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Fission rate and transient time with a quantum master equation

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The induced nuclear fission is considered a transport process over the fission barrier underlying dissipative forces. Using a quantum master equation for the reduced density matrix, the influence of microscopical diffusion coefficients on the total fission time and transient time is studied. The influence of transient effects on the probability of the first-chance fission is estimated. For different temperatures and friction coefficients, the quasistationary fission rate is compared with the analytical Kramers formula.

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The time scale of an induced fission process at moderate to high excitation energies is of interest at present from the theoretical and experimental points of view [1–3]. The experimental data indicate that the number of neutrons, γ rays, and light particles evaporated prior to fission considerably exceeds the expectations of the statistical model [4]. The possible explanation for this observation is based on the facts that the time-dependent diffusive flux over the fission barrier (saddle point) strongly depends on the nuclear viscosity and there exists a time delay (a transient time) between the beginning of the diffusion fission process and the attainment of the stationarity of probability flow [5–10]. Indeed, most of observations listed under Ref. [4] are sensitive to the whole time up to scission: the transient time plus the saddle-to-scission time. However, only the transient time affects on the survival probability of the excited nucleus [1,3]. If the transient time is of the order or longer of a nucleon evaporation time, one can expect that the nucleon emission competes favorably with fission. So, the knowledge of the total fission and transient times that depend on the nuclear viscosity is crucial for the interpretation of the experimental data.

The dissipation of energy from the collective degrees of freedom into the internal excitation of the system is a crucial but controversial problem in nuclear theory [11]. With the calculated static potential energy and experimental postscission data [12], it has been found that only rather weak dissipation is compatible with existing experimental data for the thermal-neutron induced fission. This semiempirical method avoids any assumptions about the dynamics of the nuclear motion or the dissipation mechanism in fission. In spite of intensive theoretical and experimental efforts, the conclusions on the temperature and shape dependence of nuclear friction are not convinced [1]. The question how fast a highly excited nuclear system changes its shape is important for the understanding of various nuclear reactions. The comparison of theoretical results of Ref. [13] and experimental data shows that the prescission neutron multiplicities and other observables of the fission of highly excited nuclei can be reproduced using the modified one-body mechanism with the reduction coefficients 0.25—0.5.

The experimental data on nuclear dissipation have been often interpreted by using the stochastic classical Langevin equation or classical diffusion Fokker-Planck equation, where mainly a few collective coordinates, elongation, neck, and mass asymmetry parameters, are used for describing the possible shapes of the fissioning nucleus from the ground-state configuration up to the scission configuration of two touching fragments [1,3,13–22]. Although many nuclear properties arise from quantum effects, the theory of fission is still expressed mainly in terms of classical dynamics for the one-body or two-body dissipation mechanisms. The influence of relaxation effects on the time dependence of the fission decay width has been carefully studied in Refs. [1,3,5–10,13] using the numerical solution of the classical Fokker-Planck and Langevin equations.

Our objective is to study the fission dynamics within the microscopical model to reveal the dissipative nuclear properties. The question raises to what extent the microscopic effects may play an important role in the fission process and how these effects change the characteristic fission times. The evolution of quantum system in the relevant collective fission coordinate q, which is coupled with internal degrees of freedom, can be described within a density matrix formalism. The linear coupling in q between the collective and internal subsystems is usually considered [23]. Then, the reduced density matrix ρ for the collective subsystem obeys the following equation [24–26]:

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[\tilde{H}_c, \rho] - \frac{i\lambda_p}{2\hbar}[q, \{p, \rho\}_+] - \frac{D_{pp}}{\hbar^2}[q, [q, \rho]] + \frac{D_{qp}}{\hbar^2}([p, [q, \rho]] + [q, [p, \rho]]), \tag{1}$$

where

$$\tilde{H}_c = \frac{1}{2\mu}p^2 + \tilde{U}(q), \tag{2}$$

$$D_{pp} = \frac{T\mu\gamma^2\lambda_p}{\gamma(\gamma+\lambda_p)+\omega_m{}^2} \times \left[1 + 2\sum_{k=1}^{\infty}\frac{\nu_k\gamma\lambda_p + \omega_m{}^2(\gamma+\nu_k)}{(\gamma+\nu_k)(\nu_k(\nu_k+\lambda_p)+\omega_m{}^2)}\right], \tag{3}$$

$$D_{qp} = \frac{T\gamma\lambda_p}{2[\gamma(\gamma+\lambda_p)+\omega_m{}^2]} \times \left[1 + 2\gamma\sum_{k=1}^{\infty}\frac{\omega_m{}^2 - \nu_k\gamma}{(\gamma+\nu_k)(\nu_k(\nu_k+\lambda_p)+\omega_m{}^2)}\right] \quad (4)$$

