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Long standing problem of ^{210}Bi and the realistic proton-neutron effective interaction

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The odd-odd nucleus ^{210}Bi is studied within the framework of the shell model using effective two-body matrix elements derived from the CD-Bonn nucleon-nucleon potential. The experimental energies of the proton-neutron multiplet $\pi h_{9/2}\nu g_{9/2}$ are remarkably well reproduced by the theory, which accounts for the 1^- state being the ground state instead of the 0^- predicted by the Nordheim strong coupling rule. It is shown that the core-polarization effects are crucial to produce this inversion. The similarity between proton-neutron multiplets in the ^{132}Sn and ^{208}Pb regions is discussed in connection with the effective interaction.

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A fundamental goal of nuclear physics is to understand the properties of nuclei starting from the forces between nucleons. Within the framework of the shell model, which is the basic framework for the study of complex nuclei in terms of nucleons, this implies the derivation of the two-body effective interaction $V_{\rm eff}$ from the free nucleon-nucleon (NN) potential. This has long been a main problem of microscopic nuclear structure theory. Suffice to say that since the early 1950s through the mid-1990s in the vast majority of shell-model calculations either empirical effective interactions containing adjustable parameters have been used or the two-body matrix elements themselves have been treated as free parameters.

Great progress toward "realistic" shell-model calculations has been achieved over the last decade, proving the ability of effective interactions derived from modern NN potentials to provide, with no adjustable parameters, an accurate description of nuclear properties. This makes it very interesting to put to the test realistic shell-model calculations on peculiar nuclear properties which could not be unambiguously explained by earlier calculations. One such case is provided by the nucleus ^{210}Bi.

The nucleus ^{210}Bi with one proton and one neutron outside doubly magic ^{208}Pb core has long been the subject of theoretical studies. In this nucleus there are ten states below 600 keV with angular momenta from zero to nine, which in terms of the shell model constitute a proton-neutron multiplet arising from the $\pi h_{9/2}\nu g_{9/2}$ configuration. The experimental determination of the ground-state spin of ^{210}Bi, $J^\pi = 1^-$, dates back to some 50 years ago [1] and since then it has been a challenge to shell-model theorists to explain this feature, which is at variance with the prediction, $J^\pi = 0^-$, of the Nordheim strong coupling rule [2].

Early calculations [3–5] made use of empirical two-body interactions containing adjustable parameters. These studies led to the conclusion that tensor-force components were needed to explain the inversion of the 0^- and 1^- states. Realistic shell-model calculations starting from the Hamada-Johnston NN potential [6] were performed for ^{210}Bi by Herling and Kuo [7] in the early 1970s. The hard core of the Hamada-Johnston potential was handled by using the Brueckner reaction matrix G. Then, the effective two-body interaction was constructed by adding to the G matrix certain classes of diagrams arising from core-excitation processes, accounting in this way for configurations left out of the space in which the shell-model calculation was performed. A good overall agreement between the calculated and experimental spectra was obtained in [7], but the above inversion was not explained. Some twenty years later, calculations in the ^{208}Pb region were performed by Warburton and Brown [8] with some modifications to the Kuo-Herling interaction. In particular, for the $\pi h_{9/2}\nu g_{9/2}$ multiplet, modifications of all the corresponding two-body matrix elements were introduced to reproduce the experimental energies.

In the work of [9] the results of a shell-model calculation for nuclei in the lead region including ^{210}Bi were presented together with a brief but comprehensive review of studies on this nucleus through 1972. The authors of [9] made use of an empirical effective interaction with no tensor components but containing multipole forces with adjustable strengths to account for model-space truncation effects. Their conclusion was that an unambiguous explanation of the inversion of the 0^- and 1^- states still remained to be given. In fact, they showed that their model was not able to simultaneously describe the correct ordering of the two states and the energies of the higher-lying states of the $\pi h_{9/2}\nu g_{9/2}$ multiplet. They pointed out that, while the studies [3,5] evidenced the key role of the tensor force to reproduce the whole multiplet, realistic effective interactions as that used in [7] had failed to reproduce the ^{210}Bi ground state although the starting NN potential contained explicitly tensor components.

This situation had not changed much after more than two decades, as is reflected in the work of Ref. [10], where a description of ^{210}Bi is given by using a phenomenological model, which, besides an empirical proton-neutron effective interaction, includes explicitly macroscopic degrees of freedom through a ^{208}Pb vibrating core.

