Σ-nuclear spin-orbit coupling from two-pion exchange

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Using SU(3) chiral perturbation theory we calculate the density-dependent complex-valued spin-orbit coupling strength $U_{\Sigma ls}(k_f) + i W_{\Sigma ls}(k_f)$ of a Σ hyperon in the nuclear medium. The leading long-range ΣN interaction arises from iterated one-pion exchange with a Λ or a Σ hyperon in the intermediate state. We find from this unique long-range dynamics a sizable "wrong-sign" spin-orbit coupling strength of $U_{\Sigma ls}(k_{f0}) \simeq -20$ MeV fm^2 at normal nuclear matter density $\rho_0 = 0.16$ fm^{-3}. The strong $\Sigma N \to \Lambda N$ conversion process contributes at the same time an imaginary part of $W_{\Sigma ls}(k_{f0}) \simeq -12$ MeV fm^2. When combined with estimates of the short-range contribution the total Σ-nuclear spin-orbit coupling becomes rather weak.

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Hypernuclear physics has a long and well-documented history [1–3]. One primary goal in this field is to determine from the experimental data the nuclear mean-field potentials relevant for the hyperon single-particle motion. For the Λ hyperon the situation is by now rather clear and the following quantitative features have emerged. The attractive nuclear mean-field potential for a Λ hyperon is about half as strong as the one for nucleons in nuclei: $U_\Lambda \simeq -28$ MeV [4]. With this value of the potential depth the empirical single-particle energies of a Λ bound in hypernuclei are well described over a wide range in mass number. On the other hand, the Λ-nucleus spin-orbit interaction is found to be extraordinarily weak. For example, recent precision measurements [5] of $E1$-transitions from p- to s-shell orbitals in $^{13}_{\Lambda}$C give a $p_{3/2} - p_{1/2}$ spin-orbit splitting of only (152 ± 65) keV to be compared with a value of about 6 MeV in ordinary p-shell nuclei.

In the case of the Σ hyperon recent developments have lead to a revision concerning the sign and magnitude of its nuclear mean-field potential [6]. Whereas an earlier analysis of the shifts and widths of x-ray transitions in Σ^- atoms came up with an attractive (real) Σ-nucleus optical potential of about -27 MeV [1], there is currently good experimental and phenomenological evidence for a substantial Σ-nucleus repulsion. A reanalysis of the Σ^- atom data in Ref. [7] including the then available precise measurements of W and Pb atoms and employing phenomenological density-dependent fits has lead to a Σ-nucleus potential with a strongly repulsive core (of height ~ 95 MeV) and a shallow attractive tail outside the nucleus. The inclusive (π^-, K^+) spectra on medium-to-heavy nuclear targets measured at KEK [8,9] give more direct evidence for a strongly repulsive Σ-nucleus potential. In the framework of the distorted wave impulse approximation, a best fit of the measured (π^-, K^+) inclusive spectra on Si, Ni, In, and Bi targets is obtained with a Σ-nucleus repulsion of about 90 MeV. However, the detailed description of the Σ^- production mechanism plays an important role for the extracted value of the Σ-nucleus repulsion. Within a semiclassical distorted wave model [10], which avoids the factorization approximation by an averaged differential cross section, the KEK data can also be well reproduced with a complex Σ-nucleus potential of strength $(30 - 20i)$ MeV. Concerning the Σ-nucleus spin-orbit coupling there exist so far no experimental hints for it. Most theoretical models [11,12] predict the Σ-nucleus spin-orbit coupling to be strong (i.e., comparable to the one of nucleons). The basic argument for a strong spin-orbit coupling is provided by the large and positive value of the tensor-to-vector coupling ratio of the ω meson to the Σ hyperon assuming vector meson dominance and the nonrelativistic quark model with SU(6) spin-flavor symmetry. The G-matrix calculations by the Kyoto-Niigata group [13] using the hyperon-nucleon interaction as derived from their SU(6) quark model predict a Σ-nucleus spin-orbit coupling which is about half as strong as the one of nucleons. However, due to the presence of the strong $\Sigma N \to \Lambda N$ conversion process in the nuclear medium one expects the Σ-nucleus spin-orbit coupling strength to have also an imaginary part. This possibility has generally been ignored in quark and one-boson exchange models.

