Measurement of ^{23}Mg$(p,\gamma)^{24}$Al resonance energies

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The existence of two systematically inconsistent sets of measurements of the ^{24}Al excitation energies, which are used to determine ^{23}Mg$+p$ resonance energies, results in a variation of a factor 5 in the thermonuclear ^{23}Mg$(p,\gamma)^{24}$Al reaction rate at $T = 0.25$ GK. The astrophysically important energies have been determined to an uncertainty of 6 keV by measuring triton spectra from the ^{24}Mg(^{3}He,t)^{24}Al reaction at $E(^3\text{He}) = 30$ MeV, and good general agreement is found with one previous set. The present measurement of $E_x = 2346(6)$ keV for what is thought to be the most important resonance is, however, in disagreement with *both* prior measurements of 2328(10) and 2369(4) keV, where the latter value belongs to the outlying set. The presently determined resonance energies reduce the related uncertainty in the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate by a factor of ≈3, which will constrain the determination of nuclear flow out of the NeNa cycle, and production of $A \geqslant 20$ nuclides, in explosive hydrogen burning over a temperature range $0.2 < T < 1.0$ GK.

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I. INTRODUCTION

For decades, the ^{23}Mg$(p,\gamma)^{24}$Al reaction [$Q_{p\gamma} = 1872.1(31)$ keV] [1] has been known [2,3] to be a potential means for breaking out of the NeNa cycle to heavier nuclear species in explosive hydrogen-burning stellar environments such as novae, type I x-ray bursts and accreting black holes. At stellar temperatures $T < 0.1$ GK, ^{23}Mg can be produced by the NeNa cycle, which is closed by its β^+ decay ($t_{1/2} = 11.3$ s) to ^{23}Na, followed by the ^{23}Na$(p,\alpha)^{20}$Ne reaction. At higher temperatures the ^{23}Mg$(p,\gamma)^{24}$Al$(\beta^+\nu_e)^{24}$Mg sequence is expected to become competitive with ^{23}Mg β^+ decay, providing a nucleosynthetic path to heavier species together with the ^{23}Na$(p,\gamma)^{24}$Mg reaction. Models of explosive hydrogen-burning environments, therefore, require an accurate determination of the ^{23}Mg$(p,\gamma)^{24}$Al thermonuclear reaction rate to constrain the expected production of $A \geqslant 20$ elements. For example, a recent post-processing study [4] showed that large variations in the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate could affect the production of the γ-ray astronomy targets ^{22}Na ($t_{1/2} = 2.6$ a) and ^{26}Al ($t_{1/2} = 0.7$ Ma) in classical oxygen-neon novae, which have peak temperatures of up to ≈0.35 GK. Above $T = 1$ GK, the ^{23}Mg$(p,\gamma)^{24}$Al reaction becomes less important because production of ^{23}Mg is bypassed on the proton-rich side by the sequence ^{21}Na$(p,\gamma)^{22}$Mg$(p,\gamma)^{23}$Al$(p,\gamma)^{24}$Si$(\beta^+\nu_e)^{24}$Al. A knowledge of ^{23}Mg$+p$ resonances with energy $E_{\text{c.m.}} \lesssim 1$ MeV is required to reduce the uncertainty in the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate in the interesting temperature range $0.1 < T < 1.0$ GK.

In a stellar environment where particles have a Maxwell-Boltzmann distribution of energies characterized by temperature T, the resonant ^{23}Mg$(p,\gamma)^{24}$Al reaction rate per particle pair [5] is given by a sum over narrow, isolated resonances r,

$$\langle\sigma\upsilon\rangle = \left(\frac{2\pi}{\mu kT}\right)^{3/2}\hbar^2\sum_r(\omega\gamma)_r e^{-E_r/kT}, \tag{1}$$

where $\hbar$ is the Dirac constant, k is the Boltzmann constant, μ is the reduced mass, and E_r is the resonance energy in the c.m. frame:

$$(\omega\gamma)_r = \frac{(2J_r+1)}{(2J_p+1)(2J_{\text{Mg}}+1)}\left(\frac{\Gamma_p\Gamma_\gamma}{\Gamma}\right)_r \tag{2}$$

is the resonance strength, where $J_p(=1/2)$, $J_{\text{Mg}}(=3/2)$ and J_r are the spins of the reactants and the resonance, respectively. Γ_p and Γ_γ are the proton and γ-ray partial widths of the resonance, respectively, and $\Gamma = \Gamma_p + \Gamma_\gamma$ is the total width. Each term in the sum in Eq. (1) has an exponential dependence on E_r because of the Coulomb barrier.

