Mass and width of strange baryon resonances using QCD sum rules

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The mass spectra of strange baryons in the octet family are investigated in a finite-energy QCD sum rule approach based on the Gauss-Weierstrass transform. The phenomenological form of the spectral function is saturated by the ground state and two of the lowest excited states, considered as having opposite parities. Treating the ground-state parameters as known quantities, the masses, widths, and couplings to the interpolating fields are determined and compared with experiment.

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I. INTRODUCTION

An important goal of the study of hadronic physics is to understand the baryon spectrum from QCD, the underlying theory of the strong interaction. In quantum mechanics, stationary states in bound state problems are eigenstates (eigenfunction and eigenenergy) of an appropriate Hamiltonian. Bound states such as light baryonic systems in QCD, on the other hand, are far more complicated objects. Here the excitation energy is sufficient to create several of the constituents. In hadrons, a typical excitation energy (due to QCD) is a few hundred MeV. This is sufficient to create one or more light quark-antiquark pairs. QCD is intrinsically a strongly interacting many-body theory. Its strong coupling and confinement nature makes it extremely difficult to solve at low energies relevant to hadronic binding. One theoretical tool that has enjoyed success in the low-energy regime is the QCD sum rule method [1]. It is a nonperturbative approach to QCD that reveals a direct connection between hadronic observables and the QCD vacuum structure via a few universal parameters called vacuum condensates (vacuum expectation values of QCD local operators). This method is based on the evaluation of a suitable correlation function in the deep Euclidean region using operator product expansion (OPE) on the one hand, and its phenomenological evaluation using physical hadronic states on the other hand. The method is analytical and physically transparent and has minimal model dependence with well-understood limitations inherent in the OPE. It provides a complementary view of the same nonperturbative physics to the numerical approach of lattice QCD. The method was applied to the baryon sector not long after it was introduced [2–6], and it was later improved and extended to include some excited states [7–13] and magnetic moments [14–17]. On the experimental side, the effort is fueled by data of increasing quality from the Thomas Jefferson National Accelerator Facility and other accelerator facilities. In all of the studies, however, the conventional Borel transform and pole-plus-continuum ansatz has been used in which the excited states are exponentially suppressed.

To gain access beyond the ground-state pole, one has to seek alternative ansatz in the phenomenological spectral function. Such an attempt has been made in Ref. [18], where the first two excited states of the nucleon are explicitly included, using a combined framework of Gaussian sum rules and finite energy sum rules (FESR). As advocated in Refs. [18,19], FESR is more suitable for studying the spectrum of excited states, because its spectral function has a polynomial kernel instead of an exponential one. Furthermore, the widths of the resonances can be taken into account in addition to the masses, a unique feature of this procedure. In this work, we will extend the work in Ref. [18] to the other members of the baryon octet, using an updated version of the Borel sum rules for the octet baryons as given in Ref. [12]. In effect, it is an exercise similar to the calculation of excitation energies of atoms and nuclei, but it uses fully relativistic quantum field theoretic tools. In our calculation, we consistently include operators up to dimension eight with radiative corrections, first-order strange quark mass corrections, flavor symmetry breaking of the strange quark condensates, anomalous dimension corrections, and possible factorization violation of the four-quark condensate.

II. METHOD

Hadron masses can be extracted from the time-ordered two-point correlation function in the QCD vacuum,

$$\Pi(p) = i \int d^4x\, e^{ip\cdot x} \langle 0\,|T\{\eta(x)\bar{\eta}(0)\}\,|\,0\rangle. \tag{1}$$

For the nucleon, we use the interpolating field

$$\eta^N = -2\,\epsilon^{abc}[(u^{aT}C\gamma_5 d^b)u^c + t\,(u^{aT}Cd^b)\gamma_5 u^c]. \tag{2}$$

The real parameter t allows the freedom for optimal mixing of the two operators. The choice usually used in QCD sum rules studies corresponds to $t = -1$. We will explore different values. For the other members of the octet family, we consider for Σ

$$\eta^\Sigma = -2\,\epsilon^{abc}[(u^{aT}C\gamma_5 s^b)u^c + t\,(u^{aT}Cs^b)\gamma_5 u^c], \tag{3}$$

and for Ξ

$$\eta^\Xi = -2\,\epsilon^{abc}[(s^{aT}C\gamma_5 u^b)s^c + t(s^{aT}Cu^b)\gamma_5 s^c]. \tag{4}$$

For Λ, there is the possibility of octet and flavor-singlet quantum numbers. In this work, we consider only the octet member whose interpolating field is

$$\eta^{\Lambda_o} = 2\sqrt{\frac{1}{6}}\epsilon^{abc}\{[2(u^{aT}C\gamma_5 d^b)s^c + (u^{aT}C\gamma_5 s^b)d^c - (d^{aT}C\gamma_5 s^b)u^c] + t[2(u^{aT}Cd^b)\gamma_5 s^c + (u^{aT}Cs^b)\times\gamma_5 d^c - (d^{aT}Cs^b)\gamma_5 u^c]\}. \quad (5)$$