Recently, we have applied chiral effective field theory to calculate the hyperon mean-fields in nuclear matter [14]. In this approach the small Λ-nuclear spin-orbit interaction finds a novel explanation in terms of an almost complete cancellation between short-range contributions (estimated from the known nucleonic spin-orbit coupling strength) and long-range terms generated by iterated one-pion exchange with intermediate Σ hyperons. The exceptionally small $\Sigma\Lambda$ mass splitting of $M_\Sigma - M_\Lambda = 77.5$ MeV influences hereby prominently the effect coming from the second order 1π-exchange tensor interaction. Furthermore, it has been shown in Ref. [15] that the proposed cancellation mechanism does not get disturbed by the inclusion of analogous two-pion exchange processes involving decuplet baryons [$\Delta(1232)$ and $\Sigma^*(1385)$] in the intermediate state with considerably larger mass splittings. The density-dependent complex Σ-nuclear mean-field $U_\Sigma(k_f) + i W_\Sigma(k_f)$ has also been calculated in the same framework in Ref. [16]. It has been found that genuine long-range[1] contributions

[1]Genuine long-range means that (unique) part of the pion-loop which depends exclusively on small scales (k_f, m_π, Δ), but not on any high-momentum cutoff. In case of the Σ-nuclear mean-field $U_\Sigma(k_f)$ it seems that the net short-range contribution is small [16]. For the Λ single-particle potential $U_\Lambda(k_f)$ an attractive short-range contribution [14] is however necessary in order to reproduce the empirical potential depth of -28 MeV. A deeper understanding of this feature is presently missing.

from iterated one-pion exchange with intermediate Λ and Σ hyperons sum up to a moderately repulsive (real) single-particle potential of $U_\Sigma(k_{f0}) \simeq 59\,\text{MeV}$ at normal nuclear matter density $\rho_0 = 0.16\,\text{fm}^{-3}$. The $\Sigma N \to \Lambda N$ conversion process induced by one-pion exchange generates at the same time an imaginary single-particle potential of $W_\Sigma(k_{f0}) \simeq -21.5$ MeV. This value is in fair agreement with empirical determinations [7] and quark model predictions [17]. The purpose of the present Brief Report is to calculate in the same chiral effective field theory framework the density-dependent complex-valued Σ-nuclear spin-orbit coupling strength. As for the Λ hyperon [14] we do find a sizable "wrong-sign" spin-orbit coupling from the second-order one-pion exchange tensor interaction. When combined with estimates of the short-range contribution (employing QCD sum rule predictions) the total Σ-nuclear spin-orbit coupling becomes rather weak.

Let us begin with some basic considerations. The pertinent quantity to extract the Σ-nuclear spin-orbit coupling is the spin-dependent part of the self-energy of a Σ hyperon interacting with weakly inhomogeneous isospin-symmetric (spin-saturated) nuclear matter. Let the Σ hyperon scatter from initial momentum $\vec{p} - \vec{q}/2$ to final momentum $\vec{p} + \vec{q}/2$. The spin-orbit part of the self-energy is then

$$\Sigma_{\text{spin}} = \frac{i}{2}\,\vec{\sigma}\cdot(\vec{q}\times\vec{p})[U_{\Sigma ls}(k_f) + i\,W_{\Sigma ls}(k_f)]\,, \qquad (1)$$