As mentioned above, in the last ten years or so shell-model calculations employing realistic effective interactions derived from modern NN potentials have produced results in remarkably good agreement with experiment for a number of nuclei in various mass regions. In the most recent calculations, the difficulty of dealing with potentials having a strong repulsive core has been overcome by resorting to the so called $V_{\rm low\text{-}k}$ approach [11]. Within this approach a low-momentum

NN potential, $V_{\text{low-k}}$, is obtained by integrating out the high-momentum modes of the NN potential down to a cutoff momentum Λ, with the requirement that the deuteron binding energy as well as the half-on-shell T matrix of the original potential are preserved. This T-matrix equivalence procedure, as described in detail in Ref. [11], produces a smooth potential which is suitable for being used directly in nuclear structure calculations. The shell-model effective interaction is then constructed within the framework of a $\hat{Q}$-box folded-diagrams expansion [12], namely it is expressed as a $\hat{Q}$-box, which is a vertex function composed of irreducible valence-linked diagrams at any order in $V_{\text{low-k}}$, plus the folded-diagram series. Note that the G-matrix calculation of the effective interaction performed by Herling and Kuo [7] is equivalent to retain the first three terms of the $\hat{Q}$-box without folded diagrams, i.e., the G matrix and the core-polarization diagrams corresponding to one particle-one hole (1p1h) and two particle-two hole (2p2h) excitations.

In recent years, particular attention from the experimental and theoretical point of view has been focused on nuclei around doubly magic ^{132}Sn. In this region, the counterpart of ^{210}Bi is the nucleus ^{134}Sb with one proton and one neutron in the $Z = 50$–82 and $N = 82$–126 shells, respectively. In this case, the ground state has $J^\pi = 0^-$ and is nearly degenerate with the first-excited $J^\pi = 1^-$ state, the latter lying at 13 keV. These two states are members of the lowest proton-neutron multiplet arising from the $\pi g_{7/2}\nu f_{7/2}$ configuration. Very recently, we have performed a shell-model study of this nucleus [13] making use of an effective interaction derived from the CD-Bonn NN potential [14]. Our results turned out to be in very good agreement with the experimental data including the very small energy spacing between the 0^- and 1^- states. A main finding of this study was that core polarization effects, in particular those arising from 1p1h excitations, introduce modifications in the effective interaction which are essential to reduce the spacing between the 1^- and 0^- states.

The above achievement is at the origin of the present paper, where we present the results of a realistic shell-model calculation for ^{210}Bi conducted along the same lines as that for ^{134}Sb. Note that each of the two single-particle levels composing the $\pi g_{7/2}\nu f_{7/2}$ multiplet in ^{134}Sb has a counterpart with the same radial quantum number and one more unit in the angular momenta l and j in the $\pi h_{9/2}\nu g_{9/2}$ multiplet of ^{210}Bi. As we shall see, this multiplet is very well reproduced with the 0^- and 1^- states in correct order.

Our study of ^{210}Bi has been performed assuming ^{208}Pb as a closed core and taking as model space for the valence proton and neutron the six levels of the 82–126 shell and the seven levels of the 126–184 shell, respectively.

The two-body effective interaction V_{eff} has been derived by means of the $\hat{Q}$-box folded-diagrams expansion [12] from the CD-Bonn NN potential, the short-range repulsion of the latter being renormalized by use of the $V_{\text{low-k}}$ potential [11]. As in our previous studies for nuclei around ^{132}Sn [13,15,16], the cutoff momentum Λ is given the value 2.2 fm^{-1}. This $V_{\text{low-k}}$, with addition of the Coulomb interaction for protons, is then used to calculate the $\hat{Q}$-box. In our calculation, we include one- and two-body diagrams up to second order in the interaction, as they are explicitly shown in [17]. The computation of these diagrams is performed within the harmonic-oscillator basis using intermediate states composed of all possible hole states and particle states restricted to the five proton and neutron shells above the Fermi surface. This guarantees the stability of the results when increasing the number of intermediate particle states. The oscillator parameter is 6.88 MeV, as obtained from the expression $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$. Once the $\hat{Q}$-box is calculated, the series of folded diagrams is summed up to all orders using the Lee-Suzuki iteration method [18].

TABLE I. Proton and neutron single-particle energies (in MeV).