Wallace and Woosley [2] initially evaluated the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate based on the contribution of a single resonance. By considering a direct-capture process and two additional resonances, Wiescher *et al.* [3] improved upon the calculation of Ref. [2]. Kubono *et al.* [6] then studied the ^{24}Mg(^{3}He, t)^{24}Al reaction at a beam energy of 60 MeV, and reevaluated the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate using their experimental constraints on the spins and excitation energies (±10 keV) of four ^{24}Al levels. However, a prior measurement of the ^{24}Mg(^{3}He, t)^{24}Al reaction at 81 MeV by Greenfield *et al.* [7] yielded ^{24}Al excitation energies with comparable precision that were systematically higher by ≈20 to 50 keV (Table I). Most recently, Herndl *et al.* [8] reevaluated the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate using all available (inconsistent)

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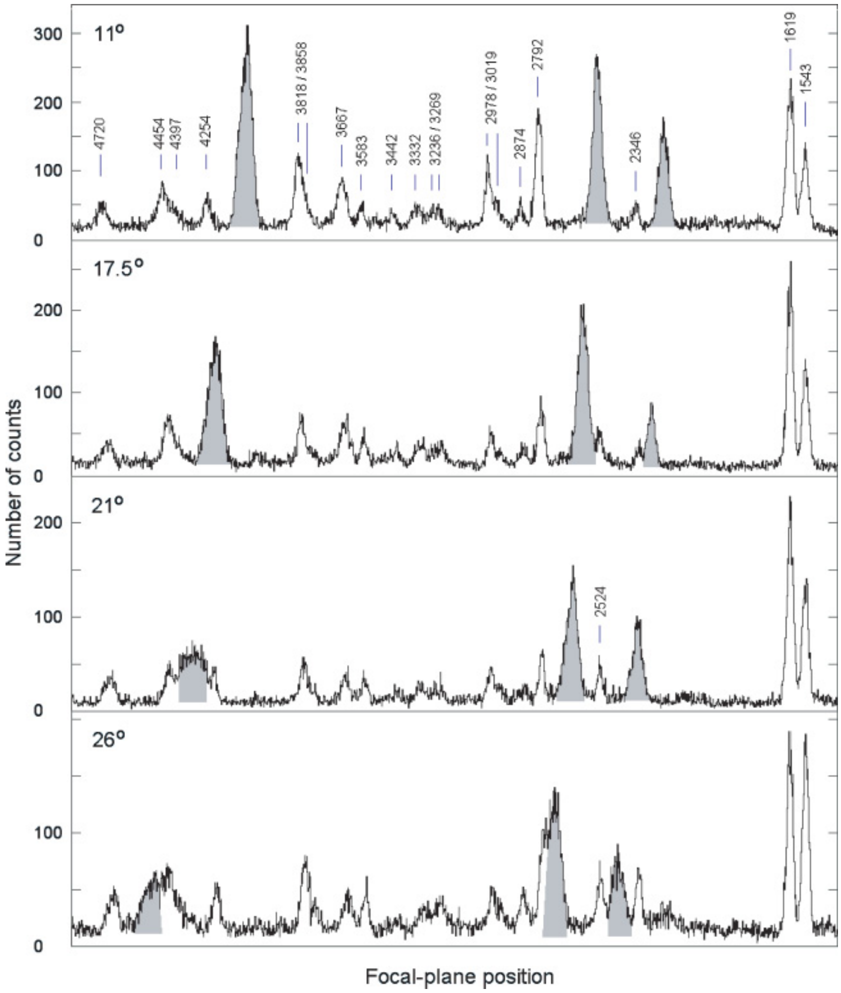

FIG. 1. (Color online) Focal-plane triton spectra from the $^{24}Mg(^3He, t)^{24}Al$ reaction at 30 MeV, corresponding to $1500 \lesssim E_x(^{24}Al) \lesssim 4800$ keV determined in the present work (labeled). The spectra were acquired with $\theta_{lab} = 11^\circ, 17.5^\circ, 21^\circ$, and 26° from top to bottom, and are shown shifted relative to one another so that the ^{24}Al excitation-energy scale is roughly matched. Background peaks of ^{12}N (g.s.), ^{16}F (721 keV), and ^{16}F (424 keV), from left to right, are shaded in gray.