where the density-dependent spin-orbit coupling strength $U_{\Sigma ls}(k_f) + i\,W_{\Sigma ls}(k_f)$ is taken in the limit of homogeneous nuclear matter (characterized by its Fermi momentum k_f) and zero external Σ-momenta: $\vec{p} = \vec{q} = 0$. The more familiar spin-orbit Hamiltonian follows from Eq. (1) by multiplication with a density form factor and Fourier transformation $\int d^3q \exp(i\vec{q}\cdot\vec{r}\,)$. For orientation, consider first the ω meson exchange between the Σ hyperon and the nucleons. The nonrelativistic expansion of the vector (and tensor) coupling vertex between Dirac spinors of the Σ hyperon gives rise to a spin-orbit term proportional to $i\,\vec{\sigma}\cdot(\vec{q}\times\vec{p}\,)/4M_\Sigma^2$. Next one takes the limit of homogeneous nuclear matter (i.e., $\vec{q} = 0$), performs the remaining integral over the nuclear Fermi sphere and arrives at the familiar result

$$U_{\Sigma ls}(k_f)^{(\omega)} = \frac{g_{\omega\Sigma}(1 + 2\kappa_{\omega\Sigma})g_{\omega N}}{2M_\Sigma^2 m_\omega^2}\,\rho\,, \qquad (2)$$

linear in density $\rho = 2k_f^3/3\pi^2$. Here, $\kappa_{\omega\Sigma}$ denotes the tensor-to-vector coupling ratio of the ω meson to the Σ hyperon.

The crucial observation is now that the (left) iterated one-pion exchange diagram in Fig. 1 generates also a (sizable) spin-orbit coupling term. The prefactor $\frac{i}{2}\vec{\sigma}\times\vec{q}$ is immediately identified by rewriting the product of $\pi\Sigma B$-interaction vertices $\vec{\sigma}\cdot(\vec{l} - \vec{q}/2)\,\vec{\sigma}\cdot(\vec{l} + \vec{q}/2) = \frac{i}{2}(\vec{\sigma}\times\vec{q}\,)\cdot(-2\vec{l}\,) + \ldots$ at the open baryon line. For all remaining parts of the diagram one can then take the limit of homogeneous nuclear matter (i.e., $\vec{q} = 0$). The other essential factor $\vec{p}$ comes from the energy denominator $-\Delta^2 + \vec{l}\cdot(\vec{l} - \vec{p}_1 + \vec{p}\,)$. The $\Sigma\Lambda$ mass splitting is rewritten here in terms of the small scale parameter $\Delta = \sqrt{M_B(M_\Sigma - M_\Lambda)} \simeq 285\,\text{MeV}$ with $M_B = (2M_N + M_\Lambda + M_\Sigma)/4 \simeq 1047\,\text{MeV}$ a mean baryon mass. It serves the purpose to average out small differences in the kinetic energies of the various baryons involved. Keeping only the term linear in the external momentum $\vec{p}$ one finds from the left diagram in Fig. 1 with a Λ hyperon in the intermediate state the following contribution to the Σ-nuclear spin-orbit coupling strength:

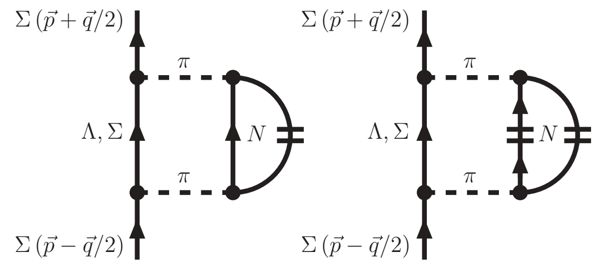

FIG. 1. Iterated one-pion exchange diagrams with Λ and Σ hyperons in the intermediate state generating a Σ-nuclear spin-orbit coupling. The horizontal double-line symbolizes the filled Fermi sea of nucleons, i.e., the medium insertion $-\theta(k_f - |\vec{p}_j|)$ in the in-medium nucleon propagator.