The most general structure of $\Pi(p)$ is

$$\Pi(p) = \hat{p}F_1(p^2) + F_2(p^2), \quad (6)$$

where $\hat{p} \equiv \gamma \cdot p$. Wilson's operator product expansion up to dimension eight gives the result in the following form for the invariant functions F_1 and F_2:

$$\begin{aligned}-F_1(p^2) &= [A + B\ln(-p^2/\mu^2)][\ln(-p^2/\mu^2)]p^4 + A_4\ln(-p^2/\mu^2)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + E_4\ln(-p^2/\mu^2)m_s\langle\bar{q}q\rangle \\ &+ [A_6 + B_6\ln(-p^2/\mu^2)](1/p^2)\kappa_v\langle\bar{q}q\rangle^2 + E_6(1/p^2)\times m_s\langle\bar{q}g\sigma\cdot Gq\rangle + A_8(1/p^4)\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma\cdot Gq\rangle + \cdots\end{aligned} \quad (7)$$

$$\begin{aligned}-F_2(p^2) &= H_1\ln(-p^2/\mu^2)p^4 m_s + C_3 p^2\ln(-p^2/\mu^2)\langle\bar{q}q\rangle + [C_5 + D_5\ln(-p^2/\mu^2)]\ln(-p^2/\mu^2)\langle\bar{q}g\sigma\cdot Gq\rangle \\ &+ H_5\ln(-p^2/\mu^2)m_s\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + C_7(1/p^2)\langle\bar{q}q\rangle\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + H_7 m_s\kappa_v\langle\bar{q}q\rangle^2 + \cdots\end{aligned} \quad (8)$$

The ellipses denote higher order terms that are ignored. Note that F_1 contains only dimension-even condensates, while F_2 contains only dimension-odd condensates. The coefficients in Eqs. (7) and (8) are given in the Appendix. To derive the finite-energy sum rules (FESR), we employ the Gauss-Weierstrass (GW) transform of F_1 and F_2, as outlined in Refs. [18,19]. We compute the lowest three Hermite moments for F_1 and F_2, yielding six equations from which six sum rules can be constructed. Using the abbreviation $(5+2t+5t^2)/[32(2\pi)^4] = \beta$,

$$\begin{aligned}\frac{1}{\pi}\int_0^{s_0} ds\,\mathrm{Im}F_1(s) &= \beta\left(1+\frac{75}{12}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{-4}{9}}\frac{s_0^3}{3} + A_4 L^{\frac{-4}{9}}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle s_0 \\ &+ E_4 L^{\frac{-4}{9}}m_s\langle\bar{q}q\rangle s_0 + \bar{A}_6\kappa_v\langle\bar{q}q\rangle^2 + E_6 L^{\frac{-26}{27}}m_s \times\langle\bar{q}g\sigma\cdot Gq\rangle,\end{aligned} \quad (9)$$

$$\begin{aligned}\frac{1}{\pi}\int_0^{s_0} ds\,s\,\mathrm{Im}F_1(s) &= \beta\left(1+\frac{25}{3}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{-4}{9}}\frac{s_0^4}{4} + A_4 L^{\frac{-4}{9}}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{s_0^2}{2} \\ &+ E_4 L^{\frac{-4}{9}}m_s\langle\bar{q}q\rangle\frac{s_0^2}{2} + A_8\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle,\end{aligned} \quad (10)$$

$$\begin{aligned}\frac{1}{\pi}\int_0^{s_0} ds\,s^2\,\mathrm{Im}F_1(s) &= \beta\left(1+\frac{367}{60}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{-4}{9}}\frac{s_0^5}{5} + A_4 L^{\frac{-4}{9}}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{s_0^3}{3} \\ &+ B_6\kappa_v\langle\bar{q}q\rangle^2\frac{s_0^2}{2} + E_4 L^{\frac{-4}{9}}m_s\langle\bar{q}q\rangle\frac{s_0^3}{3},\end{aligned} \quad (11)$$

$$\begin{aligned}\frac{1}{\pi}\int_0^{s_0} ds\,\mathrm{Im}F_2(s) &= H_1 m_s L^{\frac{-8}{9}}\frac{s_0^3}{3} + C_3\langle\bar{q}q\rangle\frac{s_0^2}{2} + \bar{C}_5\langle\bar{q}g\sigma\cdot Gq\rangle s_0 \\ &+ H_5 L^{\frac{-8}{9}}m_s\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle s_0 + C_7\langle\bar{q}q\rangle\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \\ &+ H_7 m_s\kappa_v\langle\bar{q}q\rangle^2,\end{aligned} \quad (12)$$

$$\begin{aligned}\frac{1}{\pi}\int_0^{s_0} ds\,s\,\mathrm{Im}F_2(s) &= H_1 m_s L^{\frac{-8}{9}}\frac{s_0^4}{4} + C_3\langle\bar{q}q\rangle\frac{s_0^3}{3} + \bar{C}_5\langle\bar{q}g\sigma\cdot Gq\rangle\frac{s_0^2}{2} \\ &+ H_5 L^{\frac{-8}{9}}m_s\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{s_0^2}{2},\end{aligned} \quad (13)$$

$$\begin{aligned}\frac{1}{\pi}\int_0^{s_0} ds\,s^2\,\mathrm{Im}F_2(s) &= H_1 m_s L^{\frac{-8}{9}}\frac{s_0^5}{5} + C_3\langle\bar{q}q\rangle\frac{s_0^4}{4} + \bar{C}_5\langle\bar{q}g\sigma\cdot Gq\rangle\frac{s_0^3}{3} \\ &+ H_5 L^{\frac{-8}{9}}m_s\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{s_0^3}{3}.\end{aligned} \quad (14)$$