are the renormalized collective Hamiltonian, diffusion coefficient in momentum, and mixed diffusion coefficient, respectively [27]. Here, λ_p is the reduced dissipation (or friction) coefficient and $\nu_k = 2\pi T \cdot k/\hbar$. The expressions (13) and (14) for the diffusion coefficients contain parameter γ, which characterizes the width of the internal excitations and satisfies the condition $\gamma \gg \omega_m$, i.e., the relaxation time of the internal subsystem is much shorter than the characteristic time of the collective motion [26]. We set $\hbar\gamma = 12$ MeV in our calculations. In general case, the friction and diffusion coefficients depend also on q at given coupling strength of the collective subsystem with the environment. However, we found that at moderate to high excitation energies of the heat bath this dependence is rather weak to be disregarded [27]. Because the friction and diffusion coefficients are derived with a linear coupling in the coordinate between the collective subsystem and the environment, we have zero diffusion $D_{qq} = 0$ and friction $\lambda_q = 0$ in the coordinate [25,26].

The master equation for ρ in the coordinate representation $[\rho(t,x,y) = \langle x|\rho|y\rangle]$ is written as

$$\begin{aligned}\frac{d}{dt}\rho(t,x,y) &= L(x,y)\rho(t,x,y),\\ L(x,y) &= -i\left[\frac{\hbar}{2\mu}(\partial_{x,x}-\partial_{y,y}) + \tilde{U}(x) - \tilde{U}(y)\right]\\ &\quad - \frac{1}{2}\lambda_p(x-y)(\partial_x-\partial_y) - \frac{D_{pp}}{\hbar^2}(x-y)^2\\ &\quad - \frac{iD_{qp}}{\hbar}[(\partial_x+\partial_y)(x-y)+(x-y)(\partial_x+\partial_y)].\end{aligned} \quad (5)$$

Here, we use the following notations: $\partial_k = \partial/\partial k$, $\partial_{k,k} = \partial^2/\partial^2 k$. Making the following coordinate transformations $x = q + z/2$ and $y = q - z/2$, and expanding the potential in z, we obtain the equation for $\rho(t,q,z)$:

$$\begin{aligned}\frac{d}{dt}\rho(t,q,z) &= L(q,z)\rho(t,q,z),\\ L(q,z) &= i\frac{\hbar}{\mu}\partial_{q,z} - iz\tilde{U}'(q) - i\frac{1}{24}z^3\tilde{U}'''(q)\\ &\quad - \lambda_p z\partial_z - \frac{D_{pp}}{\hbar^2}z^2 - \frac{iD_{qp}}{\hbar}[z\partial_q + \partial_q z].\end{aligned} \quad (6)$$

Here, $\tilde{U}'(q) = \partial\tilde{U}/\partial q$ and $\tilde{U}'''(q) = \partial^3\tilde{U}/\partial q^3$. Equation (6) is solved by using an oscillator basis:

$$\begin{aligned}\rho(t,q,z) &= \sum_{k=0}^{n} f_k(t,q)B_k(\sigma,z),\\ B_k(\sigma,z) &= \frac{i^k}{k!}\left(\frac{k}{2}\right)! e^{-\frac{z^2}{8\sigma^2}} H_k\left(\frac{z}{2\sigma}\right).\end{aligned} \quad (7)$$

Here, $B_k(\sigma,0) = 1$ and 0 for even and odd k, respectively. We found that the diffusion coefficient D_{pp} in the minimum of the potential, from which the escape is treated, and the optimal basis parameter σ are related: $4\sigma^2 D_{pp} = \hbar^2\lambda_p$. The proposed method allows us to obtain ρ for any continuous potential and any set of friction and diffusion coefficients. Because the used friction and diffusion coefficients are self-consistently derived through the fully coupled oscillator model [26,27], they preserve the positivity of the density matrix at any time and

$$\mathrm{Tr}\hat{\rho} = \sum_{k=0,2,4,\ldots}\int_{-\infty}^{\infty} f_k(t,q)dq = 1. \quad (8)$$

Solving the master equation (6) with the sets of diffusion coefficients mentioned above and at given λ_p, we obtain the time-dependent density matrix $\rho(t,q,0) = \langle q|\rho(t)|q\rangle$ in coordinate representation and find the probability $P(t)$ of escape of the Gaussian packet over the barrier at $q = q_b$:

$$P(t) = \int_{q_b}^{\infty}\rho(t,q,0)dq = \sum_{k=0,2,4,\ldots}\int_{q_b}^{\infty} f_k(t,q)dq \quad (9)$$

as well as the time-dependent value of the probability rate

$$\begin{aligned}\Lambda(t) &= \frac{1}{1-P(t)}\frac{dP(t)}{dt}\\ &= \frac{i\hbar}{\mu[1-P(t)]}\int_{q_b}^{\infty} dq\,\partial_{q,z}\rho(t,q,z)|_{z=0}\\ &= \frac{-i\hbar}{\mu[1-P(t)]}\sum_{k=1,3,5,\ldots} f_k(t,q_b)\partial_z B_k(\sigma,z)|_{z=0}.\end{aligned} \quad (10)$$

The time-dependent fission width $\Gamma_f(t)$, related to the escape rate $\Lambda(t)$, is then defined as $\Gamma_f(t) = \hbar\Lambda(t)$.

