Collective motion in a quantum diffusive environment

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The general problem of dissipation in macroscopic large-amplitude collective motion and its relation to energy diffusion of intrinsic degrees of freedom of a nucleus is studied. By applying the cranking approach to the nuclear many-body system, a set of coupled dynamical equations for the collective classical variable and the quantum mechanical occupancies of the intrinsic nuclear states is derived in the limit of weak coupling of the collective and intrinsic subsystems. Different dynamical regimes of the intrinsic nuclear motion and its consequences on time properties of collective dissipation are discussed.

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I. INTRODUCTION

The appearance of dissipation for large-amplitude collective motion in nuclei is still an unsolved problem. The transport models of the nuclear collective motion like the linear response theory [1] or the wall-formula approach [2] assume *a priori* that the collective dynamics is adiabatically slow, such that the fast intrinsic nucleonic subsystem has always sufficient time to adjust to the large changes of collective deformation parameters. In that case, one can say that statistical equilibrium for the fast intrinsic subsystem is established instantaneously, providing the essentially Markovian equations of motion for the collective variables.

In the general case the adiabaticity of the collective motion must not be implied *a priori*, and one should consider self-consistently dynamics of the collective and intrinsic nucleonic degrees of freedom. This is quite important when we are dealing with nuclear fission at high excitation energies or the initial stage of heavy ion collisions, i.e., when the typical times for the macroscopic collective and intrinsic nucleonic motions are of the comparable size. Here, one would rather expect a non-Markovian collective dynamics caused by the complex energy flow between the macroscopic collective and intrinsic nucleonic modes.

Memory (non-Markovian) effects in a time evolution of the collective parameters have been studied within the linear response theory [3], the time-dependent shell-model theory [4] and in the Fermi-liquid model [5,6]. While in all these approaches the main focus is on non-Markovian collective motion, we shall concentrate on the self-consistent description of the dynamics of the collective and nucleonic degrees of freedom. We start the discussion from general non-Markovian dynamics for occupancies of intrinsic quantum states, and then study how the different dynamical regimes of the intrinsic nucleonic excitations define dissipative properties of the macroscopic collective motion.

The plan of the paper is as follows. In Sec. II we start from the cranking approach to the nuclear many-body problem. Section III is devoted to the quantum-mechanical description of the intrinsic nuclear excitations. The different regimes of the intrinsic quantum dynamics are discussed in Sec. IV. In Sec. V, we derive a system of coupled equations for the slow collective and fast intrinsic modes of the nuclear many-body motion, and measure how the energy diffusion of the quantum-mechanical occupancies of the nuclear states defines the time properties of the collective friction. We apply our model to the description of nuclear fission dynamics on the part of descent from fission barrier to scission point in Sec. VI. Finally, conclusions and discussion of the main results of the paper are given in the Summary.

II. NUCLEAR MACROSCOPIC MOTION

The total energy of the nucleus under collective excitation $\Xi_{\rm tot}$ may be written as

$$\Xi_{\rm tot} = E_{\rm pot}(q) + \frac{1}{2}B(q)\dot{q}^2 + E^*(t), \tag{1}$$

where $q(t)$ is a single classical collective variable (a "nuclear deformation"), $E_{\rm pot}$ is the collective potential energy, B is the collective mass coefficient, and E^* is the excitation energy of the intrinsic nucleonic degrees of freedom. Writing the energy of the nucleus in the form of Eq. (1), we pick out explicitly the contribution from the virtual transition between the nuclear states, which gives rise to a collective kinetic energy term $(1/2)B\dot{q}^2$, and the contribution from the real nuclear transitions leading to the intrinsic excitation energy E^*.

Since the total energy of the nucleus is conserved, we can derive an equation of motion for the classical collective variable q by differentiating with respect to time the both sides of Eq. (1),

$$B(q)\ddot{q} = -\frac{1}{2}\frac{\partial B(q)}{\partial q}\dot{q}^2 - \frac{\partial E_{\rm pot}(q)}{\partial q} - \frac{1}{\dot{q}}\frac{dE^*(t)}{dt}. \tag{2}$$

To study how dissipation in the collective motion may arise, in next section we derive an expression for the intrinsic excitation energy E^*.

III. INTRINSIC QUANTUM DIFFUSIVE DYNAMICS

We treat intrinsic nucleonic motion of the nucleus quantum-mechanically and start from the Liouville equation for the

density matrix operator $\hat{\rho}$,

$$\frac{\partial \hat{\rho}(t)}{\partial t} + i\hat{L}(t)\hat{\rho}(t) = 0, \tag{3}$$

where $\hat{L}$ is the Liouville operator defined in terms of the commutator

$$\hat{L}\hat{\rho} = \frac{1}{\hbar}[\hat{H}, \hat{\rho}] \tag{4}$$

of the intrinsic nucleonic many-body Hamiltonian $\hat{H}(q[t])$.

Then, using Zwanzig's projection technique [7], we introduce a projection operator $\hat{\mathcal{P}}$ and split the density matrix operator into the diagonal and the nondiagonal parts,

$$\hat{\rho} = \hat{\rho}_d + \hat{\rho}_{od}, \tag{5}$$

where the diagonal part is defined as

$$\hat{\rho}_d = \hat{\mathcal{P}}\hat{\rho}, \tag{6}$$

and the nondiagonal part is given by

$$\hat{\rho}_{od} = (1 - \hat{\mathcal{P}})\hat{\rho}. \tag{7}$$

It is assumed that the projection operator $\hat{\mathcal{P}}$ is linear and time-independent. Acting on the Liouville equation (3) by the operators $\hat{\mathcal{P}}$ and $1 - \hat{\mathcal{P}}$ from the left, we obtain a system of equations for $\hat{\rho}_d$ and $\hat{\rho}_{od}$

$$\frac{\partial \hat{\rho}_d}{\partial t} + i\hat{\mathcal{P}}\hat{L}(\hat{\rho}_d + \hat{\rho}_{od}) = 0, \tag{8}$$

$$\frac{\partial \hat{\rho}_{od}}{\partial t} + i(1 - \hat{\mathcal{P}})\hat{L}(\hat{\rho}_d + \hat{\rho}_{od}) = 0, \tag{9}$$

with the initial condition

$$\hat{\rho}_{od}(t = 0) = 0. \tag{10}$$

Formally, a solution to Eq. (9) can be written as

$$\hat{\rho}_{od}(t) = -i\int_0^t \exp\left\{-i\int_0^{t-t'}(1 - \hat{\mathcal{P}})\hat{L}(t'')dt''\right\} \times (1 - \hat{\mathcal{P}})\hat{L}(t')\hat{\rho}_d(t')dt'. \tag{11}$$

Substituting solution (11) into Eq. (8), we obtain a closed equation for the diagonal part of the density matrix operator $\hat{\rho}_d$

$$\frac{\partial \hat{\rho}_d(t)}{\partial t} + i\hat{\mathcal{P}}\hat{L}(t)\hat{\rho}_d(t) = -\int_0^t \hat{\mathcal{P}}\hat{L}(t)\exp\left\{-i\int_0^{t-t'}(1 - \hat{P})\hat{L}dt''\right\} \times (1 - \hat{\mathcal{P}})\hat{L}(t')\hat{\rho}_d(t')dt' \tag{12}$$

which is the basic kinetic equation of Zwanzig's approach [7,8].