$\pi(n, l, j)$	ϵ	$\nu(n, l, j)$	ϵ
$0h_{9/2}$	-3.80	$1g_{9/2}$	-3.94
$1f_{7/2}$	-2.90	$0i_{11/2}$	-3.16
$0i_{13/2}$	-2.19	$0j_{15/2}$	-2.51
$1f_{5/2}$	-0.97	$2d_{5/2}$	-2.37
$2p_{3/2}$	-0.68	$3s_{1/2}$	-1.90
$2p_{1/2}$	-0.16	$1g_{7/2}$	-1.44
		$2d_{3/2}$	-1.40

The effective interaction obtained by this procedure contains one- and two-body contributions, the former determining, once summed to the unperturbed Hamiltonian, the single-particle energies. We have used the subtraction procedure of Ref. [17] so as to retain only the two-body term V_{eff}, while the single-particle energies have been taken from experiment. The single-proton and -neutron energies adopted in our calculation are reported in Table I. They are relative to ^{208}Pb as obtained from the spectra of ^{209}Bi and ^{209}Pb [19] using mass excesses from [20].

Let us now come to the results of our calculations, which have been performed by using the OXBASH shell-model code [21]. As for the binding energy of the ground state, our calculated value is 8.347 ± 0.003 MeV, which compares very well with the experimental one, 8.404 ± 0.002 MeV [20]. Note that the error on the calculated value arises from the experimental errors on the proton and neutron separation energies of ^{209}Bi and ^{209}Pb [20].

The calculated wave functions of the yrast states with J^π from 0^- to 9^- are all dominated by the $\pi h_{9/2}\nu g_{9/2}$ configuration with very little configuration mixing. In fact, the percentage of this component ranges from 96% to 100% except for the 1^- state where it has the minimum value of 91%. These states represent therefore the ten members of the $\pi h_{9/2}\nu g_{9/2}$ multiplet and their excitation energies are reported in Fig. 1, where they are compared with the experimental yrast states with the same spin and parity [19]. Note that besides the ten members of the multiplet, only a second 1^- state is found below 600 keV in both the experimental and calculated spectra.

From Fig. 1, we see that the experimental pattern is well reproduced by the theory. We correctly predict the energy decrease from the 0^- to the 1^- state as well as the sharp and slight increase occurring in the next two steps. Then, from the 3^- state on the experimental and theoretical patterns stagger with the same magnitude and phase. The results shown in Fig. 1 are also in remarkably quantitative agreement with

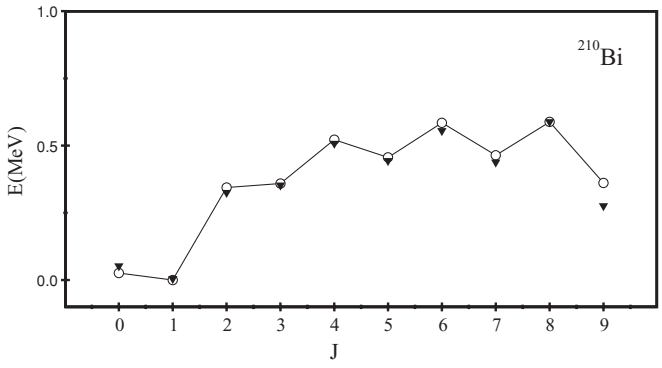

FIG. 1. Proton-neutron $\pi h_{9/2}\nu g_{9/2}$ multiplet in ^{210}Bi. The theoretical results are represented by open circles while the experimental data by solid triangles.

the experimental data. In fact, the discrepancies are less than 50 keV for all the states except the 9^-, which is predicted to lie at 90 keV above the experimental one.

Let us now discuss our effective interaction. In our study of ^{134}Sb [13] the effective two-body interaction was derived using precisely the same approach as that adopted here for ^{210}Bi and it turned out that the $\pi g_{7/2}\nu f_{7/2}$ multiplet was very well described. In this context, we analyzed the various terms contributing to the effective interaction in order to understand their relative importance in determining the proton-neutron matrix elements leading to a successful description of the multiplet. This provided clear evidence of the crucial role played by the 1p1h excitations especially in regard to the 0^-–1^- spacing.