The above six expressions form the left-hand side (LHS) of the sum rules. The two redefined coefficients appearing in the above expressions are given by, for Σ,

$$\bar{A}_6 = \frac{1}{6}\left[(6f_s+1)\left(1-\frac{43}{42}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{248}{189}} - 2t\left(1-\frac{1}{6}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{8}{27}} - (6f_s-1)t^2\left(1-\frac{29}{30}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{184}{135}}\right], \quad (15)$$

$$\bar{C}_5 = \frac{3}{4(2\pi)^2}\left[\left(1+\frac{50}{9}\frac{\bar{\alpha}_s}{\pi}\right) - t^2\left(1+\frac{62}{9}\frac{\bar{\alpha}_s}{\pi}\right)\right]L^{\frac{-14}{27}}. \quad (16)$$

For Λ_o,

$$\bar{A}_6 = \frac{1}{18}\left[(10f_s+11)\left(1-\frac{43}{42}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{248}{189}} + (2-8f_s)t\times\left(1-\frac{1}{6}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{8}{27}} - (2f_s+13)t^2\left(1-\frac{29}{30}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{184}{135}}\right], \quad (17)$$

$$\bar{C}_5 = \frac{1+2f_s}{4(2\pi)^2}\left[\left(1+\frac{179}{36}\frac{\bar{\alpha}_s}{\pi}\right) - t^2\left(1+\frac{227}{36}\frac{\bar{\alpha}_s}{\pi}\right)\right]L^{\frac{-14}{27}}. \quad (18)$$

For Ξ,

$$\bar{A}_6 = \frac{f_s}{6}\left[(f_s+6)\left(1-\frac{43}{42}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{248}{189}} - 2f_s t\left(1-\frac{1}{6}\frac{\bar{\alpha}_s}{\pi}\right)\times L^{\frac{8}{27}} + (f_s-6)t^2\left(1-\frac{29}{30}\frac{\bar{\alpha}_s}{\pi}\right)L^{\frac{184}{135}}\right], \quad (19)$$

$$\bar{C}_5 = \frac{3f_s}{4(2\pi)^2}\left[\left(1+\frac{179}{36}\frac{\bar{\alpha}_s}{\pi}\right) - t^2\left(1+\frac{227}{36}\frac{\bar{\alpha}_s}{\pi}\right)\right]L^{\frac{-14}{27}}. \quad (20)$$

For the phenomenological side of the spectral function, following Ref. [18], we take contributions from the lowest three states: the ground state with positive parity, the first excited state with positive parity, and the second excited state with negative parity. We treat the ground state as a pole described by two parameters: the coupling strength λ^2 and mass m. The two excited states are treated as resonances with finite widths, each characterized by three parameters $(\lambda_1^2, m_1, \Gamma_1)$ and $(\lambda_2^2, m_2, \Gamma_2)$. The spectral functions read

$$\frac{1}{\pi}\,\mathrm{Im}F_1(s) = \lambda^2\delta(s-m^2) + \frac{1}{\pi}\left[\frac{\lambda_1^2 m_1\Gamma_1}{\left(s-m_1^2\right)^2+m_1^2\Gamma_1^2} + \frac{\lambda_2^2 m_2\Gamma_2}{\left(s-m_2^2\right)^2+m_2^2\Gamma_2^2}\right]\theta(s-m^2), \quad (21)$$

$$\frac{1}{\pi}\,\mathrm{Im}F_2(s) = \lambda^2 M\delta(s-m^2) + \frac{1}{\pi}\left[\frac{\lambda_1^2\left(s+m_1^2\right)\Gamma_1/2}{\left(s-m_1^2\right)^2+m_1^2\Gamma_1^2} - \frac{\lambda_2^2\left(s+m_2^2\right)\Gamma_2/2}{\left(s-m_2^2\right)^2+m_2^2\Gamma_2^2}\right]\theta(s-m^2). \quad (22)$$

In the limit of zero widths ($\Gamma_i \to 0$), the excited-state contributions also reduce to δ functions. The same feature as in the Borel sum rules that the excited states with opposite parities add in the chiral-even F_1 and cancel in the chiral-odd F_2 is also preserved. The λ's are the coupling strengths of the interpolating field to the states, defined by

$$\langle 0|\eta|Bps\rangle = \lambda_B u(p,s), \quad (23)$$

where $u(p,s)$ is the spin-1/2 spinor. Applying the same GW transform, taking the first three Hermite moments of the spectral functions, and introducing the cutoff s_0, one obtains the following six phenomenological expressions [18] that match one-to-one to those on the QCD side in Eqs. (9)–(14):

$$\frac{1}{\pi}\int_0^{s_0} ds\,\mathrm{Im}F_1(s) = \lambda^2 + \frac{\lambda_1^2}{\pi}f_1 + \frac{\lambda_2^2}{\pi}f_2, \quad (24)$$

$$\frac{1}{\pi}\int_0^{s_0} ds\,s\,\mathrm{Im}F_1(s) = \lambda^2 m^2 + \frac{\lambda_1^2}{\pi}\left[\frac{m_1\Gamma_1}{2}r_1 + m_1^2 f_1\right] + \frac{\lambda_2^2}{\pi}\left[\frac{m_2\Gamma_2}{2}r_2 + m_2^2 f_2\right], \quad (25)$$