Let us write the basic kinetic equation (12) in matrix form. With that, we use an adiabatic basis of the intrinsic many-body Hamilton operator $\hat{H}(q)$,

$$\hat{H}(q)\Psi_n(q) = E_n(q)\Psi_n(q). \tag{13}$$

That is determined by a set of static many-body wave functions Ψ_n and energies E_n found at each fixed value of the collective variable q. Using the adiabatic basis (13), the time-dependence of the matrix elements of the density matrix operator $\hat{\rho}$ are given by

$$\rho_{nm}(t) = \exp\left\{-i\int_0^t \omega_{nm}(t')dt'\right\}\langle\Psi_n|\hat{\rho}|\Psi_m\rangle, \tag{14}$$

and the matrix elements of the Liouville operator $\hat{L}$ (4) are equal to

$$(\hat{L})_{nmn'm'} = \exp\left\{-i\int_0^t \omega_{nn'}(t')dt'\right\}\langle\Psi_n|\frac{\partial}{\partial t}|\Psi_{n'}\rangle\delta_{mm'} - \exp\left\{-i\int_0^t \omega_{m'm}(t')dt'\right\}\langle\Psi_{m'}|\frac{\partial}{\partial t}|\Psi_m\rangle\delta_{nn'}, \tag{15}$$

where $\omega_{nm} = (E_n - E_m)/\hbar$.

The second term on the left-hand side of Eq. (12) with the choice of the projection operator $\hat{\mathcal{P}}$,

$$(\hat{\mathcal{P}})_{nmn'm'} = \delta_{nm}\delta_{n'm'}\delta_{nn'}, \tag{16}$$

vanishes, since in this case

$$\hat{\mathcal{P}}\hat{L}\hat{\mathcal{P}} = 0. \tag{17}$$

Therefore, from Eq. (12) we obtain

$$\frac{\partial \hat{\rho}_d(t)}{\partial t} = -\int_0^t \hat{\mathcal{P}}\hat{L}(t)\exp\left\{-i\int_0^{t-t'}(1 - \hat{P})\hat{L}dt''\right\} \times (1 - \hat{\mathcal{P}})\hat{L}(t')\hat{\rho}_d(t')dt'. \tag{18}$$

Writing down the last equation in the matrix notations, we obtain

$$\frac{\partial \rho_{nn}(t)}{\partial t} = \sum_{m\neq n}\int_0^t \mathcal{H}_{nnmm}(t, s)[\rho_{mm}(s) - \rho_{nn}(s)]ds, \tag{19}$$

where the integral kernel H_{nnmm} is equal to

$$\mathcal{H}_{nnmm}(t, s) = -\left(\hat{\mathcal{P}}\hat{L}(t)\exp\left\{-i\int_0^{t-s}(1 - \hat{\mathcal{P}})\hat{L}(t')dt'\right\} \times (1 - \hat{\mathcal{P}})\hat{L}(s)\right)_{nnmm}, \tag{20}$$

where it was used that

$$\sum_m \mathcal{H}_{nnmm} = 0. \tag{21}$$

We will proceed by considering the integral kernel H_{nnmm} of our basic kinetic equation (19) and first represent the matrix elements of the Liouville operator (15) as

$$(\hat{L})_{nmn'm'} = \frac{\dot{q}}{E_n - E_{n'}}\exp\left\{-i\int_0^t \omega_{nn'}(t')dt'\right\} \times \langle\Psi_n|\frac{\partial \hat{H}}{\partial q}|\Psi_{n'}\rangle\delta_{mm'} - \frac{\dot{q}}{E_{m'} - E_m}\exp \times \left\{-i\int_0^t \omega_{m'm}(t')dt'\right\}\langle\Psi_{m'}|\frac{\partial \hat{H}}{\partial q}|\Psi_m\rangle\delta_{nn'}, \tag{22}$$

see Eq. (13). Here, the matrix elements $(\partial \hat{H}/\partial q)_{nm}$ measures the coupling between the quantum nucleonic and the macroscopic collective subsystems. Assuming that the

energy distances $E_n - E_m$ rapidly fluctuate with time, we can approximately write for the integral kernel (20)

$$\mathcal{H}_{nnmm}(t,s) \approx \sum_{abcd} \frac{\dot{q}(t)\dot{q}(s)}{(E_a - E_b)(q[t])(E_c - E_d)(q[t])} \times \left(\frac{\partial \hat{H}}{\partial q}\right)_{ab}(q[t]) \left(\frac{\partial \hat{H}}{\partial q}\right)^*_{cd}(q[s]) \times \exp(-i\omega_{ab}\cdot t)\exp(i\omega_{cd}\cdot s) G_{abcd}(t,s) \times (\delta_{bn} - \delta_{an})(\delta_{dm} - \delta_{cm}), \quad (23)$$

where star stands for the complex conjugation and

$$G_{abcd}(t,s) = \left(\exp\left\{-i(1-\hat{\mathcal{P}})\int_0^{t-s}\hat{L}(t')dt'\right\}\right)_{abcd}. \quad (24)$$

Since the integral kernel (23) already contains factors of second order in the intrinsic-collective coupling, the term G_{abcd} determines nonperturbative response of the intrinsic quantum subsystem (3) and (4).

At high excitation energies, the nuclear spectrum is very complex and can be described by random matrix theory [9]. Thus, we average the right-hand side of Eq. (19) over suitably chosen statistics of the randomly distributed coupling matrix elements and the energy spacings. In the random matrix theory, the ensemble averaging over the energy spacings and matrix elements can be performed independently. First, we perform the ensemble averaging over the matrix elements. They are treated as complex random numbers with the real and the imaginary parts which are independently Gaussian distributed, and with

$$\overline{\left(\frac{\partial \hat{H}}{\partial q}\right)_{nm}(q)\left(\frac{\partial \hat{H}}{\partial q}\right)^*_{n'm'}(q')} = \delta_{nn'}\delta_{mm'}\sigma^2(E_n,E_m)Y(q-q'), \quad (25)$$

where $Y(q-q')$ is a correlation function, measuring how strong the ensemble averaged matrix elements correlate at different collective deformations q and q', and $\sigma^2(E_n,E_m)$ is the energy distribution of the squared matrix elements. It is rather clear that at high excitation energies the coupling matrix elements between the complex many-body states should drop out with increasing energy distance between them. In order to characterize the energy distribution of the matrix elements, we introduce the strength of the distribution σ_0^2 and its width Γ. To clarify the physical meaning of the quantities σ_0^2 and Γ, one may use the random matrix approach of Ref. [10], where the nuclear many-body states are constructed on unperturbed basis states which are linearly coupled to the external time-dependent classical variable $q(t)$, and the complexity is achieved by adding the two-body interaction. In this approach, σ_0^2 is the variance of the slopes, $\partial E_n/\partial q$, of the many-body energy levels. The strength of the two-body interaction, introduced to model the effect of residual interaction between nucleons, defines the spreading width Γ of the squared of–diagonal matrix elements $|(\partial \hat{H}/\partial q)_{nm}|^2$, for example, via Fermi's Golden Rule. Thus, we have

$$\sigma^2(E_n,E_m) = \frac{\sigma_0^2}{\sqrt{\Omega(E_n)\Omega(E_m)}\Gamma} f(|E_n - E_m|/\Gamma), \quad (26)$$

where $\Omega(E)$ is the average level-density at given excitation energy and f is a shape of the energy distribution of the coupling matrix elements with the natural boundary conditions, $f(0) \sim 1$ and $f(\infty) = 0$.