Here, we study the escape of the Gaussian packet from a left well to a deeper right well of an asymmetric bistable potential:

$$\tilde{U}(q) = -0.838q - 0.118q^2 + 0.01q^4. \quad (11)$$

In Fig. 1, q_m and $q = q_b = 0$ are the positions of the left minimum and the barrier, respectively, B_f is the depth of

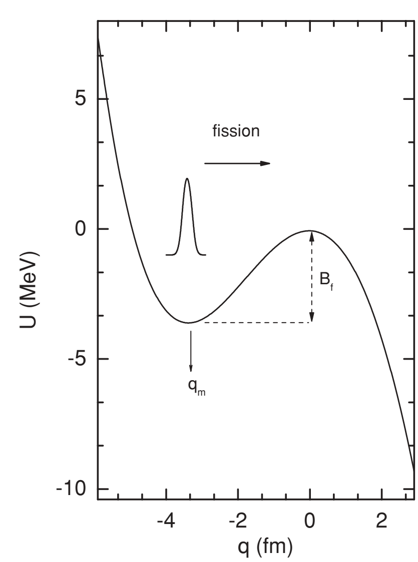

FIG. 1. The used asymmetric bistable potential is partly shown. The schematically shown Gaussian packet in the left well decays into the right-hand side of the barrier.

the left well. We identify this barrier with the saddle point, the minimum with the initial configuration of the fissioning nucleus, and the much deeper minimum at $q > 0$ with the completed fission process. The depth of the second minimum is 116.7 MeV at $q_R = 12.23$ fm to ensure that at the temperature considered here no backflow into the shallow minimum at q_m is possible. Of course, in reality there is no potential increase at large positive values of q. This choice of $\tilde{U}(q)$ was dictated to keep the bound domain of the numerical integration of Eq. (1) in q. We set the shallow minimum at $q_m = -3.41$ fm of $B_f = 3.7$ MeV, which is the height of the fission barrier of ^{248}Cm. We consider the mass parameter $\mu = Am_0/4$ (m_0 is the nucleon mass), for which the corresponding frequencies in the left potential minimum and on the top of the barrier are $\hbar\tilde{\omega}_m = 1.2$ MeV and $\hbar\tilde{\omega}_b = 1.06$ MeV, respectively, at $A = 248$. The chosen parameters, which are related to the nuclear fission of ^{248}Cm are taken the same as in Ref. [1,6,8]. The coordinate q is related to the deformation coordinate.

With the sets of microscopic diffusion coefficients given in Eqs. (3) and (4) we consider the escape of the initial Gaussian packet, which is centered at the left potential well at $q(0) = q_m$ and $p(0) = 0$, and has the variances $\sigma_{qp}(0) = 0$, $\sigma_{qq}(0) = T^*/[\mu\omega_m^2]$, and $\sigma_{pp}(0) = \hbar^2/[4\sigma_{qq}(0)]$, where $T^* = (\hbar\omega_m/2)\coth[\hbar\omega_m/(2T)]$ is the quantum effective temperature. Due to the quantum-mechanical uncertainty principle, the initial equilibrium probability distribution has a minimal width. It should be noted that the results are not sensitive to a reasonable variation of the initial variances at fixed collective energy.

We apply the initial conditions and the numerical procedure used for solving the quantum master equation (1) to study the fission of the nucleus ^{248}Cm. As follows from our estimations, the viscous dissipation smeares the packet toward the quasiequilibrium in the left minimum long before it reaches the saddle point, and the flux over the saddle point slowly rises toward its quasistationary value. The quasistationary regime is finally established when the distribution settles around the position in between the bottom of the left well and the saddle point. The mean value $\langle q(t)\rangle$ and variance $\sigma_{qq}(t) = \langle q^2(t)\rangle - \langle q(t)\rangle^2$ of q increase with time and their slopes increase with decreasing λ_p. Here, we treat the values of $\hbar\lambda_p$ from 0.66 to 6.6 MeV. We attribute the friction dependence to the fact that decreasing λ_p enhances the mobility of the system. From the calculations we conclude that the spreading of the probability distribution during the time in which the quasistationary flux sets in is rather insensitive to the width of the initial distribution.