$$\frac{1}{\pi}\int_0^{s_0} ds\,s^2\,\mathrm{Im}F_1(s) = \lambda^2 m^4 + \frac{\lambda_1^2}{\pi}m_1\Gamma_1 \times\left[(s_0-m^2) + \frac{m_1^2\left(m_1^2-\Gamma_1^2\right)}{m_1\Gamma_1}f_1 + m_1^2 r_1\right] + \frac{\lambda_2^2}{\pi}m_2 \times\Gamma_2\left[(s_0-m^2) + \frac{m_2^2\left(m_2^2-\Gamma_2^2\right)}{m_2\Gamma_2}f_2 + m_2^2 r_2\right], \quad (26)$$

$$\frac{1}{\pi}\int_0^{s_0} ds\,\mathrm{Im}F_2(s) = \lambda^2 m + \frac{\lambda_1^2}{\pi}\left[\frac{\Gamma_1}{4}r_1 + m_1 f_1\right] - \frac{\lambda_2^2}{\pi}\left[\frac{\Gamma_2}{4}r_2 + m_2 f_2\right], \quad (27)$$

$$\frac{1}{\pi}\int_0^{s_0} ds\,s\,\mathrm{Im}F_2(s) = \lambda^2 m^3 + \frac{\lambda_1^2}{\pi}\left[(s_0-m^2)\frac{\Gamma_1}{2} + m_1\left(m_1^2-\Gamma_1^2/2\right)f_1 + \frac{3\Gamma_1 m_1^2}{4}r_1\right] - \frac{\lambda_2^2}{\pi}\left[(s_0-m^2)\frac{\Gamma_2}{2} + m_2\left(m_2^2-\Gamma_2^2/2\right)f_2 + \frac{3\Gamma_2 m_2^2}{4}r_2\right], \quad (28)$$

$$\frac{1}{\pi}\int_0^{s_0} ds\,s^2\,\mathrm{Im}F_2(s) = \lambda^2 m^5 + \frac{\lambda_1^2}{\pi}\frac{\Gamma_1}{2}\left[\frac{s_0^2-m^4}{2} + 3m_1^2(s_0-m^2) + \frac{2m_1^6-4m_1^4\Gamma_1^2}{m_1\Gamma_1}f_1 + \frac{5m_1^4-m_1^2\Gamma_1^2}{2}r_1\right] - \frac{\lambda_2^2}{\pi}\frac{\Gamma_2}{2} \times\left[\frac{s_0^2-m^4}{2} + 3m_2^2(s_0-m^2) + \frac{2m_2^6-4m_2^4\Gamma_2^2}{m_2\Gamma_2}f_2 + \frac{5m_2^4-m_2^2\Gamma_2^2}{2}r_2\right], \quad (29)$$

where

$$f_i \equiv \tan^{-1}\left(\frac{s_0-m_i^2}{m_i\Gamma_i}\right) + \tan^{-1}\left(\frac{m_i^2-m^2}{m_i\Gamma_i}\right), \quad (i=1,2), \quad (30)$$

$$r_i \equiv \ln\frac{\left(s_0-m_i^2\right)^2+m_i^2\Gamma_i^2}{\left(m_i^2-m^2\right)^2+m_i^2\Gamma_i^2}, \quad (i=1,2). \quad (31)$$

The six expressions in Eqs. (24)–(29) form the right-hand side (RHS) of the sum rules. Note that the couplings appear linearly, while the masses and widths highly nonlinearly.

A theoretical issue in the construction of the phenomenological side of the spectral function is the potential contribution of the pion nucleon continuum to the time-ordered two-point correlation. A plausible parametrization of this contribution in the case of the nucleon may be given as in Ref. [20], that is,

$$\Pi_N^{\pi N_{\rm cont}}(p) = -\epsilon\gamma_5\Pi_{\rm pole}(p)\gamma_5, \quad (32)$$

where $\Pi_{\rm pole}(p)$ is the nucleon and antinucleon pole contributions to the correlator. The quantity ϵ, which has small values on the order of a few percent, is given as

$$\epsilon = \frac{3}{64\pi^2 f_\pi^2} m_\pi^2 \ln \frac{m_\pi^2}{M_0^2}, \tag{33}$$

where M_0 is an arbitrary constant. (Changes in M_0 can always be absorbed into changes in an analytic term proportional to m_π^2 as given in Ref. [20].) Thus the inclusion of the pion-nucleon continuum simply results in a slight modification in the couplings $\lambda^2 \to \lambda^2(1+\epsilon)$ in Eqs. (24)–(26) and $\lambda^2 \to \lambda^2(1-\epsilon)$ in Eqs. (27)–(29). That is, it introduces a few percent error in the couplings λ^2's which can be easily absorbed into the uncertainties for the parameters at the current level of accuracy in the solutions. A similar conclusion can be drawn for other strange baryons.