In ensemble averaging the integral kernel (23), we assume that

$$\overline{\left(\frac{\partial \hat{H}}{\partial q}\right)_{ab}\left(\frac{\partial \hat{H}}{\partial q}\right)^*_{cd} G_{abcd}} \approx \overline{\left(\frac{\partial \hat{H}}{\partial q}\right)_{ab}\left(\frac{\partial \hat{H}}{\partial q}\right)^*_{ab}} \cdot \bar{G}_{abab}, \quad (27)$$

leading us to [see Eqs. (19), (25), and (26)]

$$\frac{\partial \bar{\rho}(E_n,t)}{\partial t} = \frac{2\sigma_0^2\dot{q}(t)}{\sqrt{\Omega(E_n)}\Gamma}\int_0^t ds\dot{q}(s)Y(q[t]-q[s]) \times \int_{E_{\text{g.s.}}}^{+\infty} dE_m\sqrt{\Omega(E_m)}f(|E_n - E_m|/\Gamma)\bar{G}_{nmnm} \times (E_n,E_m,t-s)\frac{\cos\{[E_n - E_m][t-s]/\hbar\}}{(E_n - E_m)^2} \times [\bar{\rho}(E_m,s) - \bar{\rho}(E_n,s)], \quad (28)$$

where $E_{\text{g.s.}}$ is the nuclear ground-state energy and the summation over all descrete states m was replaced by the integration over the corresponding continuous energy variable E_m.

The energy spacings part of the ensemble averaging procedure is defined through the two-level correlation function, $R(\Omega|E_n - E_m|)$, that is the probability density to find the state m with energy E_m within the interval $[E_m, E_m + dE_m]$ at the average distance $|E_n - E_m|$ from the given state n with energy E_n. In the nuclear case, the many-body Hamiltonian $\hat{H}$ obeys time-reversal symmetry implying the usage of Gaussian orthogonal ensemble (GOE) to model the nuclear spectrum. For a general mesoscopic system [11], $\hat{H}$ may not have a time-reversal symmetry and one has to use Gaussian unitary ensemble (GUE) of many-body levels. Correspondingly,

(i) For the GOE statistics [12]

$$R_{\text{GOE}}(x) = 1 - \left(\frac{\sin(\pi x)}{\pi x}\right)^2 + \left(\int_0^1 dy\frac{\sin(\pi xy)}{y} - \frac{\pi}{2}\right) \times \left(\frac{\cos(\pi x)}{\pi x} - \frac{\sin(\pi x)}{(\pi x)^2}\right), \quad (29)$$

(ii) while in the GUE case

$$R_{\text{GUE}}(x) = 1 - \left(\frac{\sin(\pi x)}{\pi x}\right)^2, \quad (30)$$

where $x \equiv |E_n - E_m|\Omega(E_n)$. The behavior of the two-level correlation function $R(x)$ with the normalized level spacing x for the different statistical ensembles (29) and (30) is shown in Fig. 1. The main difference between the GOE and GUE cases, seen in Fig. 1, is the behavior of $R(x)$ at small energy spacings x. For the GOE statistics one has the linear repulsion between levels, $R_{\text{GOE}} \sim x$, while the GUE statistics implies the quadratic level repulsion, $R_{\text{GUE}} \sim x^2$. On the other hand, R_{GOE} and R_{GUE} are similar at moderate spacings x, when the spectral correlations between levels consistently disappear.

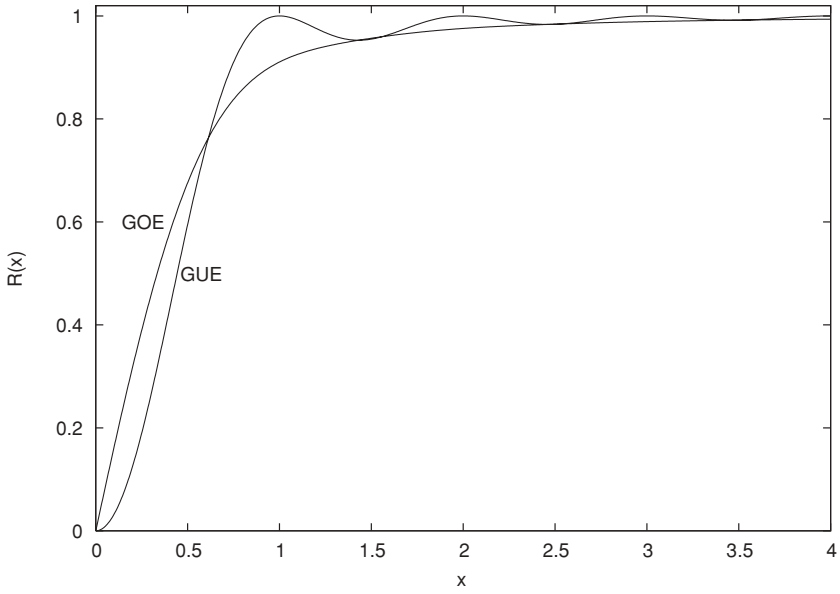

FIG. 1. The two-level correlation function $R(x)$ vs the normalized level spacing x for the different Gaussian ensembles of Eqs. (29) and (30) of energy levels.

Introducing the new energy variables,

$$E \equiv E_n, \quad e \equiv E_n - E_m, \tag{31}$$

we rewrite the dynamical equation (28) for the occupancies of the intrinsic many-body states within the random matrix approach as

$$\begin{aligned}\frac{\partial \bar{\rho}(E,t)}{\partial t} = {} & \frac{2\sigma_0^2 \dot{q}[t]}{\sqrt{\Omega(E)}\Gamma} \int_0^t ds \dot{q}[s] Y(q[t]-q[s]) \\ & \times \int_{-\infty}^{+\infty} \sqrt{\Omega(E-e)} R(\Omega|e|) f(|e|/\Gamma) \\ & \times \bar{G}(E,e,t-s) \frac{\cos(e[t-s]/\hbar)}{e^2} \\ & \times [\bar{\rho}(E-e,s) - \bar{\rho}(E,s)]. \end{aligned} \tag{32}$$

