

Cross sections of neutron-rich nuclei from projectile fragmentation: Canonical thermodynamic model estimates

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A remarkably simple dependence of fragmentation cross section on average binding energy has been established in experimental data. This dependence was empirically parametrized leading to a very useful formula for extrapolation. We find that the canonical thermodynamic model, which has been used in the past for successful computations of many observables resulting from multifragmentation, reproduces the salient features of fragmentation cross sections of very neutron-rich nuclei very well. This contributes to a theoretical understanding of the observed data.

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As part of a drive toward understanding the production mechanisms of rare isotopes, fragmentation cross sections of many neutron-rich isotopes have recently been measured from the ⁴⁸Ca and ⁶⁴Ni beams at 140 MeV per nucleon on ⁹Be and ¹⁸¹Ta targets [1]. Copper isotope cross sections have been measured in projectile fragmentation of ⁸⁶Kr at 64 MeV per nucleon [2]. A remarkable feature is the correlation between the measured fragment cross section and the average binding energy (Fig. 1) [3]. This observation has prompted attempts at parametrization of cross sections. One very successful parametrization [3] is

$$\sigma = C \exp(A^{-1}(B - \varepsilon_{\text{pair}})/\tau). \quad (1)$$

Here B is the binding energy of the nucleus with mass number A ; $\varepsilon_{\text{pair}} = \kappa \varepsilon A^{-3/4}$; and κ is 1 for even-even nuclei, 0 for odd-even nuclei, and -1 for odd-odd nuclei. The pairing term smooths the straggling seen in the data when logarithms of cross sections are plotted against B/A for even-even and odd nuclei (or odd-even and odd-odd nuclei). The theoretical basis for the simple appearance of B/A or the pairing term correction is not transparent. Another equation that is highly successful is

$$\sigma = cA^{3/2} \exp[(N\mu_n + Z\mu_p + B - \varepsilon_{\text{pair}} + E_s \cdot \min(S_n, S_p, S_\alpha))/T]. \quad (2)$$

For values of parameters and details, see Ref. [4].

Here we do calculations for the production cross sections of silicon isotopes from projectile fragmentation of ⁴⁸Ca and of copper isotopes from projectile fragmentation of ⁸⁶Kr using the canonical thermodynamic model. Some of these cross sections are very small and they serve as very stringent tests of the model. The model has been extensively applied for production cross sections of other particles that are more numerous [5] and agreements are good. The basic physics of the model is the

same as in many other models of intermediate energy heavy-ion collisions: the statistical multifragmentation model (SMM) [6] or the microcanonical simulations of heavy-ion collisions [7,8]. But SMM or the microcanonical simulations are totally impractical for calculations of very small cross sections as they rely on Monte-Carlo simulations. The canonical model gives closed expressions and calculations can be made as accurate as desired. The grand canonical model is unsuitable for exploring these furthest limits of the phase space and is expected to be very unreliable [5,9]. We come back to this point later.

We consider production of silicon isotopes from the statistical breakup of ⁴⁸Ca. We denote the average number (multiplicity) of ¹⁴Si_{*n*} by $\langle n_{14,n} \rangle$. Then the cross section is $\sigma(14, n) = C \langle n_{14,n} \rangle$, where C is a constant not calculable from the thermodynamic model. It depends upon dynamics that are outside the scope of the model. Similarly we consider the production of Cu isotopes ²⁹Cu_{*n*} from a source that has 36 protons and 50 neutrons, i.e., ⁸⁶Kr. The source sizes adopted for the calculation are zero order guesses. It could be sometimes smaller or greater depending on the diffusion from the target.

We write down the formulas used for the calculations but we do not derive them as they can be found elsewhere [5,10]. The fragmenting system (⁴⁸Ca or ⁸⁶Kr) has Z_0 protons and N_0 neutrons. The canonical partition function is given by

$$Q_{Z_0, N_0} = \sum \prod \frac{\omega_{i,j}^{n_{i,j}}}{n_{i,j}!}. \quad (3)$$

Here the sum is over all possible channels of breakup (the number of such channels is enormous) that satisfy $Z_0 = \sum i \times n_{i,j}$ and $N_0 = \sum j \times n_{i,j}$; $\omega_{i,j}$ is the partition function of one composite with proton number i and neutron number j , respectively; and $n_{i,j}$ is the number of this composite in the given channel. The one-body partition function $\omega_{i,j}$ is a product of two parts, one arising from the translational motion of the composite and another from the intrinsic partition function of the composite,

$$\omega_{i,j} = \frac{V_f}{h^3} (2\pi m(i+j)T)^{3/2} \times z_{i,j}(\text{int}). \quad (4)$$