$$\begin{aligned} &U_{\Sigma ls}(k_f)^{(2\pi\Lambda)} + i\,W_{\Sigma ls}(k_f)^{(2\pi\Lambda)} \\ &\quad = -\frac{2D^2g_A^2}{9f_\pi^4}\int_{|\vec{p}_1|<k_f}\frac{d^3p_1d^3l}{(2\pi)^6} \\ &\qquad \times \frac{M_B\vec{l}^4}{(m_\pi^2 + \vec{l}^2)^2[-\Delta^2 - i0 + \vec{l}^2 - \vec{l}\cdot\vec{p}_1]^2} \\ &\quad = \frac{2}{3}\frac{\partial}{\partial\Delta^2}[U_\Sigma(k_f)^{(2\pi\Lambda)} + i\,W_\Sigma(k_f)^{(2\pi\Lambda)}]\,. \end{aligned} \qquad (3)$$

Here, $D = 0.84$ and $F = 0.46$ [14] denote the SU(3) axial vector coupling constants together with $g_A = D + F = 1.3$ the nucleon axial vector coupling constant. $f_\pi = 92.4$ MeV is the pion decay constant and $m_\pi = 138\,\text{MeV}$ the average pion mass. Note that the loop integral in Eq. (3) is convergent as its stands. Most useful is actually the representation of the spin-orbit coupling strength as a derivative of the Σ-nuclear potential $U_\Sigma(k_f) + i\,W_\Sigma(k_f)$ with respect to the (mass splitting) parameter Δ^2. Using the analytical expressions in Ref. [16] to evaluate this derivative we find for the real and imaginary part

$$\begin{aligned} U_{\Sigma ls}(k_f)^{(2\pi\Lambda)} &= \frac{D^2g_A^2M_Bm_\pi^2}{72\pi^3f_\pi^4}\left\{(4 + 2\delta)\arctan\frac{\sqrt{u}}{1+\delta}\right. \\ &\quad \left. - \frac{3u + (1+\delta)(4+2\delta)}{u + (1+\delta)^2}\sqrt{u}\right\}, \end{aligned} \qquad (4)$$

$$\begin{aligned} W_{\Sigma ls}(k_f)^{(2\pi\Lambda)} &= \frac{D^2g_A^2M_Bm_\pi^2}{72\pi^3f_\pi^4}\left\{-\frac{u + (1+\delta)(2+\delta)}{u + (1+\delta)^2}\right. \\ &\quad \times\sqrt{u(4\delta + u)} + (4 + 2\delta) \\ &\quad \left.\times\ \ln\frac{u + 2 + 2\delta + \sqrt{u(4\delta+u)}}{2[u + (1+\delta)^2]^{1/2}}\right\}, \end{aligned} \qquad (5)$$

with the abbreviations $u = k_f^2/m_\pi^2$ and $\delta = \Delta^2/m_\pi^2$. The right diagram in Fig. 1 with two medium insertions represents the Pauli blocking correction. In comparison to the expression in Eq. (3) the sign is reverse and the momentum transfer $\vec{l}$ gets replaced by $\vec{l} = \vec{p}_1 - \vec{p}_2$ with $\vec{p}_2$ to be integrated over a Fermi sphere of radius k_f, i.e., $|\vec{p}_2| < k_f$. In case of the real part one is left with a double-integral of the form

$$U_{\Sigma ls}(k_f)^{(2\pi\Lambda)}_{\text{Pauli}} = \frac{D^2 g_A^2 M_B m_\pi^2}{36\pi^4 f_\pi^4} \fint_0^u dx \int_0^u dy \times \frac{1}{(2\delta+1+x-y)^2}\left\{\frac{(2\delta+x-y)^2\sqrt{xy}}{2(\delta-y)^2-2xy} + \frac{2\sqrt{xy}}{(1+x+y)^2-4xy} + \frac{2\delta+x-y}{2\delta+1+x-y} \times \ln\frac{|\delta-y-\sqrt{xy}|(1+x+y-2\sqrt{xy})}{|\delta-y+\sqrt{xy}|(1+x+y+2\sqrt{xy})}\right\}, \quad (6)$$