The integration limits over the energy spacing e were extended to infinities since the time changes of the occupancy $\bar{\rho}(E,t)$ of the given intrinsic state with the energy E are mainly due to the direct interlevel transitions from the close-lying states located at the distances $|e| \ll E$. The same assumptions enable us to truncate expansion to e^3-order terms,

$$\begin{aligned}&\sqrt{\Omega(E-e)}[\bar{\rho}(E-e,s) - \bar{\rho}(E,s)] \\ &= -\sqrt{\Omega(E)} \frac{\partial \bar{\rho}(E,s)}{\partial E} e + \frac{1}{2\sqrt{\Omega(E)}} \frac{d\Omega(E)}{dE} \frac{\partial \bar{\rho}(E,s)}{\partial E} e^2 \\ &\quad + \frac{\sqrt{\Omega(E)}}{2} \frac{\partial^2 \bar{\rho}(E,s)}{\partial E^2} e^2 + O(e^3). \end{aligned} \tag{33}$$

Substituting the expansion (33) into Eq. (32), the odd-e terms drop out and we obtain the diffusion-like equation of motion for the occupancy $\bar{\rho}(E,t)$,

$$\begin{aligned}\Omega(E) \frac{\partial \bar{\rho}(E,t)}{\partial t} \approx {} & \sigma_0^2 \dot{q}(t) \int_0^t ds K(t,s) \dot{q}(s) \\ & \times \frac{\partial}{\partial E} \left[\Omega(E) \frac{\partial \bar{\rho}(E,s)}{\partial E} \right]. \end{aligned} \tag{34}$$

In Eq. (34), the memory kernel, $K(t,s)$, is defined as

$$\begin{aligned}K(t,s) = {} & \frac{1}{\Gamma} Y(q[t]-q[s]) \int_{\infty}^{+\infty} de \bar{G}(\dot{q},e,t-s) f(|e|/\Gamma) \\ & \times R(|e|\Omega(E)) \cos(e[t-s]/\hbar). \end{aligned} \tag{35}$$

To define the nonperturbative factor $\bar{G}_{nmnm}$ in Eq. (35), we use an expansion of Eq. (24) in powers of $(1-\hat{\mathcal{P}})\hat{L}$,

$$\begin{aligned}G_{nmnm}(t) = {} & \left(1 + (-i) \int_0^t dt_1 (1-\hat{\mathcal{P}}) \hat{L}(t_1) + (-i)^2 \right. \\ & \times \int_0^t dt_1 (1-\hat{\mathcal{P}}) \hat{L}(t_1) \\ & \left. \times \int_0^{t_1} dt_2 (1-\hat{\mathcal{P}}) \hat{L}(t_2) + \ldots \right)_{nmnm}. \end{aligned} \tag{36}$$

By applying the ensemble averaging procedure (25) to Eq. (36), we see that $\bar{G}_{nmnm}$ contains only even powers of the Liouville operator $\hat{L}$ and such an expansion of $\bar{G}_{nmnm}$ in terms of $\hat{L}$ is determined by a perturbation parameter

$$\alpha \sim \left| \left(\int_0^t dt_1 \int_0^{t_1} dt_2 \overline{(1-\hat{\mathcal{P}}) \hat{L}(t_1) (1-\hat{\mathcal{P}}) \hat{L}(t_2)} \right)_{nmnm} \right|. \tag{37}$$

With the help of Eqs. (16), (22), and (25), one can show that

$$\begin{aligned}\alpha = {} & \left| \int_0^t dt_1 \dot{q}(t_1) \int_0^{t_1} dt_2 \dot{q}(t_2) Y(q[t_1]-q[t_2]) \right. \\ & \times \left\{ \sum_c \frac{\sigma^2(E_c,E_n)}{(E_c-E_n)^2} \cos([E_c-E_n][t_1-t_2]/\hbar) \right. \\ & + \sum_d \frac{\sigma^2(E_d,E_m)}{(E_d-E_m)^2} \cos([E_d-E_m][t_1-t_2]/\hbar) \\ & \left. \left. - 2 \frac{\sigma^2(E_n,E_m)}{(E_n-E_m)^2} \cos([E_n-E_m][t_1-t_2]/\hbar) \right\} \right|. \end{aligned} \tag{38}$$

Attempting to estimate the maximal value of the perturbation parameter (37), we assume at the moment equidistant spectrum

of the nucleonic many-body states with the average level-density Ω. Thus, we obtain

$$\alpha \sim \frac{\sigma_0^2}{\Omega\Gamma}\Omega^2\dot{q}^2\tau_{\rm coll}^2, \tag{39}$$

where $\tau_{\rm coll}$ is the typical time of the nuclear collective motion (a duration of the physical process).

IV. DIFFERENT REGIMES OF THE INTRINSIC QUANTUM DIFFUSIVE DYNAMICS

Three different timescales enter the intrinsic nucleonic diffusive dynamics (34)–(35). The first timescale, $\tau_{\rm cor}$, is the timescale characterizing the decay of time correlations of the coupling matrix elements. It can simply be estimated from perturbation theory as

$$\tau_{\rm cor} \sim \frac{1}{(\sigma_0/\sqrt{\Omega\Gamma})\Omega\dot{q}}. \tag{40}$$

The second one is the characteristic timescale $\hbar/\Gamma$ caused by the finite width Γ of the energy distribution (26) of the ensemble averaged matrix elements (25). And the third one is the typical time of the nuclear collective motion $\tau_{\rm coll}$. In the paper, we do not investigate the effect of the time correlations of the coupling matrix elements by putting

$$Y(q-q') = 1. \tag{41}$$

The assumption (41) may be justified either by neglecting the time variations of the coupling matrix elements $((\partial\hat{H}/\partial q)_{nm}(q[t]) \approx (\partial\hat{H}/\partial q)_{nm}(q[t=0]))$, or by considering the correlation time $\tau_{\rm cor}$ (40) of the matrix elements to be the largest timescale in the system

$$\tau_{\rm cor} \gg \tau_{\rm coll}\left(\frac{1}{(\sigma_0/\sqrt{\Omega\Gamma})\Omega\dot{q}} \gg \tau_{\rm coll}\right). \tag{42}$$

Moreover, we treat the intrinsic diffusive dynamics in weak-coupling limit implying that the perturbation parameter α (39) is quite small. In that case, the nonperturbative factor $\bar{G}_{nmnm}$ can be well approximated by one,

$$\bar{G}_{nmnm} \approx 1 \quad \text{for} \quad \alpha \sim \frac{\sigma_0^2}{\Omega\Gamma}\Omega^2\dot{q}^2\tau_{\rm coll}^2 \ll 1. \tag{43}$$

Such a weak-coupling limit of the nucleonic motion is fulfilled if either the displacements $\Delta q = q[t] - q[t=0] \approx \dot{q}\tau_{\rm coll}$ of the collective deformation is small, or the coupling between the quantum intrinsic and the macroscopic collective subsystems is weak (σ_0^2 is small). Thus, we obtain for the memory kernel (35)

$$K(t-s) = \frac{1}{\Gamma}\int_{\infty}^{+\infty} de\, f(|e|/\Gamma)R(|e|\Omega(E)) \times \cos(e[t-s]/\hbar), \tag{44}$$

which is the decaying function of $|t-s|$ and whose time spread is given by $\hbar/\Gamma$.