The time dependence of the fission rate over the saddle point is shown in Fig. 2 for several values of nuclear friction coefficient related to the underdamped ($\lambda_p < 2\omega_m$) or overdamped ($\lambda_p > 2\omega_m$) regime and different temperatures. One can see that the fission rate or width as a function of time can be characterized by three main features: a delayed onset, a rising part, and a stationary value. The transition effects take a so-called transient time τ, until the fission rate (or decay width) reaches 90% of its stationary value. The overdamped motion in q leads to a much later onset of the fission process and to smaller value of the quasistationary flow over the barrier. The initial suppression of the fission rate during the transient time τ may increase the chance of excited nucleus to emit a particle at the earliest times. As a consequence of the transient behavior of the fission, the survival probability of highly excited compound nucleus can be strongly enhanced.

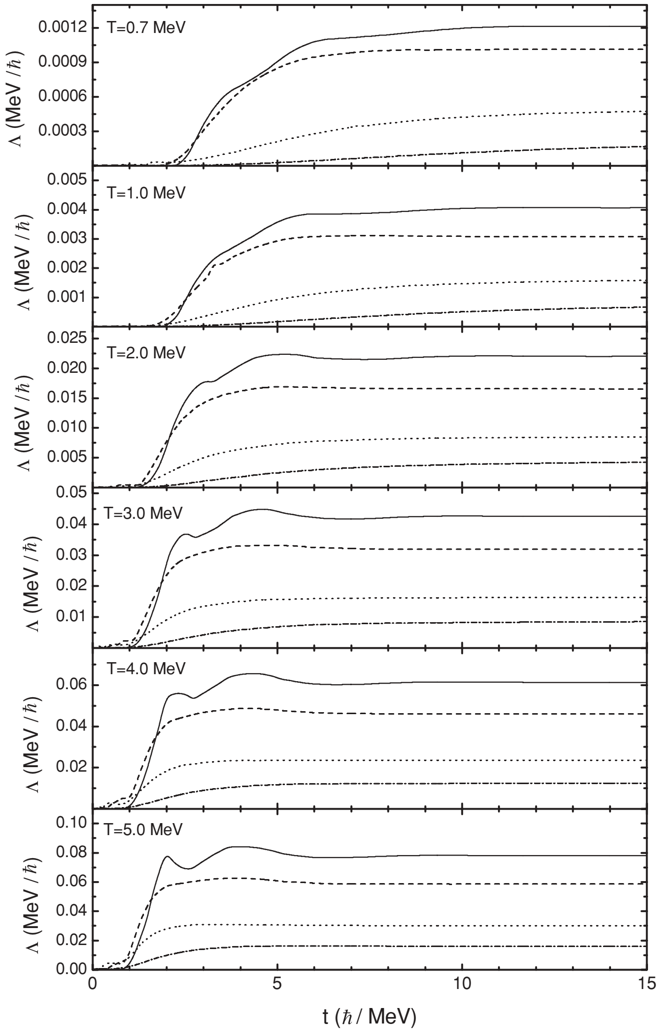

FIG. 2. The fission rate $\Lambda(t)$ defined in Eq. (10) for various values of the reduced dissipation coefficients and indicated temperatures. The results obtained for the $\hbar\lambda_p = 0.66$, 1.32, 3.3, and 6.6 MeV are presented by solid, dashed, dotted, and dash-dotted lines, respectively.

The numerical solutions of classical Langevin and Fokker-Planck equations show also that the value of $\Lambda(t)$ does not strongly rise already at very early times [1,3,6,8]. However, in the quantum case the transient times is larger up to factor of 2 than those in the classical case based on Langevin equation [1]. With quantum treatment the transient time is longer than with classical one because the negative value of the mixed diffusion coefficient D_{pq} keeps the fission rate smaller. At the same time the asymptotic fission rates in both cases are almost the same, especially at high temperatures. Therefore, in the quantum treatment of the time evolution of fission the neutrons have more chance to be emitted at the beginning of the process than in the classical treatment.