experimental information on ^{24}Al excitation energies. Shell-model correspondences were determined for known ^{24}Al excited states using the isobaric multiplet mass equation, and proton and γ-ray partial widths were derived using shell-model calculations. It was concluded that a single resonance at $E_r = 478(20)$ keV [$E_x(^{24}Al) = 2349(20)$ keV] with $J^\pi = 3^+$ determines the $^{23}Mg(p, \gamma)^{24}Al$ reaction rate for temperatures $0.2 < T < 1.0$ GK. The large uncertainty in this resonance energy is due to the inconsistent $(^3He, t)$ measurements mentioned above, and leads to a factor 5 variation in the reaction rate at $T = 0.25$ GK—a typical nova peak temperature—because of its exponential dependence on E_r. A better determination of this resonance energy would reduce the related uncertainty in the reaction rate, and aid future experiments that attempt to measure resonance strengths.

II. EXPERIMENTAL PROCEDURE

To resolve the inconsistencies in the measured level energies of ^{24}Al, the energies of known $^{23}Mg+p$ resonances have been remeasured, and new resonances searched for, using the $^{24}Mg(^3He, t)^{24}Al$ reaction in conjunction with calibrations based on the $^{28}Si(^3He, t)^{28}P$ reaction at Yale University's Wright Nuclear Structure Laboratory.

Natural Mg (174 μg/cm^2) and Si (303 μg/cm^2) target foils were mounted on aluminum frames by a commercial supplier.[1] The Si foil was self-supporting, and the Mg was supported by an 11-μg/cm^2 parylene-N ($-C_8H_8-$) substrate. The quoted thickness measurements were specified by the supplier to have <10% tolerance, with surface non-uniformity <1%. The energy loss of 5.486-MeV ^{241}Am-decay α particles through the targets was measured with a silicon surface-barrier detector immediately following the experiment because the foils were stored in air for several months prior to the experiment and may have oxidized. By this method, the thickness of the Si foil was determined to be 302 ± 20 μg/cm^2, consistent with negligible oxidation. Visual inspection of the Mg target indicated that it had oxidized significantly. Under the assumption of one oxygen atom per magnesium atom, its thickness was measured to be 267 ± 20 μg/cm^2 MgO, which is consistent with the inoxidized Mg foil thickness specified by the supplier.

The Yale tandem Van de Graaff accelerated a beam of ^{3}He ions to a fixed energy of 30 MeV, which impinged on either of the aforementioned targets. An Enge magnetic spectrograph accepted light reaction products through a rectangular aperture, and momentum analyzed them. Tritons were focused on

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TABLE I. ^{24}Al-level energies (keV) with corresponding ^{23}Mg + p resonance energies (keV), spins and parities.

$E_x(^3\mathrm{He},t)$ [7]	$E_x(^3\mathrm{He},t)$ [6]	E_x [8,12]	$E_x(^3\mathrm{He},t)$ present	E_r present	J^π [12]
1563(7)	1535(10)	1559(13)	1543(6)		5^+
1638(8)	1614(10)	1634(11)	1619(6)		3^+
2369(4)	2328(10)	2349(20)	2346(6)	474(6)	3^{+a}
2546(7)	2521(10)	2534(13)	2524(6)	652(6)	4^{+a}
2832(6)	2787(10)	2810(20)	2792(6)	920(6)	2^+
2920(23)	2876(10)	2900(20)	2874(6)	1002(6)	3^{+a}
3037(16)	3002(10)	3020(20)	2978(6) 3019(6)	1106(6) 1147(6)	$(1-3)^{+b}$
3291(12)	3247(10)	3270(20)	3236(6) 3269(6)	1364(6) 1397(6)	3^{+b}
3384(16)	3330(10)	3360(25)	3332(6)	1460(6)	(2^+)
3500(19)	3444(10)	3470(30)	3442(7)	1570(6)	2
3608(16)	3590(10)	3600(10)	3583(7)	1711(6)	$(0,1)^+$
3716(10)	3674(10)	3695(20)	3667(7)	1795(7)	3^+
3911(6)	3860(10)	3885(20)	3818(7) 3858(7)	1945(7) 1986(7)	$(2-4)^{-b}$
4057(17)	4061(10)	4059(10)			(1^+)
4129(24)		4120(25)			2
4301(31)	4253(10)	4275(25)	4254(8)	2382(8)	4^-
	4386(10)	4385(20)	4397(8)	2525(8)	
4485(10)	4445(10)	4465(20)	4454(9)	2582(8)	3^+
4764(8)	4709(10)	4735(25)	4720(10)	2848(10)	(4^+)