Equations (34) and (44) describe the process of energy diffusion within the space of highly excited many-body states. The intrinsic energy diffusion has the retarded character, giving rise both to the characteristic short-time oscillations of the occupation probabilities $\bar{\rho}(E,t)$ and to its relaxation for long times. The relative size of the memory effects in the intrinsic diffusive motion is defined by the counterplay of the time-spread $\hbar/\Gamma$ of the memory kernel (44) and the characteristic time of the collective motion $\tau_{\rm coll}$. It is important that we do not assume *a priori* that the time $\hbar/\Gamma$ should be the smallest timescale in the system, such that any peculiarities in the time evolution of the occupation probabilities can be neglected and we are dealing with ordinary (Markovian) master equation for $\bar{\rho}(E,t)$ describing its relaxation to equilibrium for each value of the collective deformation $q[t]$. In contrast to that, our aim is to investigate the complex self-consistent dynamics of the intrinsic nucleonic excitations and macroscopic collective deformation as a function of the width Γ of the energy distribution (26) of the squared coupling matrix elements on the nuclear many-body dynamics (see Sec. V for that discussion).

In our model, the intrinsic quantum subsystem (3)–(4) can be excited due to the coupling between different complex many-body states determined by the width Γ. The latter may be interpreted such that each many-body state, in some sense, has some finite width Γ, caused by its interaction with the neighboring states, and which decays over time of the order $\hbar/\Gamma$, leading to the excitation of the nucleonic subsystem. In this respect, Γ defines the time-irreversible energy flow (dissipation) from the collective modes of the nuclear many-body motion to the intrinsic nucleonic ones. By varying Γ, one can distinguish different regimes of the intrinsic energy diffusion (34):

(i) $\hbar/\Gamma \ll \tau_{\rm coll}$. In this case, $K(t-s)$ is sharply peaked around $t=s$, and one can integrate by parts the right-hand side of Eq. (34) and keep only leading order term in a small parameter $\hbar/\Gamma$. Thus, we obtain a Markovian limit of the intrinsic dynamics (34):

$$\Omega(E)\frac{\partial\bar{\rho}(E,t)}{\partial t} \approx \frac{\hbar\sigma_0^2 f(0)}{\Gamma}\dot{q}^2(t)\frac{\partial}{\partial E} \times\left[\Omega(E)\frac{\partial\bar{\rho}(E,t)}{\partial E}\right]. \tag{45}$$

Here, the intrinsic energy diffusion is determined by the diffusion coefficient $D_E = \hbar\sigma_0^2 f(0)\dot{q}^2/\Gamma$ which grows with the square of the collective velocity $\dot{q}$ and drops out with the increase of the width Γ. The latter feature of the quantum mechanical energy diffusion can be understood as follows. The width Γ of the energy distribution of the squared matrix elements (26) defines an effective number of states $N \sim \Gamma\Omega(E)$ coupled by the transition operator $\partial\hat{H}/\partial q$ at the given excitation E. The initially occupied many-body state with energy E will spread out over neighboring states, resulting in a gradual equilibration of the quantum mechanical intrinsic subsystem. The larger Γ, the closer the intrinsic subsystem to the equilibrium and therefore, the weaker the energy diffusion.

(ii) $\hbar/\Gamma \gg \tau_{\rm coll}$. Now, we can put approximately $K(t-s) \approx K(0)$ for the memory kernel in Eq. (34) and obtain

$$\Omega(E)\frac{\partial \bar{\rho}(E,t)}{\partial t} \approx \sigma_0^2 K(0)\dot{q}(t)\Delta q(t)\frac{\partial}{\partial E} \times \left[\Omega(E)\frac{\partial \bar{\rho}(E,t)}{\partial E}\right]. \quad (46)$$

Here, the diffusion coefficient $D_E = \sigma_0^2 K(0)\dot{q}\Delta q$ is linearly proportional to the collective velocity $\dot{q}$ and does not depend on the width Γ.

(iii) $\hbar/\Gamma \sim \tau_{\rm coll}$. In the intermediate case, the memory effects in the intrinsic energy diffusion (34) will be of maximal size.

We should underline the fact that the above considered different dynamical regimes (45)–(46) of the intrinsic energy diffusion were derived in the weak-coupling limit (43). Although the width Γ enters the condition for the weak-coupling limit, nevertheless we believe that Eq. (43) is still fulfilled even at the Markovian regime (46) of the intrinsic energy diffusion, i.e., when the width Γ is implied to be relatively small.

It is natural to address a question of the effect of level statistics (29)–(30) on the intrinsic energy diffusion. We believe that the energy diffusion will differ significantly for the statistical ensembles of levels (29)–(30) only at quite small values of the width Γ, $\Gamma \leqslant 1/\Omega$, i.e., when the features of the nuclear spectrum at small spacings between levels show up; see Fig. 1. On the other hand, at quite large widths $\Gamma \gg 1/\Omega$ the spectral statistics effect is of a minor role as far as the statistical ensembles of levels (29)–(30) show the universal behavior at large level spacings. The latter regime is realized for the highly excited nuclei provided that the width Γ of the energy distribution of the coupling matrix elements (26) may lie in a quite wide energy window $\Gamma \sim (10^0$–$10^6)$ eV.

We may illustrate quantitatively our general discussion of the intrinsic energy diffusion by calculating the memory kernel (44) for a Lorentzian shape of the energy distribution (26) of the coupling matrix elements,

$$f(|e|/\Gamma) = \frac{1/\pi}{(e/\Gamma)^2 + 1}. \quad (47)$$

To estimate the spectral statistics effect, we evaluated the memory kernel $K(t-s)$ at $s = t$ for the different levels ensembles (29)–(30). The corresponding results for $K(0)$ as a function of the reduced width $\Gamma\Omega(E)$ are shown in Fig. 2. As was discussed above, the level statistics play a role only for quite small parameters $\Gamma\Omega(E)$ and the effect from the spectral statistics on the intrinsic energy diffusion (34) disappears at large widths $\Gamma\Omega(E)$.

For $\Gamma\Omega \gg 1$, one can find analytically the memory kernel (44)

$$K(t-s) = \exp\left(-\frac{|t-s|}{\hbar/\Gamma}\right), \quad (48)$$

leading to the following non-Markovian equation of motion for the occupancy $\bar{\rho}(E,t)$ of the given nuclear state E:

$$\Omega(E)\frac{\partial \bar{\rho}(E,t)}{\partial t} = \frac{\sigma_0^2}{\Gamma}\dot{q}(t)\int_0^t \exp\left(-\frac{|t-s|}{\hbar/\Gamma}\right)\dot{q}(s)\frac{\partial}{\partial E} \times \left[\Omega(E)\frac{\partial \bar{\rho}(E,s)}{\partial E}\right] ds. \quad (49)$$

V. INTRINSIC DIFFUSION AND COLLECTIVE DISSIPATION

Now we are able to obtain a dynamical equation for the intrinsic excitation energy of the nucleus $E^*(t)$,

$$E^*(t) = \sum_n E_n \bar{\rho}_{nn}(t) = \int_0^{+\infty} dE\,\Omega(E)E\bar{\rho}(E,t), \quad (50)$$

which enters the equation of motion (2) for the classical collective variable $q(t)$. By using Eq. (34), one gets after partial