Here, we have performed the same analysis for the $\pi h_{9/2}\nu g_{9/2}$ multiplet in ^{210}Bi. In Fig. 2, we report the diagonal matrix elements of the effective interaction as a function of J for the $\pi h_{9/2}\nu g_{9/2}$ configuration. As expected, we see that their behavior is quite similar to that shown in Fig. 1 for the energies of the multiplet. Then, we show in Fig. 3 the four two-body contributions to the effective interaction, namely the corresponding matrix elements of $V_{\text{low-k}}$ and the renormalizations due to core polarization through 1p1h and 2p2h excitations as well as to the ladder diagrams which account for excluded configurations above the chosen model space. As in the case of ^{134}Sb, the folding contribution turns

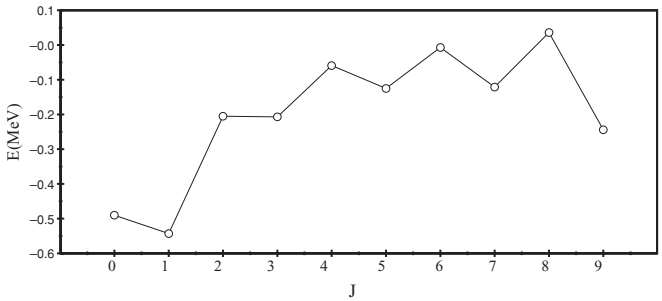

FIG. 2. Diagonal matrix elements of the two-body effective interaction for the $\pi h_{9/2}\nu g_{9/2}$ configuration.

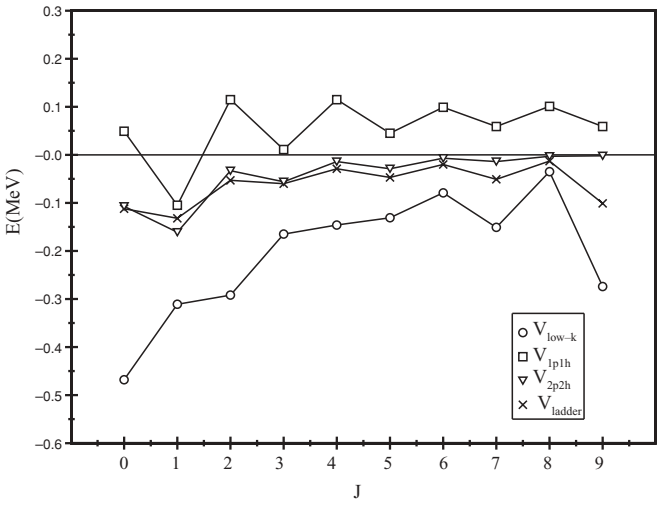

FIG. 3. Diagonal matrix elements of $V_{\text{low-k}}$ and contributions from the two-body second-order diagrams for the $\pi h_{9/2}\nu g_{9/2}$ configuration. See text for comments.

out to be quite irrelevant for the behavior of the proton-neutron multiplet. From Fig. 3 it is clear that the features of the proton-neutron effective interaction evidenced in our study of ^{134}Sb persist in the ^{208}Pb region. In fact, we see that also in this region the proton-neutron matrix elements of $V_{\text{low-k}}$ do not follow the experimental behavior, the most attractive one being that for $J^\pi = 0^-$. This behavior is significantly modified by the V_{1p1h} term, which is always repulsive except for $J^\pi = 1^-$.

It is worth mentioning that in a preliminary calculation on ^{210}Bi [22] the quality of agreement between theory and experiment was not as good as that obtained here. In particular, we did not predict the 0^- and 1^- levels in correct order. However, also in that case the contribution from 1p1h excitations was essential to reduce the spacing between the 1^- and 0^- states, although not sufficient to produce the inversion of these two states. It turned out that the correct ground state could be obtained increasing by a factor of about 2.5 the diagonal matrix elements of V_{1p1h} for the $\pi h_{9/2}\nu g_{9/2}$ configuration.

In the present calculation, the 1p1h contribution brings about the right effect without the introduction of any modification. This is traced to the improvement in the calculation of the effective interaction resulting from the increased number of intermediate states used in the computation of the $\hat{Q}$-box diagrams. The same result was also found for ^{134}Sb [13].

At this point, we may conclude that our effective interaction contains the noncentral components needed to give the 0^- and 1^- levels in correct order. These arise from virtual interactions of the core particles induced by the NN potential and their weight with respect to the central components may be quite different from that relative to the NN potential.

In summary, we have shown that a shell-model interaction derived from a modern NN potential without any modification leads to a direct solution of the long standing problem of ^{210}Bi.

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