[a]Based in part on the $A = 24$, $T = 1$ isospin-triplet identifications of Ref. [8].
[b]Questionable in light of the presently resolved doublet.

a detection plane spanned by a position-sensitive ionization drift chamber [9] over radii $70 < \rho < 87$ cm. It measured the position and the energy loss, ΔE, of the particles. The residual energy, E, of particles was deposited into a plastic scintillator.

The ^{24}Mg$(^3$He$,t)^{24}$Al and ^{28}Si$(^3$He$,t)^{28}$P reactions were measured over a five-day period using a fixed magnetic-field strength $B = 11$ kG, at spectrograph angles $\theta_{\mathrm{lab}} = 11^\circ, 17.5^\circ, 21^\circ$, and 26°, and with horizontal and vertical entrance-aperture settings of $\Delta\theta = \pm 30$ mrad and $\Delta\phi = \pm 40$ mrad, respectively.

III. DATA AND ANALYSIS

Particle groups (p, d, t, α) were identified by combining focal-plane position (~momentum), ΔE and E in 2D histograms. Tritons were selected cleanly by sorting the data offline through software gates in these histograms, and spectra of focal-plane position were plotted for the ^{24}Mg$(^3$He$,t)^{24}$Al (Fig. 1) and ^{28}Si$(^3$He$,t)^{28}$P reactions at each spectrograph angle.

Background peaks from the ^{16}O$(^3$He$,t)^{16}$F and ^{12}C$(^3$He$,t)^{12}$N$^{\mathrm{g.s.}}$ reactions were identified kinematically in the ^{24}Al spectra. These were expected, and the spectrograph angles were chosen so that the location of the background peaks would allow a clear observation of each astrophysically important ^{24}Al level at a minimum of three angles. An additional, roughly flat, background precluded the observation of levels that were relatively weakly populated. This background was likely from the unbound ^{25,26}Al, $E_x >$ 10 MeV continuum excited by the ^{25,26}Mg$(^3$He$,t)^{25,26}$Al reactions, and broad peaks from unbound ^{13}N levels with $E_x > 11$ MeV excited by the ^{13}C$(^3$He$,t)^{13}$N reaction. The ^{28}P spectra were cleaner, and simple to interpret because of their similarity to the 35 MeV ^{28}Si$(^3$He$,t)^{28}$P spectrum in Ref. [10].

The spectra were analyzed using a least-squares fit of multiple gaussian functions of typical FWHM ≈40 keV, from which peak centroids were determined. Isolated, easily identifiable peaks corresponding to known excited states [11] of ^{28}P with $E_x < 5$ MeV, and with uncertainties as low as 0.5 keV (but typically 5 keV) were used for momentum calibration of the focal plane at each spectrograph angle. Third-order polynomial least-squares fits of ρ to focal-plane position $(0.25 \leqslant \chi^2_\nu \leqslant 0.55)$ were derived from this information. These fits were used to identify peaks with ^{24}Al levels and determine their excitation energies at each spectrograph angle. A statistically weighted average excitation energy was calculated for each level and rounded to the nearest keV, as reported in Table I.

A universal uncertainty of 3 keV was determined from a combination of statistical uncertainty and reproducibility. In addition, there was a 3 keV uncertainty from the uncertainty in relative ^{24}Mg to ^{28}Si target thickness, and a 4.1 keV uncertainty from the relative Q values of the ^{24}Mg$(^3$He$,t)^{24}$Al and ^{28}Si$(^3$He$,t)^{28}$P reactions, arising mostly from the uncertainties in the masses of ^{24}Al (2.8 keV/c^2) and ^{28}P

(3 keV/c^2) [1]. Under the assumptions that the above uncertainties are mutually independent and gaussian distributed, they may be added in quadrature, which results in a 5.9 keV uncertainty. For $E_x < 2.5$ MeV the calibration was heavily weighted by the energies of well-known [11] ^{28}P levels at $E_x = 1134.0(5)$, 1313(2), 1516(2), 1568(3), and 2104(1) keV. Excitation-energy uncertainties above $E_x =$ 2.5 MeV were inflated gradually from the base value of 5.9 keV because the calibration became increasingly dependent on a single set of measurements [10] of ^{28}P-level energies with 5 or 10 keV uncertainties.