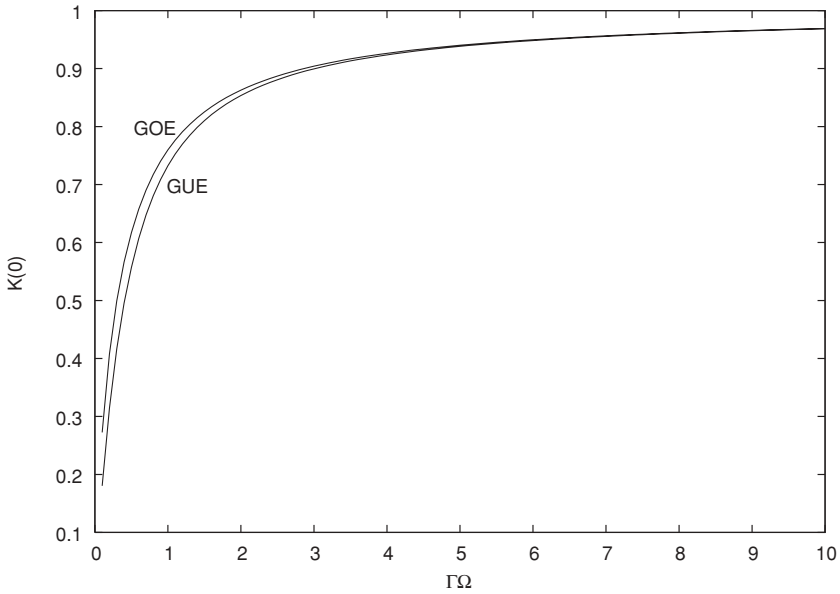

FIG. 2. Dependence of the non-Markovianity of the intrinsic nucleonic dynamics (34) on the reduced width $\Gamma\Omega(E)$ of the Lorentzian energy distribution (47) of the coupling matrix elements. The dependence is shown for the different spectral statistics (29) and (30).

integration

$$\frac{dE^*}{dt} = \frac{\sigma_0^2}{\Gamma}\dot{q}(t)\int_0^t \exp\left(-\frac{|t-s|}{\hbar/\Gamma}\right)\dot{q}(s) \times \int_0^{+\infty}\frac{d\Omega(E)}{dE}\bar{\rho}(E,s)dEds. \quad (51)$$

We stress immediately that the collective motion is undamped for the constant nuclear level-density, $\Omega(E) = \text{const}$. In that case the intrinsic subsystem is not excited during the collective motion, $E^*(t) = E^*(t=0)$ and therefore, due to the energy conservation condition (1), the collective energy is constant in time. This means that the growth of the average nuclear level-density Ω with energy is the necessary condition for the collective dissipation. In the sequel, we will use the constant–temperature level-density,

$$\Omega(E) = c\cdot\exp(E/T), \quad (52)$$

where T is the temperature of the nucleus, and which leads us to non-Markovian collective dynamics,

$$B(q)\ddot{q}(t) = -\frac{1}{2}\frac{\partial B(q)}{\partial q}\dot{q}^2(t) - \frac{\partial E_{\text{pot}}(q)}{\partial q} - \frac{\sigma_0^2}{T}\int_0^t \exp\left(-\frac{|t-s|}{\hbar/\Gamma}\right)\dot{q}(s)ds. \quad (53)$$

We see from Eq. (53) that the non-Markovian character of the intrinsic nuclear dynamics (34) gives rise to the presence of memory effects in the macroscopic collective motion. Correspondingly, the Markovian limits of the intrinsic energy diffusion (45) and (46) would correspond to the Markovian collective motion. Indeed, for the quite broad energy distributions of the squared coupling matrix elements (26), $\hbar/\Gamma \ll \tau_{\text{coll}}$, we obtain

$$B(q)\ddot{q}(t) = -\frac{1}{2}\frac{\partial B(q)}{\partial q}\dot{q}^2(t) - \frac{\partial E_{\text{pot}}(q)}{\partial q} - \frac{\hbar\sigma_0^2}{\Gamma T}\dot{q}(t). \quad (54)$$

Here an ordinary friction force with the friction coefficient $\hbar\sigma_0^2/(\Gamma T)$ appears as a result of the Markovian intrinsic energy diffusion (45).

In the opposite case of the intrinsic dynamics (46), when the energy distribution of the matrix elements is strongly peaked, $\hbar/\Gamma \gg \tau_{\text{coll}}$, we obtain a frictionless limit of the collective motion,

$$B(q)\ddot{q}(t) = -\frac{1}{2}\frac{\partial B(q)}{\partial q}\dot{q}^2(t) - \frac{\partial E_{\text{pot}}(q)}{\partial q} - \frac{\sigma_0^2}{T}(q(t)-q_0), \quad (55)$$

when the retarded force in the right-hand side of Eq. (53) is reduced to a pure conservative force $\sigma^2(q-q_0)/T$.

VI. NUCLEAR FISSION CALCULATIONS

Even within a very simple one-dimension model for the collective dynamics (53), we may calculate quantities which can be estimated from experimental observables. Let us consider a symmetric fission of the highly excited ^{236}U. The classical collective variable $q(t)$ can be chosen as the elongation of axial symmetric nuclear shape measured in units of the radius $R_0 = r_0 A^{1/3}$ of the nucleus. The collective potential energy from saddle point to scission $E_{\text{pot}}(q)$, shown in Fig. 3, is approximated by an inverted parabolic potential [13,14]

$$E_{\text{pot}}(q) = 8\,\text{MeV} - \tfrac{1}{2}\hbar\omega_f B(q_0)(q-q_0)^2, \quad (56)$$

where $\hbar\omega_f = 1.16$ MeV, q_0 is the initial (saddle-point) deformation of the nucleus, $q_0 = q(t=0) = 1.6$, and $B(q)$ is the collective mass coefficient derived for the incompressible and irrotational nuclear fluid,

$$B(q) = \frac{1}{5}AmR_0^2\left(1+\frac{1}{2q^3}\right), \quad (57)$$

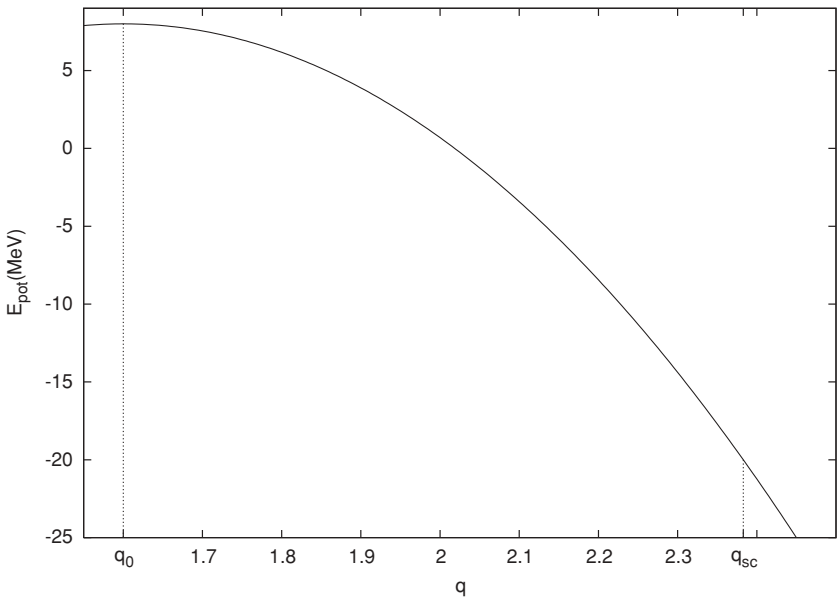

FIG. 3. Dependence of the collective potential energy E_{pot} on the nuclear shape parameter q during the descent from the top of fission barrier q_0 to the scission point q_{sc} (58).