III. RESULTS AND DISCUSSIONS

By equating the two sides of the moments of the spectral functions, we get six sum rules. Mathematically, the problem boils down to finding the solutions to a system of six simultaneous, nonlinear equations of the form

$$\text{LHS}_i(s_0, \text{QCD}) = \text{RHS}_i\left(s0, \lambda^2, \lambda_1^2, \lambda_2^2, m, m_1, m_2, \Gamma_1, \Gamma_2\right), \tag{34}$$

with $i = 1, 6$. They can be used to solve for six unknowns, which we take as the two masses (m_1, m_2), two widths (Γ_1, Γ_2), and two couplings $(\lambda_1^2, \lambda_2^2)$ for the two excited states. The remaining parameters are taken as input: the ground-state pole (m, λ^2) as well as the continuum cutoff s_0. The ground-state pole has been well studied under the Borel sum rules using generalized interpolating fields [12]. For the cutoff s_0, we take values consistent with the observed spectrum [21] and vary it to see any sensitivity to this parameter. For the QCD parameters in the left-hand side, we use the following notations: $a = -(2\pi)^2 \langle \bar{u}u \rangle$, $m_0^2 = \langle \bar{q} g\sigma \cdot Gq \rangle / \langle \bar{q}q \rangle$, $b = \langle g_c^2 G^2 \rangle$, and $f_s = \langle \bar{s}s \rangle / \langle \bar{u}u \rangle$. The four-quark condensate is parametrized using the factorization approximation: $\langle \bar{u}u\bar{u}u \rangle = \kappa_v \langle \bar{u}u \rangle^2$, where the parameter κ_v accounts for its violation. The anomalous dimension corrections of the various operators are taken into account via the factors $L^\gamma = [\alpha_s(\mu^2)/\alpha_s(s_0)]^\gamma = [\ln(s_0/\Lambda_{\text{QCD}}^2)/\ln(\mu^2/\Lambda_{\text{QCD}}^2)]^\gamma$, where γ is the appropriate anomalous dimension, $\mu = 500$ MeV is the renormalization scale, and $\Lambda_{\text{QCD}} = 0.15$ GeV is the QCD scale parameter. The function $r(s_0) = \ln(s_0/\mu^2) - \gamma_{\text{EM}}$ with $\gamma_{\text{EM}} = 0.577$, the Euler-Mascheroni constant. The numerical values we use are $a = 0.52$ GeV^3, $b = 1.2$ GeV^4, $m_0^2 = 0.8$ GeV^2, $\kappa_v = 2$, $\bar{\alpha}_s/\pi = 0.1$, $m_s = 0.15$ GeV, and $f_s = 0.8$. In addition, we rescale the couplings to their more natural values by $c_1 = (2\pi)^4 \lambda_1^2$ and so on. There is also the issue of optimal mixing parameter t in the interpolating field. We take $t = -0.8$ to get enhanced contributions of excited states, since $t = -1$ has been found to not couple strongly to the negative-parity state in the nucleon channel [10]. Since the couplings in Ref. [12] were obtained for $t = -1$, we take the lowest values of the coupling in the given range. The six equations are solved simultaneously by the multidimensional secant Broyden's method from *Numerical Recipes* [22]. We also use the globally convergent Newton-Raphson method for checking purposes. As a consistency check, we also solved the system of four equations by eliminating explicitly λ_1^2 and λ_2^2 using the sum rules corresponding to Eqs. (24) and (27). All these methods give consistent results, which are summarized in Table I.

TABLE I. Six calculated parameters $(\lambda_1^2, m_1, \Gamma_1)$ and $(\lambda_2^2, m_2, \Gamma_2)$ for the first two excited states in the baryon octet. The ground-state pole (λ^2, m) is taken as input to the calculation. The c's are rescaled couplings: $c = (2\pi)^4\lambda^2$, $c_1 = (2\pi)^4\lambda_1^2$, and $c_2 = (2\pi)^4\lambda_2^2$. The cutoff threshold s_0 is varied in each case to show sensitivity of the results to this parameter. The last two columns display the percentage difference to which the equations are satisfied and the contribution of the second excited state in the solutions. For comparison purposes, experimental values taken from the PDG [21] are also displayed.

Baryon	s_0 (GeV)	c (GeV^6)	m (GeV)	m_1 (GeV)	m_2 (GeV)	Γ_1 (GeV)	Γ_2 (GeV)	c_1 (GeV^6)	c_2 (GeV^6)	Difference (%)	2nd (%)
N	2.4	1.20	0.94	1.47–1.50	1.46–1.60	0.01–0.17	0.01–0.35	0.8–1.2	0.03–0.09	<5.0	>2.0
	2.5	1.20	0.94	1.47–1.52	1.44–1.60	0.01–0.21	0.01–0.35	0.9–1.4	0.05–0.09	<4.0	>2.0
	2.6	1.20	0.94	1.49–1.55	1.44–1.60	0.01–0.21	0.01–0.35	1.0–1.6	0.03–0.07	< 2.5	>1.0
Expt.			0.94	1.44	1.54	0.35	0.15				
Σ	3.1	2.50	1.19	1.73	1.64–1.76	0.01	0.07–0.35	0.75–0.8	0.1–0.2	< 6.75	>2.0
	3.2	2.50	1.19	1.74–1.76	1.68–1.78	0.01–0.04	0.09–0.35	0.95–1.0	0.2–0.3	< 6.0	>3.0
	3.3	2.50	1.19	1.75–1.81	1.72–1.82	0.01–0.07	0.01–0.35	1.1–1.5	0.1–0.4	< 5.0	>3.0
Expt.			1.19	1.66	1.75	0.10	0.09				
Λ_o	3.3	2.66	1.12	1.81	1.72–1.83	0.01–0.02	0.01–0.17	1.3–1.5	0.2–0.6	< 5.0	>3.0
	3.4	2.66	1.12	1.80–1.84	1.66–1.84	0.01–0.045	0.01–0.35	1.2–1.8	0.2–0.7	< 5.0	>3.0
	3.5	2.66	1.12	1.81–1.86	1.66–1.88	0.01–0.05	0.01–0.35	1.4–2.0	0.3–1.0	< 4.5	>3.0
Expt.			1.12	1.60	1.67	0.15	0.035				
Ξ	3.9	3.56	1.32	1.97	1.56–1.72	0.01	0.01–0.35	1.8	0.6–1.0	< 3.5	>2.0
	4.0	3.56	1.32	1.98–1.99	1.68–1.96	0.01–0.03	0.01–0.35	1.7–2.3	0.6–1.2	< 3.0	>2.0
	4.15	3.56	1.32	1.99–2.01	1.7–2.02	0.01–0.04	0.01–0.35	2.0–2.4	0.7–1.7	< 2.5	>1.0
Expt.			1.32	1.95		0.06					