In Fig. 3 one can see that the transient time firstly decreases with increasing friction, attains a minimum near $\lambda_p \approx \omega_m$,

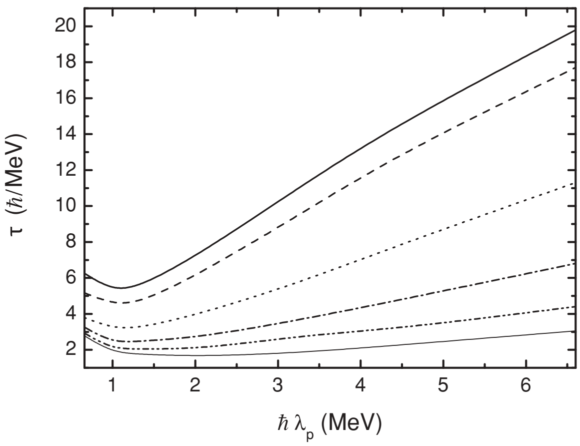

FIG. 3. The transient time evaluated at the saddle point as a function of the reduced dissipation coefficients. The results obtained for the $T = 0.7$, 1, 2, 3, 4, and 5 MeV are presented by solid, dashed, dotted, dash-dotted, dash-dot-dotted, and thin solid lines, respectively.

and then increases again as a consequence of damping. The quantum results reproduce the trend of the classical ones [8]. The transient time varies from $\sim 10^{-21}$ to $\sim 10^{-20}$ s for the wide ranges of friction and temperature considered. The value of τ decreases slowly with increasing temperature. For example, at $\lambda_p \approx \omega_m$ we have $\tau \approx 10^{-21}$ s for $T = 5$ MeV and $\tau \approx 4 \times 10^{-21}$ s for $T = 0.7$ MeV. Figures 4 and 5 show that the transient time is about 5–230 times smaller than the fission time $\tau_f \approx 1/\Lambda(\infty)$ $[\int_0^{\tau_f} \Lambda(t)dt = 1]$ evaluated at the saddle point. In Fig. 4 the total lifetime of the fissioning nucleus increases with the reduced dissipation coefficient from 5.4×10^{-19} s to 3×10^{-18} s at $T = 0.7$ MeV (the excitation energy $E^*_{\rm CN} = 12.2$ MeV), from 1.7×10^{-19} s to 8.3×10^{-19} s at $T = 1$ MeV ($E^*_{\rm CN} = 24.8$ MeV), from 3×10^{-20} s to 1.5×10^{-19} s at $T = 2$ MeV ($E^*_{\rm CN} = 99.2$ MeV), and from 8.5×10^{-21} s to 4.2×10^{-20} s at $T = 5$ MeV ($E^*_{\rm CN} = 620$ MeV). Thus, with increasing T from $T = 0.7$ MeV ($T < B_f$) to 5 MeV ($T > B_f$) the fission time decreases faster than the transient time in the underdamped regime. In the overdamped regime the transient time contribution to the fission time increases with T for $T \leqslant B_f$ and decreases with T for $T > B_f$. So, the contribution of τ in τ_f is pronounced only for small values of λ_p (the underdamped regime) and large values of T.

In Fig. 4 the analytical Kramers quasistationary fission rate

$$\Lambda^{Kr} = \frac{\omega_m}{2\pi\omega_b}\left(\left[\omega_b^2 + \lambda_p^2/4\right]^{1/2} - \lambda_p/2\right)\exp[-B_f/T] \quad (12)$$

is proven to reproduce rather closely the exact solution in the underdamped regime as well as in the overdamped regime. The largest deviations between Λ^{Kr} and $\Lambda(\infty)$ are about of 71% at $\hbar\lambda_p = 0.66$ MeV and 44% at $\hbar\lambda_p = 6.6$ MeV for $T = 0.7$ MeV. The modification $T \to T^*$ in Eq. (12) leads to better agreement at $\hbar\lambda_p = 0.66$ MeV: the deviation is about of 37%. This modification permits to overcome the limitations of Kramers description at low temperatures in the underdamped regime. Although in literature the validity limit of the Kramers stationary solution of the Fokker-Planck equation is given by the condition $T/B_f < 1$, one can see that the relative deviation between Λ^{Kr} and $\Lambda(\infty)$ for temperature $T = 5$ MeV