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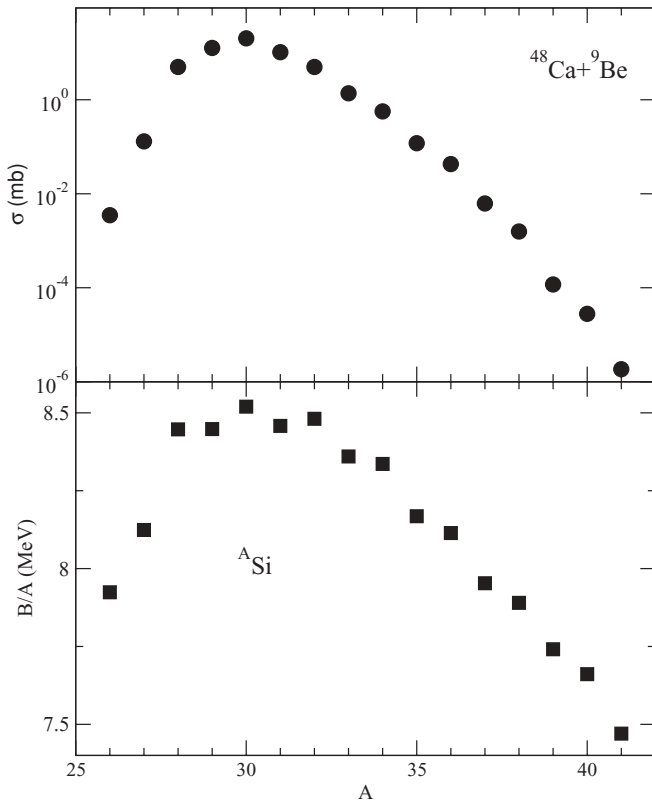


FIG. 1. Fragment cross section and average binding energy plotted as a function of mass number for silicon isotopes.

Here $m(i+j)$ is the mass of the composite and V_f is the volume available for translational motion; V_f will be less than V , the volume to which the system has expanded at breakup. We use $V_f = V - V_0$, where V_0 is the normal volume of the nucleus with Z_0 protons and N_0 neutrons. In this calculation we have used a fairly typical value $V = 6V_0$.

The probability of a given channel $P(\vec{n}_{i,j}) \equiv P(n_{0,1}, n_{1,0}, n_{1,1}, \dots, n_{i,j}, \dots)$ is given by

$$P(\vec{n}_{i,j}) = \frac{1}{Q_{Z_0, N_0}} \prod \frac{\omega_{i,j}^{n_{i,j}}}{n_{i,j}!}. \quad (5)$$

The average number of composites with i protons and j neutrons is seen easily from the above equation to be

$$\langle n_{i,j} \rangle = \omega_{i,j} \frac{Q_{Z_0-i, N_0-j}}{Q_{Z_0, N_0}}. \quad (6)$$

The constraints $Z_0 = \sum i \times n_{i,j}$ and $N_0 = \sum j \times n_{i,j}$ can be used to obtain different looking but equivalent recursion relations for partition functions. For example,

$$Q_{Z_0, N_0} = \frac{1}{Z_0} \sum_{i,j} i \omega_{i,j} Q_{Z_0-i, N_0-j}. \quad (7)$$

These recursion relations allow one to calculate Q_{Z_0, N_0} .

We list now the properties of the composites used in this work. The proton and the neutron are fundamental building blocks, thus $z_{1,0}(\text{int}) = z_{0,1}(\text{int}) = 2$, where 2 takes care of the spin degeneracy. For deuteron, triton, ^3He , and ^4He , we use $z_{i,j}(\text{int}) = (2s_{i,j} + 1) \exp(-\beta e_{i,j}(\text{gr}))$, where $\beta =$

$1/T$, $e_{i,j}(\text{gr})$ is the ground state energy of the composite and $(2s_{i,j} + 1)$ is the experimental spin degeneracy of the ground state. Excited states for these very low mass nuclei are not included. For Si and Cu nuclei whose production cross sections are sought in this work we use the experimental binding energies tabulated in Ref. [11] but also include a term for contribution from excited states (see the discussion following). For mass number $A = 5$ and greater [but charge $\neq 14(29)$], we use the liquid-drop formula. For nuclei in isolation, this reads ($a = i + j$)

$$z_{i,j}(\text{int}) = \exp \frac{1}{T} \left[W_0 a - \sigma(T) a^{2/3} - \kappa \frac{i^2}{a^{1/3}} - s \frac{(i-j)^2}{a} + \frac{T^2 a}{\epsilon_0} \right]. \quad (8)$$

The derivation of this equation is given in several places [5,6] so we do not repeat the arguments here. The expression includes the volume energy, the temperature-dependent surface energy, the Coulomb energy, and the symmetry energy. The term $\frac{T^2 a}{\epsilon_0}$ represents contribution from excited states because the composites are at a nonzero temperature. This form was used in other applications of the model and we have kept this unchanged. This term is also included in $z_{14,n}(\text{int})(z_{29,n}(\text{int}))$.