Resonance energies for the ^{23}Mg$(p,\gamma)^{24}$Al reaction were determined from the relation $E_r = E_x - Q_{p\gamma}$. The mass of ^{24}Al used to determine E_x as described in this section cancels with itself (as does its uncertainty) when E_r is determined from E_x and $Q_{p\gamma}$. However an additional, smaller uncertainty of 1.3 keV is introduced by the mass of ^{23}Mg. For these reasons, the uncertainties in E_r reported in Table I are slightly smaller than the corresponding uncertainties in E_x.

IV. DISCUSSION

All but two known ^{24}Al levels [12] in the range $1.5 < E_x < 4.8$ MeV were observed. The FWHM energy resolution of 40 keV was an improvement over that of Ref. [6] (50 keV) and Ref. [7] (70 keV), and enabled the discovery of doublets at $E_x = 2978$–3019, 3236–3269, and 3818–3858 keV. Constraints on J^π for these levels based on DWBA analysis in previous (^{3}He, t) work [6,7,12] should therefore be taken with caution. None of these previously unresolved levels are expected to have astrophysical relevance for hydrogen-burning temperatures $T < 2$ GK. The levels observed at $E_x = 3332(6)$ and 3667(7) keV were broader than the instrumental resolution, which suggests that either they are doublets or have intrinsic widths $\gtrsim$20 keV.

The uncertainties in E_x and E_r have been reduced by a factor of ≈3 over the most recent compilations [8,12]. Much of the prior uncertainty was due to the systematically inconsistent results of Kubono *et al.* [6] and Greenfield *et al.* [7]. The present measurements are in good agreement with those of Ref. [6], and poor agreement with Ref. [7] which makes the existence of an unaccounted-for systematic error in Ref. [7] probable. The magnitude of this error ranges from $\Delta E_x \approx$ +20 keV to +50 keV through the energy range $1.5 < E_x <$ 4.8 MeV.

The excitation energy of (what is thought to be) the most important resonance astrophysically was measured in the present work to be 2346(6) keV, in disagreement with the measurements of both Refs. [6] [2328(10) keV] and [7] [2369(4) keV]. The disagreement with Ref. [6] is surprising since the error bars in the present work overlap with those in Ref. [6] for every other level, besides the newly resolved doublets. The disagreement with Ref. [7] for this resonance is not surprising considering that the excitation energies in that work are systematically higher than the present results. The present value agrees well with the most recent compilation [12] value of $E_x = 2349(20)$ keV—which was used in Ref. [8] to calculate the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate—and has a substantially reduced uncertainty. It should also be noted that the (^{3}He, t) cross section for the 2346(6) keV level was observed to decrease significantly at far-forward spectrograph angles in a separate measurement using the same apparatus.

TABLE II. ^{23}Mg$(p,\gamma)^{24}$Al resonance parameters.

E_x (keV)[a]	E_r (keV)[a]	Γ_p (meV)[b]	Γ_γ (meV)[c]	$\omega\gamma$ (meV)
2346(6)	474(6)	173	33	25
2524(6)	652(6)	2.2×10^3	58	58
2792(6)	920(6)	8.6×10^5	52	52
2874(6)	1002(6)	2.9×10^4	12	12

[a]Present work.
[b]Scaled from Ref. [8] using $P_\ell(E_r, R_n)$ and E_r from the present work (see text).
[c]Adopted from Ref. [8].

Adopting the J^π assignments of Ref. [8], the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate was recalculated using Eq. (1) with the four presently determined resonance energies shown in Table II. Γ_p was scaled with E_r from Ref. [8] using the penetration factor $P_\ell(E_r, R_n)$, which was determined by computing the regular and irregular Coulomb wave functions [5,13] [$R_n = 1.25(1^{1/3} + 23^{1/3})$ fm is the interaction radius and ℓ is the proton orbital angular momentum]. Doing so does not have a significant effect on $\omega\gamma$ since all resonances considered are expected to have $\Gamma_p \gg \Gamma_\gamma$. Γ_γ was also adopted from Ref. [8] without scaling for its (relatively weak) energy dependence, and the direct-capture contribution to the rate was adopted from that work. The resonance parameters are summarized in Table II.