TABLE II. Similar to Table I, but with the constraint Γ_1 relatively large and $m_2 \geqslant m_1$ if possible for the first two excited states in the baryon octet.

Baryon	s_0 (GeV)	c (GeV^6)	m (GeV)	m_1 (GeV)	m_2 (GeV)	Γ_1 (GeV)	Γ_2 (GeV)	c_1 (GeV^6)	c_2 (GeV^6)	Accuracy (%)	2nd (%)
N	2.5	1.20	0.94	1.44–1.48	1.49–1.57	0.23	0.15–0.17	1.4	0.04–0.08	<4.75	>2.0
	2.5	1.30	0.94	1.50–1.53	1.53–1.59	0.25–0.27	0.15–0.17	1.5–1.6	0.04–0.08	<4.75	>2.0
Expt.			0.94	1.44	1.54	0.35	0.15				
Σ	3.2	2.50	1.19	1.78	1.78–1.79	0.1	0.09	1.6	0.1	<5.5	>3.0
Expt.			1.19	1.66	1.75	0.10	0.09				
Λ_o	3.5	2.66	1.12	1.83–1.84	1.84–1.87	0.11	0.035	2.1–2.4	0.3–0.6	<5.5	>3.0
	3.5	2.66	1.12	1.84	1.84–1.87	0.12	0.035	2.4	0.3–0.5	<5.5	>3.0
Expt.			1.12	1.60	1.67	0.15	0.035				
Ξ	4.1	3.56	1.32	1.99–2.01	1.69–1.96	0.04–0.06	0.02–0.05	2.2–2.8	0.7–0.9	<3.0	>2.0
Expt.			1.32	1.95		0.06					

As a first step, we revisited the nucleon channel using $t = -0.8$ rather than $t = -1$ as done in Ref. [18]. We also took $c = 1.2$ GeV^6, whereas in Ref. [18] $c \approx 0.8$ GeV^6 was used. Still, our results come close to those obtained in Ref. [18]. We solved the equations by satisfying them within a global accuracy of a few percent. We also monitored the average contribution of the second resonance as a percentage of the respective right-hand side to make sure it was capable of making a significant contribution. In this way, each parameter was allowed to vary in a certain range reflecting the stability of these parameters. We found it difficult to achieve better accuracies than the ones listed. Overall, the results for the first excited state (m_1, Γ_1) are better than those for the second excited state (m_2, Γ_2). The m_1 is fairly stable. The Γ_1 is better constrained in the strange channels than in the nucleon channel. The c_1 values, which are a measure of the ability of the interpolating fields to excite the state from the QCD vacuum, are first-time information from the perspective of QCD. For the negative-parity states, whose contribution appears with the same signs as for positive-parity states in F_1 and opposite signs in F_2 sum rules, the scatter in the numerical values obtained is larger, as noted earlier as well [18]. We restricted the values of Γ_2 from the upper side in each case to about 0.35 GeV, while the values of other parameters are as obtained without any restriction. The results are fairly stable against variations in the cutoff parameter s_0. The s_0 in each channel in Table I for the given ground-state parameters (m, λ^2) is the lowest possible value of s_0 in that channel for which one starts getting solutions of the six simultaneous equations at the considered accuracy. One can see that the lowest s_0 in each channel corresponds to a mass value $(\sqrt{s_0})$ sightly above the value of m_2 obtained in that channel, which is quite reasonable, and gives the lowest scatter in the values of physical parameters. In the Σ and Λ channels, m_1 is overestimated as compared to the experimental values. In the Ξ channel, two spin-1/2 excited states are listed in PDG at 1690 and 1950 MeV with three stars and unassigned spin-parity. Our result favors the 1950 MeV state as the first excited state with positive parity.

One issue with the results in Table I is that the calculated width of the first excited state (Γ_1) is too small compared to those given in the PDG. So we repeated the calculation with the constraint Γ_1 large and also sought $m_2 > m_1$ if possible (this can be done within a little larger error). The results are given in Table II. This is a better set of parameters than those in Table I, with smaller scatter in the parameters. We could obtain consistent solutions with $m_2 > m_1$ for the N, Σ, and Λ_o channels. For the Ξ channel, however, m_2 comes out to be smaller than m_1, which suggests that the state at 1950 MeV has positive parity, and the state at 1690 MeV negative parity.