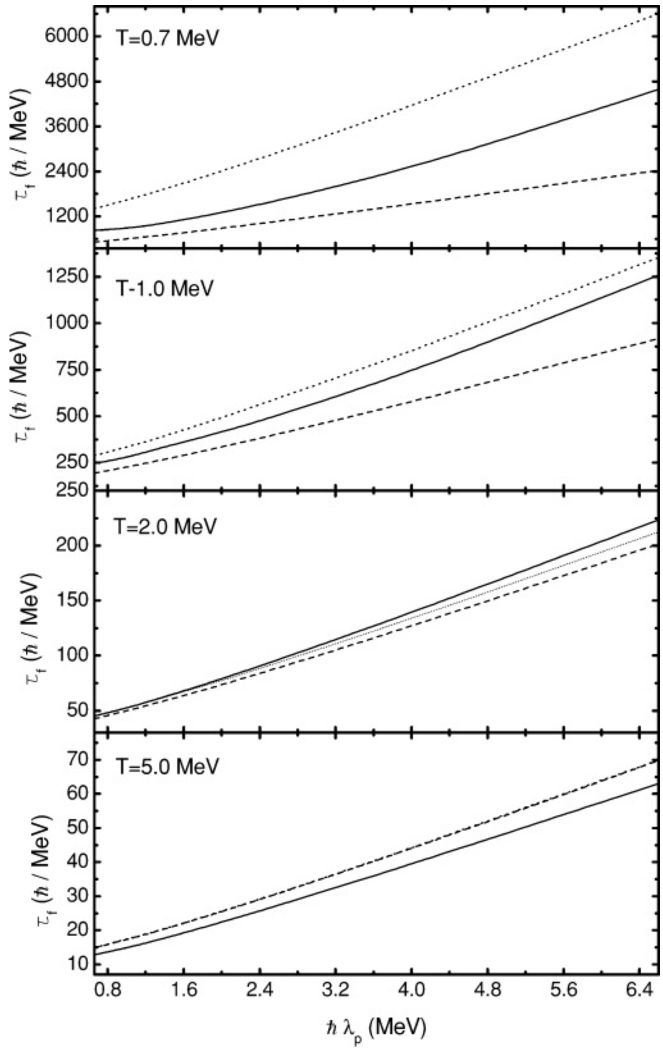

FIG. 4. The fission time evaluated at the saddle point as a function of the reduced dissipation coefficients at indicated temperatures. The results obtained with the time-dependent fission rate, Kramers quasistationary fission rate, and modified Kramers quasistationary fission rate ($T \to T^*$) are presented by solid, dotted, and dashed lines, respectively.

($T > B_f$) is about of 16% at $\hbar\lambda_p = 0.66$ MeV and about of 11% at $\hbar\lambda_p = 6.6$ MeV. The Kramers rate becomes smaller than $\Lambda(\infty)$ for temperatures exceeding the fission barrier.

From Fig. 5 one can see that in a highly excited ($E^*_{\rm CN} \geqslant 100$ MeV) heavy nucleus the width of neutron emission $\Gamma_n = \hbar/\tau_n$ becomes comparable to or larger than the transient width $\hbar/\tau$. In the contrast to the transient time, the neutron emission time decreases roughly exponentially with increasing excitation energy: from 1.5×10^{-17} s at $T = 0.7$ MeV to 1.5×10^{-22} s at $T = 5$ MeV. We apply in the calculations the following analytical expression for the neutron emission width [28]$\Gamma_n = (T^2 A^{2/3}/[20\pi])\exp(-B_n/T)$, where $B_n = 6.2$ MeV is the binding energy of neutron in nucleus ^{248}Cm.

The average neutron multiplicity during the transient time can be estimated by a simple formula: $\nu \approx \tau/\tau_n$ (Fig. 5). The emitted neutron during the transient time carries off energy, cools the compound nucleus, and thereby terminates

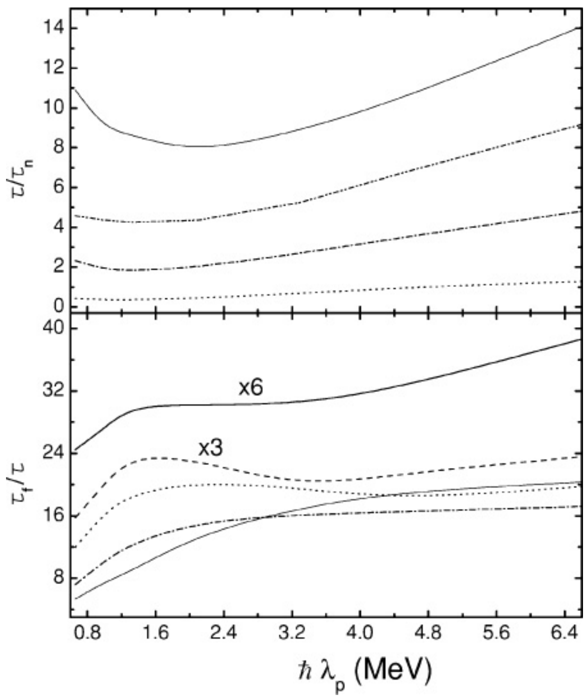

FIG. 5. The calculated ratios τ/τ_n and τ_f/τ as functions of the reduced dissipation coefficients. The results obtained for the $T = 0.7, 1, 2, 3, 4$, and 5 MeV are presented by solid, dashed, dotted, dash-dotted, dash-dot-dotted, and thin solid lines, respectively.

the occurrence of first-chance fission. The emission during the saddle-to-scission time is also possible. However, only the transient time has an influence on the choice of the system to undergo fission or survival. A significant reduction of the probability of the first-chance fission

$$P_f(E^*_{\rm CN}, \lambda_p) = \frac{\Gamma_n}{\hbar}\int_0^\infty dt \exp[-\Gamma_n t/\hbar]P(t) \qquad (13)$$