with the nucleonic mass m. The scission point q_{sc} can be obtained from the following condition [14]:

$$E_{\rm pot}(q_0) - E_{\rm pot}(q_{sc}) = 20\,{\rm MeV}. \tag{58}$$

The initial collective kinetic energy is taking to be equal to 1 MeV.

Characterizing the intrinsic nuclear motion, we adopt the initial temperature of the nucleus $T = 2$ MeV and estimate the strength σ_0^2 of the EASME's distribution within the Nilsson model for single-particle nuclear states in an anisotropic harmonic oscillator potential, see Ref. [15]:

$$\sigma_0^2 = \frac{3m^2\omega_0^3 A R_0^4}{560\pi\hbar}, \tag{59}$$

with $\hbar\omega_0 = 41/A^{1/3}$ MeV.

Using Eq. (53), we calculated numerically from Eq. (53) the time, t_{sc}, of the nuclear descent from the top of fission barrier q_0 to the scission point q_{sc} (58). The corresponding results for the saddle-to-scission time t_{sc} are plotted in Fig. 4 as a function of the width Γ of the Lorentzian energy distribution of the squared coupling matrix elements (26). As can be seen from Fig. 4, the time for the nuclear descent t_{sc} decreases with the increase of the width Γ of the matrix elements' energy distribution. In order to explain such kind of behavior of t_{sc}, we represent the retarded force in the right-hand side of Eq. (53) as a sum,

$$-\frac{\sigma_0^2}{T}\int_0^t \exp\left(-\frac{|t-s|}{\hbar/\Gamma}\right)\dot{q}(s)ds = -\gamma(t,\hbar/\Gamma)\dot{q}(t) - \tilde{C}(t,\hbar/\Gamma)(q(t)-q_0), \tag{60}$$

where γ and $\tilde{C}$ are the time-dependent friction and stiffness coefficients, respectively. The separation (60) of the retarded force is general in the sense that it always contains the time-irreversible (the friction part) and time-reversible (the conservative part) contributions. In fact, the memory effects in the collective dynamics (53) give rise to the friction, $\gamma(t)\dot{q}(t)$, and lead to the renormalization of the stiffness of the nuclear many-body system,

$$C = -B(q_0)(\hbar\omega_f)^2 + \tilde{C}(t,\hbar/\Gamma), \tag{61}$$

see Eqs. (53) and (60). It is important that $\tilde{C}$ is always positive resulting in the additional hinders of the nuclear descent from the fission barrier, see Ref. [16]. The relative sizes of the friction and the dynamic conservative forces in Eq. (60) are defined by the time-spread of the exponential kernel, $\hbar/\Gamma$. If the dynamic stiffness $\tilde{C}$ is expected to increase monotonically with $\hbar/\Gamma$, the friction coefficient γ is a nonmonotonic function of the memory time $\hbar/\Gamma$. At the limit of relatively small values of $\hbar/\Gamma$ (the large-widths limit which we consider here), both the friction and the dynamic conservative contributions drop out with the memory time explaining the decay of the saddle-to-scission time t_{sc} with the width Γ of the EASME's distribution.

By using our previous estimations of the saddle-to-scission time done in Ref. [16] for the same one-parametric nuclear shape parametrization (56)–(58), $t_{sc} \sim (6\text{–}12) \times 10^{-21} s$, we can conclude from Fig. 4 that the width Γ of the Lorentzian energy distribution of the coupling matrix elements lies in the interval $10\,{\rm MeV} \leqslant \Gamma \leqslant 20\,{\rm MeV}$.

We also calculated the dependence of collective kinetic energy at the scission point E_{ps} on the width Γ, see Fig. 5. As far as the nuclear descent gets faster with the width of the matrix elements' energy distribution, the collective energy of the nucleus at the scission point will increase with Γ. The estimated interval for the width, $10\,{\rm MeV} \leqslant \Gamma \leqslant 20\,{\rm MeV}$, obtained from our saddle-to-scission calculations (see Fig. 4), gives realistic values of the pre-scission kinetic energy $1\,{\rm MeV} \leqslant E_{ps} \leqslant 3\,{\rm MeV}$ [16].

VII. SUMMARY

In attempt to describe self-consistently the nuclear many-body dynamics undergoing along macroscopic collective path and intrinsic excitations, we have applied the cranking approach (1)–(2) to the nucleus. We have introduced a single

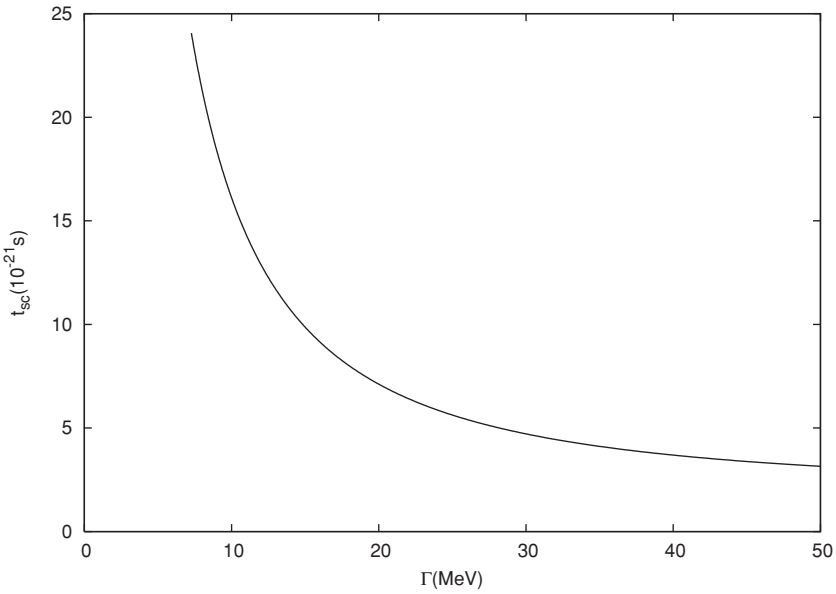

FIG. 4. The saddle-to-scission time t_{sc} of the symmetric fission of the ^{236}U, calculated from Eq. (53), is shown as a function of the width Γ of the Lorentzian energy distribution (47) of the coupling matrix elements.