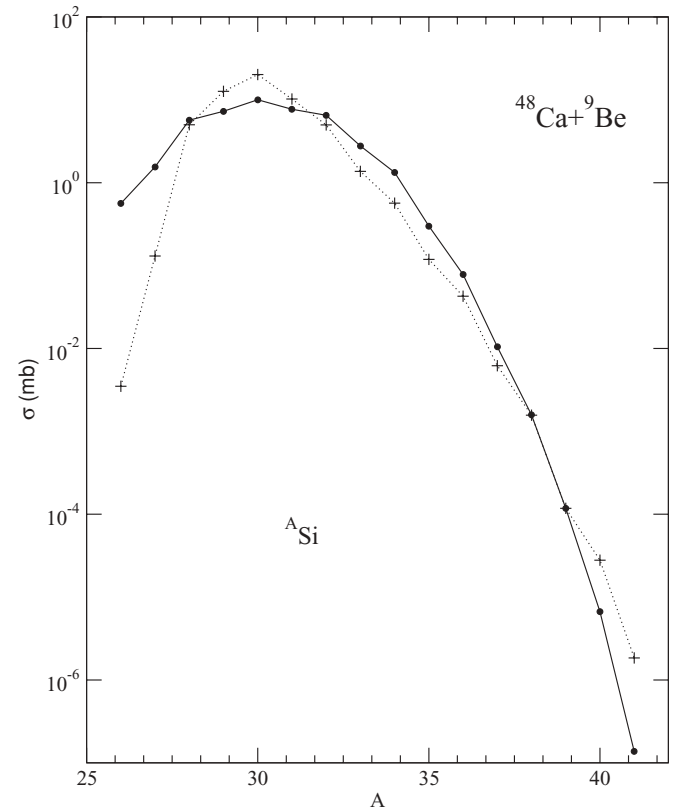


FIG. 2. Experimental cross sections for silicon isotopes (points) compared with theoretical results (crosses). A dotted line is drawn through the experimental data and a solid line through the calculated values. The temperature used is 9.5 MeV. The normalization constant for theory is chosen by fitting to the experimental cross section of ^{39}Si .

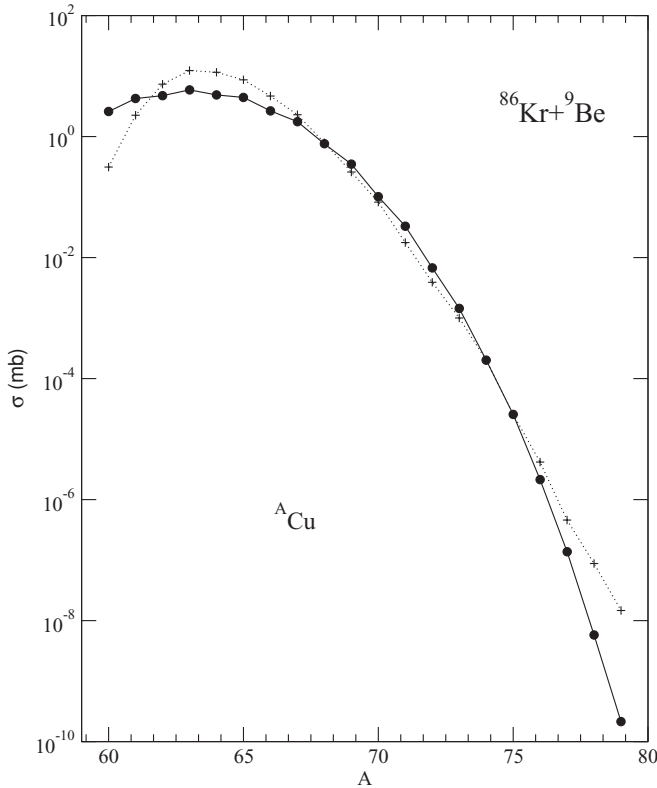


FIG. 3. Same as in Fig. 2 but for Cu isotopes. Normalization for theory is chosen from the experimental value for ^{75}Cu .

Note that in the fitting formula of Eq. (2) a different expression for contribution from excited states is used.