$\{\eta(t) = \frac{\Gamma_n}{\hbar}\exp[-\Gamma_n t/\hbar]$ is the probability that a neutron is emitted at time $t\}$ due to transient effects can be analytically shown. Using analytical approximate expression [$\theta(t)$ is the step function]

$$P(t) = 1 - [\theta(\tau - t) + \exp[-\Lambda(\infty)t]\theta(t - \tau)],$$

Eq. (13) is reduced to [6]

$$P_f(E^*_{\rm CN}, \lambda_p) = P_f^{st}(E^*_{\rm CN}, \lambda_p)\exp[-\Gamma_n\tau/\hbar] = \frac{\hbar\Lambda(\infty)}{\hbar\Lambda(\infty) + \Gamma_n}\exp[-\Gamma_n\tau/\hbar]. \qquad (14)$$

In the case of $\tau \geqslant \tau_n = \hbar/\Gamma_n$ the transient behavior of the fission width effectively reduces the fission probability P_f with respect to its stationary model value P_f^{st}. The exact numerical results of Eq. (13) show in Fig. 6 that the condition $\Gamma_n\tau/\hbar \geqslant 1$ occurs at $E_{\rm CN} \geqslant 100$ MeV for underdamped motion as well as for overdamped motion. The deviation between $P_f(E^*_{\rm CN}, \lambda_p)$ and $P_f^{st}(E^*_{\rm CN}, \lambda_p)$ increases with the excitation energy and friction: $P_f^{st}(E^*_{\rm CN} = 99.2$ MeV, $\lambda_p = 0.66\hbar^{-1}$ MeV$)/P_f(E^*_{\rm CN} = 99.2$ MeV, $\lambda_p = 0.66\hbar^{-1}$ MeV$) = 1.7$, $P_f^{st}(E^*_{\rm CN} = 620$ MeV, $\lambda_p = 0.66\hbar^{-1}$ MeV$)/P_f(E^*_{\rm CN} = 620$ MeV, $\lambda_p = 0.66\hbar^{-1}$ MeV$) = 11.7$, and $P_f^{st}(E^*_{\rm CN} = 99.2$ MeV, $\lambda_p = 6.6\hbar^{-1}$ MeV$)/P_f(E^*_{\rm CN} = 99.2$ MeV, $\lambda_p = 6.6\hbar^{-1}$ MeV$) = 1.8$, $P_f^{st}(E^*_{\rm CN} = 620$ MeV, $\lambda_p = 6.6\hbar^{-1}$ MeV$)/P_f(E^*_{\rm CN} = 620$ MeV, $\lambda_p = 6.6\hbar^{-1}$ MeV$) = 128.5$. The sensitivity of the dependence of the fission probability on λ_p increases with excitation energy. It should be noted that at higher excitation energies the transient behavior affects more steps of the de-excitation cascade.

