassical Methods in Nucleus-Nucleus Scattering (Cambridge University Press, Cambridge, 1985).