where the first term in brackets has to be treated as a principal value integral. In practice this is done by solving the $\int_0^u dx$-integral analytically and converting the occurring logarithms into logarithms of absolute values. The Pauli blocking correction to the imaginary part $W_{\Sigma ls}(k_f)$ can even be written in closed analytical form

$$W_{\Sigma ls}(k_f)^{(2\pi\Lambda)}_{\text{Pauli}} = \frac{D^2 g_A^2 M_B m_\pi^2}{72\pi^3 f_\pi^4}\theta(\sqrt{2}k_f-\Delta)\left\{\frac{u}{2}-\delta-1 + \frac{1}{1+2\delta} + \frac{u\delta}{u+\delta^2} + \frac{u(1-\delta)}{2u+2(1+\delta)^2} + \frac{u+(1+\delta)(2+\delta)}{2u+2(1+\delta)^2}\sqrt{u(4\delta+u)} + 2\ln(2+4\delta) + \delta\ln(2+2\delta^2u^{-1}) - (2+\delta) \times \ln[u+2+2\delta+\sqrt{u(4\delta+u)}\,]\right\}. \quad (7)$$

Interestingly, there is a threshold condition $k_f > \Delta/\sqrt{2}$ for Pauli blocking to become active in the imaginary part. The threshold opens at about one half of nuclear matter saturation density $\rho_{\text{th}} = 0.072\,\text{fm}^{-3} = 0.45\rho_0$.

The additional contributions from the iterated one-pion exchange diagrams with a Σ hyperon in the intermediate state are obtained by substituting axial vector coupling constants, $D^2 \to 6F^2$, and dropping the $\Sigma\Lambda$ mass splitting, $\delta \to 0$. The explicit expressions for these contributions to the complex Σ-nuclear spin-orbit coupling strength read

$$U_{\Sigma ls}(k_f)^{(2\pi\Sigma)} = \frac{F^2 g_A^2 M_B m_\pi^2}{12\pi^3 f_\pi^4}\left\{4\arctan\sqrt{u} - \frac{4+3u}{1+u}\sqrt{u}\right\}, \quad (8)$$

$$W_{\Sigma ls}(k_f)^{(2\pi\Sigma)} = -W_{\Sigma ls}(k_f)^{(2\pi\Sigma)}_{\text{Pauli}} = \frac{F^2 g_A^2 M_B m_\pi^2}{12\pi^3 f_\pi^4}\left\{2\ln(1+u) - \frac{2u+u^2}{1+u}\right\}, \quad (9)$$

$$U_{\Sigma ls}(k_f)^{(2\pi\Sigma)}_{\text{Pauli}} = \frac{F^2 g_A^2 M_B m_\pi^2}{12\pi^4 f_\pi^4}\left\{6\sqrt{u}\arctan(2\sqrt{u}) - 2u - \frac{2\sqrt{u}}{\sqrt{1+u}}\ln(\sqrt{u}+\sqrt{1+u}) - \frac{3}{2}\ln(1+4u) + \int_0^u dx\,\frac{1+2u-2x}{(1+u-x)^2} \times \ln\frac{(\sqrt{u}-\sqrt{x})(1+u+x+2\sqrt{ux})}{(\sqrt{u}+\sqrt{x})(1+u+x-2\sqrt{ux})}\right\}, \quad (10)$$