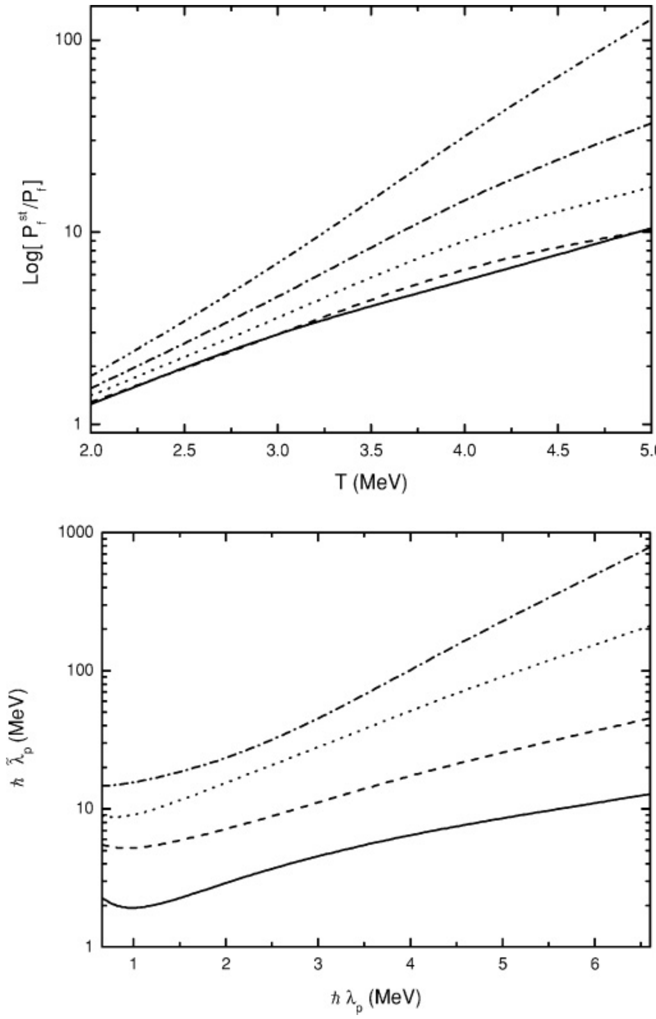

FIG. 6. The calculated $\log[P_f^{st}/P_f]$ (upper part) as a function of the temperature at $\hbar\lambda_p = 0.99$ (solid line), 2.1 (dashed line), 3.3 (dotted line), 4.5 (dash-dotted line), and 6.6 (dash-dot-dotted line) MeV. The calculated effective reduced dissipation coefficient (lower part) as a function of friction at $T = 2$ (solid line), 3 (dashed line), 4 (dotted line), and 5 (dash-dotted line) MeV.

The requirement $P_f(E^*_{\rm CN}, \lambda_p) = P_f^{st}(E^*_{\rm CN}, \tilde\lambda_p)$ can be satisfied with the stationary fission width $\hbar\Lambda(\infty)$ by using the effective friction coefficient

$$\tilde\lambda_p = \frac{\omega_b^2}{[\omega_b^2 + \lambda_p^2/4]^{1/2} - \lambda_p/2}\frac{P_f^{st}(E^*_{\rm CN}, \lambda_p)}{P_f(E^*_{\rm CN}, \lambda_p)} \qquad (15)$$

in the case of $\tilde\lambda_p \gg \omega_b$, $\Gamma_f \gg \Gamma_n$ and a_f/a_n=1, where $a_f = a$ and $a_n = a$ are the level-density parameters for fission and neutron emission, respectively. At fixed a_f/a_n the first-chance fission probability can be reproduced within the stationary statistical model (with Kramers modification of the Bohr-Wheeler statistical model result but without transients) by increasing effectively the reduced dissipation coefficient ($\tilde{\lambda}_p > \lambda_p$) (Fig. 6). At $\lambda_p \approx \omega_m$ the value of $\tilde{\lambda}_p$ is minimal that correlates with the behavior of the transient time. Note that the coupling of the main fission coordinate to other degrees of freedom is approximately equivalent to an extra friction and potential and mass corrections in the corresponding one-dimensional problem.

The fission of excited nuclei was considered a consequence of quantum statistical fluctuations across the saddle point. With the exact numerical solution of the quantum master equation for the reduced density matrix we found a influence of the quantum statistical effects on the time dependence of the fission process. In the quantum case the transient times are larger by a factor of about 2 than those in the classical case based on the Langevin equation. We calculated the fission rate for the simple potential with one pronounced minimum and one barrier. For the potentials having an additional structure (for example, a potential barrier at the scission point), the fission lifetime can be considerably longer than the calculated one. It should be noted that the fission rate and the transient time are not sensitive to the reasonable changes of the initial Gaussian packet at fixed collective energy. At moderate to high excitation energies the asymptotic fission rates in classical and quantum cases are almost the same. The most realistic friction coefficients in the range of $\hbar\lambda_p \approx$ 1–2 MeV were suggested from the study of deep inelastic collisions [29]. At $\hbar\lambda_p \approx$ 1 MeV the transient time changes from 10^{-21} s to 4×10^{-21} s with decreasing temperature from $T = 5$ MeV to $T =$ 0.7 MeV. At the same time the fission lifetime varies from 10^{-21} to 5.4×10^{-19} s. The main contribution to the fission time comes from the time spent by the fissioning nucleus before the saddle point.

At large excitation energies ($E^*_{\rm CN} \geqslant 100$ MeV) the average neutron emission time becomes comparable to or smaller than the transient time, and the deviation of the fission probability from the value of the statistical model becomes considerably large in the first steps of the de-excitation chain. It appears that in the highly excited nuclei the fission is hindered by the transient effects. To take the hindrance effectively into consideration one can increase effectively the friction coefficient in the stationary statistical model with Kramers modification to reproduce the fission probability and particle multiplicities. However, some rather detailed observables (for example, the distribution of excitation energies at the moment of fission) could not be correctly reproduced by strongly increasing the friction coefficient.

The analytical Kramers formula with thermodynamics temperature or with quantum effective temperature is rather well suited in the under- and overdamped regimes. Kramers rate is capable for reproducing the fission rate with sufficient accuracy for a range of $\hbar\lambda_p$ values of main physical interest, 0.66 MeV $\leqslant \hbar\lambda_p \leqslant$ 6.6 MeV and 0.7 MeV $\leqslant T \leqslant$ 5 MeV. We verified the existence of a quasistationary regime of the probability flow over the barrier at temperatures $T > B_f$.

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