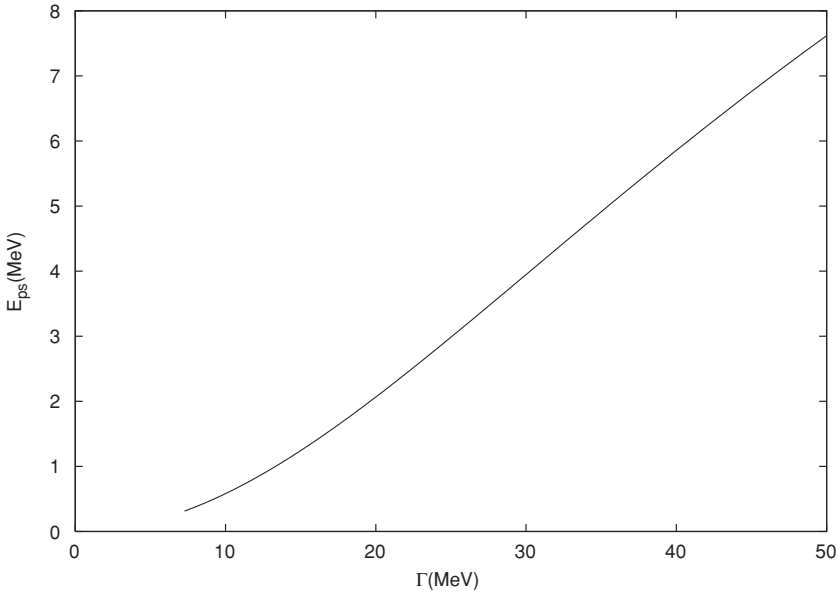

FIG. 5. The collective kinetic energy at the scission point E_{ps} vs the width Γ of the Lorentzian energy distribution (47) of the coupling matrix elements.

time-dependent classical variable $q(t)$ to characterize the slow collective nuclear motion, while the fast intrinsic modes of the motion have been treated quantum-mechanically within the Liouville equation (3) for the nuclear density matrix. Applying the Zwanzig's projection method [7], the intrinsic nucleonic dynamics has been reduced to the equation of motion for the occupancies of the nucleonic many-body states. The basic kinetic Eq. (32) has been treated within the random matrix approach (25)–(28), where the intrinsic dynamics is averaged over the randomly distributed coupling matrix elements $(\partial \hat{H}/\partial q)_{nm}$ of the nucleonic many-body Hamiltonian $\hat{H}$ and energy-level spacings $e \equiv E_n - E_m$. The used distribution of the coupling matrix elements (25) takes into account both the correlations of the matrix elements at different collective deformations $q[t]$ and its dependence on the distance between complex many-body states. The time correlations of $(\partial \hat{H}/\partial q)_{nm}(q[t])$ have been described by the correlation time (40), that is the characteristic time interval over which the coupling matrix elements correlate effectively. The energy distribution of the ensemble averaged matrix elements (26) has been characterized with the help of two parameters, the strength of the distribution σ_0^2 and its width Γ.

Our further investigation of the intrinsic nucleonic dynamics has been done in the limit of weak-coupling (43) of the quantum intrinsic and classical collective subsystems. We have derived the non-Markovian diffusion-like equation (34) of motion for the time evolution of the occupancies of the highly excited nucleonic many-body states. We have not considered the effect of the time correlations of the coupling matrix elements (25) on the intrinsic diffusion dynamics which is justified in the weak-coupling limit [see Eq. (42)] or, simply because of the neglecting of the time variations of the matrix elements $((\partial \hat{H}/\partial q)_{nm}(q[t]) \approx (\partial \hat{H}/\partial q)_{nm}(q[t=0])$. In that case, memory effects in the intrinsic energy diffusion (34) are caused by the finite width Γ of the energy distribution of the coupling matrix elements (26) that may lead to the macroscopic retardation of the intrinsic nucleonic dynamics. The relative size of the memory effects is defined by the relation between the characteristic time scale of the nucleonic motion $\hbar/\Gamma$ and the typical time of the collective motion $\tau_{\rm coll}$. We have found that at fairly broad energy distributions of the coupling matrix elements (26), i.e., when $\hbar/\Gamma \ll \tau_{\rm coll}$, Markovian regime (45) of the intrinsic energy diffusion is observed with the diffusion coefficient quadratically depending on the collective velocity $\dot{q}$ and inversely proportional to the width Γ. In the opposite case of quite small widths Γ, $\hbar/\Gamma \gg \tau_{\rm coll}$, we also found the normal (Markovian) regime of the intrinsic energy diffusion but with the diffusion coefficient linearly proportional to the collective velocity $\dot{q}$ and not depending on the width Γ.

We have investigated how the level spacing statistics can influence the intrinsic energy diffusion (34). Only in the case of quite small widths of the matrix elements' energy distribution, $\Gamma\Omega \leqslant 1$, the significant difference of the intrinsic dynamics for the Gaussian orthogonal (GOE) (29) and Gaussian unitary (GUE) (30) ensembles of levels is expected. Such a difference would disappear as far as the product $\Gamma\Omega$ becomes larger and larger. We may explain that by the fact that the transitions between the nuclear states may be sensitive to the level statistics only when the coupling between states is of order of the average level spacing, $\Gamma \sim 1/\Omega$, i.e., when the different small-spacing behavior of the GOE and GUE statistics may show up. At high nuclear excitations, we have believed that the product $\Gamma\Omega \gg 1$ and therefore, one can neglect the role of the spectral statistics on the intrinsic energy diffusion (34). We have illustrated quantitatively this feature of the intrinsic dynamics by applying the Lorentzian energy distribution (47) of the coupling matrix elements, see Fig. 3.

Our next goal was to calculate the nuclear fission's characteristics within our approach. By using the constant-temperature level-density (52), we have derived non-Markovian equation of motion (53) for the classical collective variable $q(t)$, where the influence of the intrinsic quantum motion on the collective dynamics is determined by the retarded friction force. Then the non-Markovian collective dynamics (53) has been applied to describe descent of the

nucleus ^{236}U from the top of fission barrier to the scission point approximating the collective potential energy on this path by the inverted parabolic potential (56) [13,14]. We have calculated the time of the nuclear descent, t_{sc} (Fig. 4), and the collective kinetic energy at the scission point, E_{ps} (Fig. 5), as a function of the width Γ of the energy distribution of the coupling matrix elements. We have found that the nuclear descent is hindered with the decrease of Γ due to the ordinary friction force contribution and the additional conservative dynamic force caused by the presence of memory effects in Eq. (53) [16]. The relative size of the memory effects decreases with the width of the matrix elements' energy distribution and, at $\Gamma \to \infty$, we have frictionless limit of the collective motion, see Eq. (55). From the calculations of the saddle-to-scission time and the pre-scission kinetic energy we have estimated the value of the width Γ, $10\,\text{MeV} \leqslant \Gamma \leqslant 20\,\text{MeV}$, which is consistent with the previous estimations of the analogous quantity done in Refs. [17,18].

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