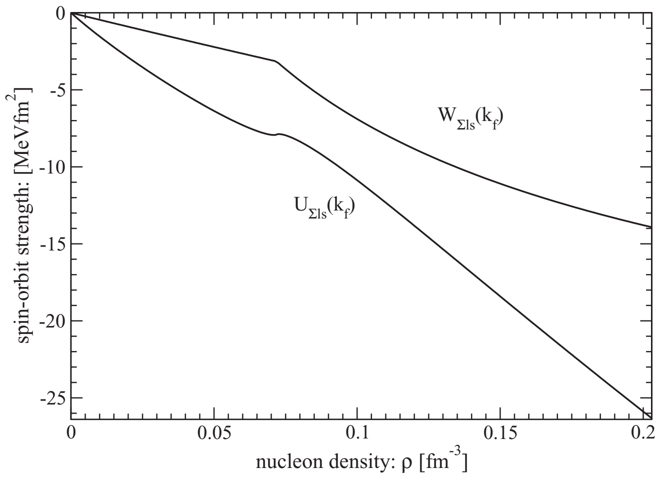

FIG. 2. The complex-valued Σ-nuclear spin-orbit coupling strength $U_{\Sigma ls}(k_f) + i\,W_{\Sigma ls}(k_f)$ generated by iterated 1π-exchange as a function of the nucleon density $\rho = 2k_f^3/3\pi^2$. The imaginary part $W_{\Sigma ls}(k_f)$ originates from the conversion process $\Sigma N \to \Lambda N$ induced by 1π-exchange.

where now almost all integrals could be solved for the Pauli blocking correction.

Summing up all calculated two-loop terms written in Eqs. (4)–(10) we show in Fig. 2 the resulting complex Σ-nuclear spin-orbit coupling strength $U_{\Sigma ls}(k_f) + i\,W_{\Sigma ls}(k_f)$ as a function of the nucleon density in the region $0 \leqslant \rho \leqslant 0.2\,\text{fm}^{-3}$ (corresponding to Fermi momenta $k_f \leqslant 283$ MeV). It is expected that higher-loop contributions related to pion-absorption on two nucleons, in-medium nucleon and pion self-energy corrections etc. are small in this low-density region. The upper curve for the imaginary part $W_{\Sigma ls}(k_f)$ clearly displays the onset of the Pauli blocking effect at the threshold density $\rho_{\text{th}} = 0.072\,\text{fm}^{-3}$. It may come as a surprise that Pauli blocking increases the magnitude of the negative imaginary part. But going back to the original expression Eq. (3) one sees that the squared energy denominator introduces as a weight function for imaginary part the derivative of a delta-function. Therefore the usual argument of phase space reduction by Pauli blocking becomes insufficient even for a qualitative estimate. At normal nuclear matter density $\rho_0 = 0.16\,\text{fm}^{-3}$ (corresponding to a Fermi momentum of $k_{f0} = 263$ MeV) one finds for the total imaginary part $W_{\Sigma ls}(k_{f0}) = (-6.83\text{–}4.89)\,\text{MeV fm}^2 = -11.7\ \text{MeV fm}^2$, where the second entry stems from Pauli blocking. The physics behind this imaginary spin-orbit coupling strength is, of course, the $\Sigma N \to \Lambda N$ conversion process induced by 1π-exchange. One can also see from Fig.2 that the cusp effect in the imaginary part $W_{\Sigma ls}(k_f)$ causes some nonsmooth behavior of the real part $U_{\Sigma ls}(k_f)$. The almost linear decrease with density gets interrupted at the threshold density $\rho_{\text{th}} = 0.072\,\text{fm}^{-3}$. At saturation density one finds a "wrong-sign" Σ-nuclear spin-orbit coupling strength of $U_{\Sigma ls}(k_{f0}) = [(-1.83-2.32)+(-18.21+2.43)]\,\text{MeV fm}^2 = -19.9$ MeV fm^2, where the individual entries correspond to respective terms written in Eqs. (4), (6), (8), (10), in that order. It is somewhat larger than the "wrong-sign" spin-orbit coupling of a Λ hyperon, $U_{\Lambda ls}(k_{f0}) = -15\,\text{MeV fm}^2$ [14]. This is our major result: The second order 1π-exchange tensor interaction generates sizable "wrong-sign" spin-orbit couplings for the Λ and the Σ hyperon together. The negative sign in case of the Σ hyperon is however less obvious, because the relevant loop integrals are derivatives of six-dimensional principal value integrals [see Eq. (3)]. As an aside we note that in the chiral limit ($m_\pi = 0$) the Σ-nuclear spin-orbit coupling strength changes to $U_{\Sigma ls}(k_{f0}) + i\,W_{\Sigma ls}(k_{f0}) = (-25.0\text{–}13.0i)\,\text{MeV fm}^2$, with the real part coming now entirely from the Pauli blocking corrections.