IV. CONCLUSIONS

We have extended the study of nucleon excited states in Ref. [18] to the strange members of the baryon octet, using the combination of Gauss-Weierstrass (GW) transform and finite energy sum rules (FESR). Taking the ground-state pole as input, the masses and widths of the first two excited states of opposite parity can be computed in this approach, a possibility not afforded in the conventional Borel-based QCD sum rules. Furthermore, by changing the overall sign of the phenomenological side, the same approach can be used to study a particle channel that has the reverse parity ordering: ground state (negative), first excited (negative), second excited (positive). A case in point in the physical spectrum is the flavor-singlet, spin-1/2 sector where the ground state $\Lambda_S(1405)1/2-$ has negative parity. A calculation is underway to understand this channel in the present approach. We presented two sets of solutions with different constraints. Overall, our results show that the first excited state is well probed by this method. The mass is stable and consistent with experiment, and the width has a relatively small uncertainty range. On the other hand, the second excited state is less well constrained; both the mass and width have larger scatter than those for the first excited state. On the other hand, the second excited state can be well described by a different method: the Borel-based parity-projected QCD sum rules [10,13]. Taken together, the QCD sum rule method in its three variants (Borel-based conventional, Borel-based parity-projected, and GW-based

FESR) gives access to the lowest three states in a given particle channel from a nonpertubative perspective of QCD.

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APPENDIX: WILSON COEFFICIENTS

Here we list the coefficients appearing in Eqs. (7) and (8). Those which are common for all the members in the octet family are (ignoring flavor breaking in radiative corrections) [12,18]

$$A = \frac{1}{32(2\pi)^4}(5+2t+5t^2)\left(1+\frac{71}{12}\frac{\alpha_s}{\pi}\right),\quad B = \frac{-1}{64(2\pi)^4}(5+2t+5t^2)\left(\frac{\alpha_s}{\pi}\right), \tag{A1}$$

$$A_4 = \frac{1}{64(2\pi)^2}(5+2t+5t^2),\quad B_6 = \frac{1}{18}(62-4t-46t^2)\left(\frac{\alpha_s}{\pi}\right). \tag{A2}$$

The other coefficients are member dependent. For the Σ,

$$H_1 = \frac{1}{4(2\pi)^4}(1-t)^2,$$

$$C_3 = \frac{-1}{4(2\pi)^2}\left[(6+f_s)\left(1+\frac{15}{14}\frac{\alpha_s}{\pi}\right) - 2f_s t\left(1+\frac{3}{2}\frac{\alpha_s}{\pi}\right) - (6-f_s)t^2\left(1+\frac{7}{10}\frac{\alpha_s}{\pi}\right)\right], \tag{A3}$$

$$E_4 = \frac{-1}{8(2\pi)^2}[(12-5f_s) - 2f_s t - (12+5f_s)t^2],\quad C_5 = \frac{3}{4(2\pi)^2}\left[\left(1+\frac{79}{18}\frac{\alpha_s}{\pi}\right) - t^2\left(1+\frac{103}{18}\frac{\alpha_s}{\pi}\right)\right], \tag{A4}$$

$$D_5 = \frac{3}{4(2\pi)^2}\left(-\frac{7}{12}\right)(1-t^2)\left(\frac{\alpha_s}{\pi}\right),\quad H_5 = \frac{-1}{32(2\pi)^2}(1-t)^2, \tag{A5}$$

$$A_6 = \frac{1}{6}\left[(6f_s+1)\left(1-\frac{43}{42}\frac{\alpha_s}{\pi}\right) - 2t\left(1-\frac{1}{6}\frac{\alpha_s}{\pi}\right) - (6f_s-1)t^2\left(1-\frac{29}{30}\frac{\alpha_s}{\pi}\right)\right], \tag{A6}$$

$$E_6 = \frac{-1}{24(2\pi)^2}[(4f_s+21+18r(s_0)) + 4f_s t + (4f_s-21-18r(s_0))t^2], \tag{A7}$$

$$C_7 = \frac{-1}{288}[(24-5f_s) + 10f_s t - (24+5f_s)t^2],\quad H_7 = \frac{1}{6}[(5-3f_s) + 2t + (5+3f_s)t^2], \tag{A8}$$

$$A_8 = \frac{-1}{24}[(1+12f_s) - 2t - (12f_s-1)t^2]. \tag{A9}$$

For the octet Λ_O,

$$H_1 = \frac{11+2t-13t^2}{12(2\pi)^4},$$

$$C_3 = \frac{-1}{12(2\pi)^2}\left[(10+11f_s)\left(1+\frac{15}{14}\frac{\alpha_s}{\pi}\right) + (-8+2f_s)t \times\left(1+\frac{3}{2}\frac{\alpha_s}{\pi}\right) - (2+13f_s)t^2\left(1+\frac{7}{10}\frac{\alpha_s}{\pi}\right)\right], \tag{A10}$$

$$E_4 = \frac{-1}{24(2\pi)^2}[(20-15f_s) - (16+6f_s)t - (4+15f_s)t^2], \tag{A11}$$

$$C_5 = \frac{1+2f_s}{4(2\pi)^2}\left[\left(1+\frac{79}{18}\frac{\alpha_s}{\pi}\right) - t^2\left(1+\frac{103}{18}\frac{\alpha_s}{\pi}\right)\right],\quad D_5 = \frac{1+2f_s}{4(2\pi)^2}\left(-\frac{7}{12}\right)(1-t^2)\left(\frac{\alpha_s}{\pi}\right),\quad H_5 = \frac{1}{96(2\pi)^2}(13-2t-11t^2), \tag{A12}$$