In Fig. 2 the ratio of the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate from the present work to that determined in Ref. [8] is plotted for stellar temperatures $0.15 < T < 2.0$ GK. The uncertainty bands for both rates are derived by using the upper and lower limits of the resonance-energy uncertainties, and show that the rates are generally in agreement. The present measurements increase the recommended rate by 5–20% in the temperature range $0.19 < T < 1.9$ GK, and the threefold reduction in resonance-energy uncertainties reduces the related uncertainty in the reaction rate by a factor ≈3 in the temperature range $0.2 < T < 2.0$ GK. The $E_r = 474$-keV resonance dominates the reaction rate for $0.2 < T < 1.9$ GK. Below 0.2 GK, the rate is dominated by direct capture. The $E_r = 652$-keV resonance makes contributions of 1, 10, 22, 35, and 40% to the rate at temperatures $T = 0.38$, 0.68, 1.0, 1.5, and 2.0 GK, respectively. The $E_r = 920$ and 1002-keV resonances contribute <8% and <2% respectively to the rate for $T < 2.0$ GK.

The present reduction in the uncertainty of the ^{23}Mg$(p,\gamma)^{24}$Al reaction rate, which is now certainly dominated by the unmeasured resonance strengths, will constrain the determination of nuclear flow out of the NeNa cycle during explosive hydrogen burning for $T < 1$ GK. However, the present reaction rate is sufficiently similar to past results [6,8] that it should not directly change the expected production of ^{22}Na and ^{26}Al in classical novae [4,14].

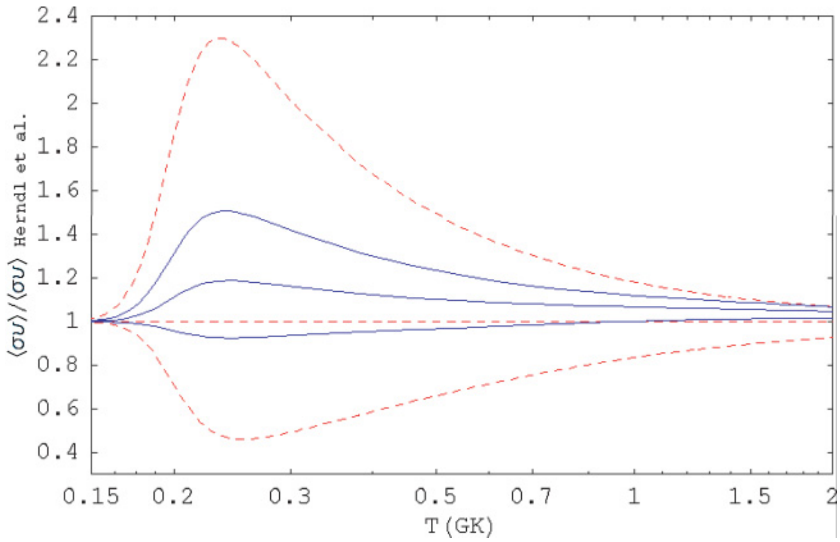

FIG. 2. (Color online) Ratio of the ^{23}Mg$(p, \gamma)^{24}$Al rate from the present work (solid—blue online) to that of Herndl *et al.* [8] (dashed—red online). The uncertainty bands represent only the uncertainty in the rate derived by taking upper and lower limits on E_r.

V. OUTLOOK

The excitation energies and uncertainties from the present work could be adjusted and reduced, respectively, by a precise determination of a single, or several, value(s) of E_x using well calibrated, high-resolution γ-ray detectors. Such a measurement would eliminate the uncertainties originating from target thicknesses and relative $(^3\text{He}, t)$ Q value. Alternatively, the present values and uncertainties for E_x and E_r could be adjusted in the future by precise mass measurements of ^{24}Al, ^{28}P and, to a lesser extent, ^{23}Mg.

Direct experimental determinations of the spins and strengths of the $E_r = 474$ and 652-keV resonances are needed to test the isobaric-triplet assignments and calculations of Ref. [8]. These resonance strengths could be measured directly using a high intensity, radioactive, ^{23}Mg-ion beam. In the absence of a ^{23}Mg beam, the 474-keV resonance strength could be determined by measuring the lifetime and spin of the 2346-keV ^{24}Al level and either its γ-ray or proton branching ratio. The planning, execution and analysis of any such measurements will be simplified with the present results in hand.

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