We have to state which nuclei are included in computing Q_{Z_0, N_0} . For i, j (the proton and the neutron number), we

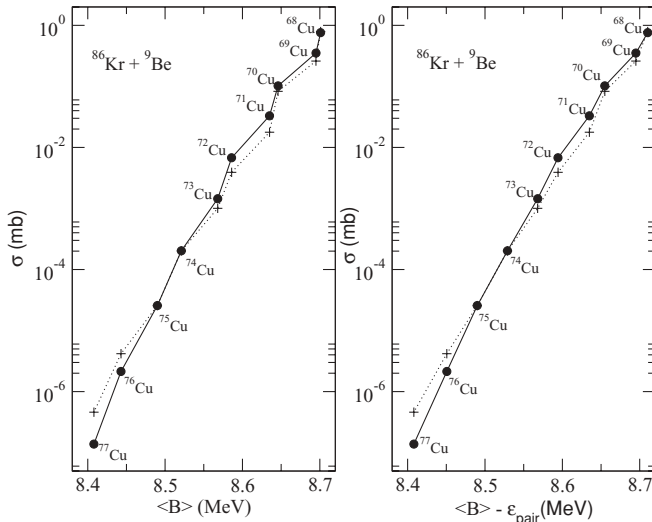


FIG. 4. Straggling of data between odd-odd and odd-even cases (points) when plotted against the average binding energy $\langle B \rangle = B/A$. Similar scatter is seen in theoretical calculation (crosses). In the right-hand panel the cross sections are plotted against $\langle B \rangle - \epsilon_{\text{pair}}$ [see Eq. (1)]. The value of ϵ used here is 30 MeV. This decreases the straggling significantly both for data and theory.

include a ridge along the line of stability. The liquid-drop formula above also gives neutron and proton drip lines and the results shown here include all nuclei within the boundaries.

The long-range Coulomb interaction between different composites can be included in an approximation called the Wigner-Seitz approximation. We incorporate this following the scheme set up in Ref. [6].

It remains now to state the results. Figure 1 taken from Ref. [3] shows the remarkable correlation between experimental values of cross sections of silicon isotopes (from ^{48}Ca on ^9Be reaction at 140 MeV per nucleon) and average binding energies. The experimental data on cross sections (shown as solid symbols in this article) span about seven orders of magnitude. In Fig. 2 we show the results of our calculations (crosses). There are basically two parameters: an overall normalization factor (chosen in the figure to give the correct value of cross section for ^{28}Si) and the temperature (taken here to be 9.5 MeV, which is within the range of temperatures expected for this reaction). Except at the tails of the distribution, the agreement is fair and the calculation does indeed give the very rapid decrease of the cross section for large A . The straggling in values of the cross sections between even-even and odd nuclei is also reflected in the calculation. In Fig. 3 we compare data and calculations for the case of production of copper isotopes. The data here span more than 8 decades and the calculation, except for the tails, does very well. The straggling between cross section values for odd-odd and odd copper isotopes is highlighted in Fig. 4. In the same figure we show that for both data and calculation the straggling disappears if the cross section is plotted against $\langle B \rangle - \epsilon_{\text{pair}}$ rather than against just $\langle B \rangle$ (see also Ref. [4]). We find it gratifying that the model is able to reproduce such fine details.

Last, we make a connection with grand canonical fitting of the data [4]. In our model we use $\sigma(z, n) = C \langle n_{z,n} \rangle$, where the value of C has to be taken from experiment. Thus in a model of this type what we need is the ratio $\langle n_{z,n+1} \rangle / \langle n_{z,n} \rangle$ to be predicted correctly. From Eq. (6) this is

$$\frac{\langle n_{z,n+1} \rangle}{\langle n_{z,n} \rangle} = \frac{\omega_{z,n+1}}{\omega_{z,n}} \frac{Q_{Z_0-z, N_0-n-1}}{Q_{Z_0-z, N_0-n}}. \quad (9)$$

Here Z_0, N_0 are the charge and neutron number of the projectile that is fragmenting (^{86}Kr or ^{48}Ca) and (z, n) denotes Cu or Si isotopes. The right-hand side of Eq. (9) is very simple in the grand canonical ensemble. The ratio of the two canonical partition functions is replaced by a term independent of n , i.e., the right-hand side is simply $\frac{\omega_{z,n+1}}{\omega_{z,n}} \exp(\beta \mu_n)$, where μ_n is a constant for all n, z . That would be correct if Z_0 and N_0 are large and also $Z_0 \gg z$ and $N_0 \gg n$, whereas in small systems such as here the ratio of the partition functions varies as n changes. As we approach the neutron drip line it drops fast with n . To approximate the right-hand side of Eq. (9) by the simpler expression $\frac{\omega_{z,n+1}}{\omega_{z,n}} \exp(\beta \mu_n)$, we need to choose μ_n judiciously and further alter the value of the temperature from the one used in the canonical model. In that case the temperature must be significantly reduced.

In conclusion, the canonical model reproduces the salient features of production cross sections of very neutron-rich nuclei. The empirical formulas for extrapolation [3,4] are very useful and the canonical thermodynamic model cannot hope to replace these but it contributes to a theoretical understanding of the data. The same parameters that we have used here can be used to predict the production cross sections of intermediate

mass fragments or the properties of the largest fragment after multifragmentation [12].

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