$$A_6 = \frac{1}{18}\left[(10f_s+11)\left(1-\frac{43}{42}\frac{\alpha_s}{\pi}\right) - (2-8f_s)t \times\left(1-\frac{1}{6}\frac{\alpha_s}{\pi}\right) - (2f_s+13)t^2\left(1-\frac{29}{30}\frac{\alpha_s}{\pi}\right)\right], \tag{A13}$$

$$E_6 = \frac{1}{24(2\pi)^2}[(4f_s-5-6r(s_0)) + (4+4f_s)t + (4f_s+1+6r(s_0))t^2], \tag{A14}$$

$$C_7 = \frac{-1}{864}[(4+53f_s) + (40-10f_s)t - (44+33f_s)t^2], \tag{A15}$$

$$H_7 = \frac{1}{18}[(15-5f_s) + (6+4f_s)t + (15+f_s)t^2],\quad A_8 = \frac{-1}{72}[(23+16f_s) + (2-8f_s)t - (25+8f_s)t^2]. \tag{A16}$$

For the Ξ,

$$H_1 = \frac{3}{2(2\pi)^4}(1-t^2), \tag{A17}$$

$$C_3 = \frac{-1}{4(2\pi)^2}\left[(6f_s+1)\left(1+\frac{15}{14}\frac{\alpha_s}{\pi}\right) - 2t\left(1+\frac{3}{2}\frac{\alpha_s}{\pi}\right) - (6f_s-1)t^2\left(1+\frac{7}{10}\frac{\alpha_s}{\pi}\right)\right],\quad E_4 = \frac{-3}{4(2\pi)^2}\left[(2-f_s) - 2f_s t - (2+f_s)t^2\right], \tag{A18}$$

$$C_5 = \frac{3f_s}{4(2\pi)^2}\left[\left(1+\frac{79}{18}\frac{\alpha_s}{\pi}\right) - t^2\left(1+\frac{103}{18}\frac{\alpha_s}{\pi}\right)\right],\quad D_5 = \frac{3f_s}{4(2\pi)^2}\left(-\frac{7}{12}\right)(1-t^2)\left(\frac{\alpha_s}{\pi}\right),\quad H_5 = \frac{3}{16(2\pi)^2}(1-t^2), \tag{A19}$$

$$A_6 = \frac{f_s}{6}\left[(f_s+6)\left(1-\frac{43}{42}\frac{\alpha_s}{\pi}\right) - 2f_st\left(1-\frac{1}{6}\frac{\alpha_s}{\pi}\right) + (f_s-6)t^2\left(1-\frac{29}{30}\frac{\alpha_s}{\pi}\right)\right], \quad \text{(A20)}$$

$$E_6 = \frac{-1}{24(2\pi)^2}[(15 - f_s + 18r(s_0)) - 10f_st - (15 + f_s + 18r(s_0))t^2], \quad \text{(A21)}$$

$$C_7 = \frac{-1}{288}[(24f_s - 5) + 10t - (24f_s + 5)t^2],$$
$$H_7 = \frac{f_s}{2}[(3 - f_s) + 2t + (3 + f_s)t^2], \quad \text{(A22)}$$

$$A_8 = \frac{-f_s}{24}[(f_s + 12) - 2f_st - (f_s - 12)t^2]. \quad \text{(A23)}$$

[1] M. A. Shifman, A. I. Vainshtein, and Z. I. Zakharov, Nucl. Phys. **B147**, 385, 448 (1979).
[2] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591(E) (1981).
[3] D. Espriu, P. Pascual, and R. Tarrach, Nucl. Phys. **B214**, 285 (1983).
[4] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Lett. **B120**, 209 (1983); **B122**, 487(E) (1983).
[5] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[6] Y. Chung, H. G. Dosch, M. Kremer, and D. Schall, Z. Phys. C **57**, 151 (1984).
[7] H. G. Dosch, M. Jamin, and S. Narison, Phys. Lett. **B220**, 251 (1989).
[8] D. B. Leinweber, Ann. Phys. (NY) **198**, 203 (1990); **254**, 328 (1997).
[9] D. B. Leinweber, Ann. Phys. (NY) **254**, 328 (1997).
[10] D. Jido, N. Kodama, and M. Oka, Phys. Rev. D **54**, 4532 (1996); hep-ph/9611322.
[11] F. X. Lee, Phys. Rev. C **57**, 322 (1998).
[12] F. X. Lee and X. Liu, Phys. Rev. D **66**, 014014 (2002).
[13] F. X. Lee, Nucl. Phys. **A791**, 352 (2007).
[14] J. Pasupathy, J. P. Singh, S. L. Wilson, and C. B. Chiu, Phys. Rev. D **36**, 1442 (1987).
[15] C. B. Chiu, S. L. Wilson, J. Pasupathy, and J. P. Singh, Phys. Rev. D **36**, 1553 (1987).
[16] F. X. Lee, Phys. Rev. D **57**, 1801 (1998).
[17] F. X. Lee, Phys. Lett. **B419**, 14 (1998).
[18] J. P. Singh, Phys. Rev. D **50**, 468 (1994).
[19] R. A. Bertlmann, G. Launer, and E. de Rafael, Nucl. Phys. **B250**, 61 (1985).
[20] S. H. Lee *et al.*, Phys. Lett. **B348**, 263 (1995).
[21] S. Eidelman *et al.*, Phys. Lett. **B592**, 1 (2004).
[22] *Numerical Recipes in Fortran*, 2nd ed. (Cambridge University, Cambridge, England, 1992).