It is expected that the additional 2π-exchange effects of Ref. [15] including decuplet baryons in the intermediate state do not change the present results in a significant way. Firstly, the additional mass splittings in the energy denominators are so high that no new contribution to the imaginary part $W_{\Sigma ls}(k_f)$ is generated for $\rho \leqslant \rho_0$. Secondly, the approximate cancellation between the contributions from $\Delta(1232)$ and $\Sigma^*(1385)$ intermediate states works for Λ and Σ hyperons together, since it is based on different signs of spin-sums [15].

The short-range part of the Σ-nuclear spin-orbit interaction results from a variety of processes, one of them being the ω-exchange piece presented in Eq. (2). Following Ref. [14], we relate the short-distance spin-orbit coupling of the Σ hyperon to the one of the nucleon as follows:

$$U_{\Sigma ls}(k_f)^{(\text{sh})} = C_{ls}\frac{M_N^2}{M_\Sigma^2}U_{Nls}(k_f)^{(\text{sh})}\,. \tag{11}$$

The factor $(M_N/M_\Sigma)^2 = 0.62$ results from the replacement of the nucleon by a Σ hyperon in these relativistic spin-orbit terms. The coefficient C_{ls} parametrizes the ratio of the relevant coupling constants. The expectation from the naive quark model would be $C_{ls} = 2/3$. On the other hand, QCD sum rule calculations of Σ hyperons in nuclear matter [18] indicate that the Lorentz scalar and vector mean fields of a Σ hyperon are similar to the corresponding ones of a nucleon, i.e., $C_{ls} \simeq 1$. In case of the Lorentz scalar mean field, the QCD sum rule calculations are subject to uncertainties due to poorly known contributions from four-quark condensates. Reference [18] concludes that due to a significant SU(3) symmetry breaking in nuclear matter the short-range spin-orbit term of a Σ hyperon may be comparable to the one of a nucleon. For the further discussion we take for the short-range nucleonic spin-orbit coupling strength $U_{Nls}(k_f)^{(\text{sh})} = 3\rho W_0/2 = 30\,\text{MeV fm}^2\rho/\rho_0$ with $W_0 = 124\,\text{MeV fm}^5$ the spin-orbit parameter in the Skyrme phenomenology [19]. Employing $C_{ls} \simeq 1$, as indicated by the sum rule calculations, one estimates the short-range Σ-nuclear spin-orbit coupling strength to $U_{\Sigma ls}(k_{f0})^{(\text{sh})} \simeq 18.6\,\text{MeV fm}^2$. This would lead to an almost complete cancellation of the long-range component generated by iterated one-pion exchange, resulting in a rather weak Σ-nuclear spin-orbit coupling (admittedly with large uncertainties). Finally, we note that the long-range and short-range pieces are distinguished by markedly different dependences on the pion mass m_π (or light quark mass $m_q \sim m_\pi^2$) and the density $\rho = 2k_f^3/3\pi^2$. Therefore, there seems to be no double counting problem when adding long-range and short-range components.

In summary, we have calculated in this work the Σ-nuclear spin-orbit coupling generated by iterated one-pion exchange with a Λ or a Σ hyperon in the intermediate state. We find from this unique long-range dynamics a sizable "wrong-sign" spin-orbit coupling strength of $U_{\Sigma ls}(k_{f0}) \simeq -20$ MeV fm^2. When combined with estimates of the short-range component a weak Σ-nuclear spin-orbit coupling will result in total. Unfortunately, the prospects for an experimental check of this feature are poor. The recently established repulsive nature of the Σ-nucleus optical potential [6] precludes a rich spectroscopy of heavy Σ-hypernuclei which could reveal spin-orbit splittings.

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