nd scattering lengths from a quark-model based NN interaction

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We calculate the doublet and quartet neutron-deuteron scattering lengths using a nonlocal nucleon-nucleon interaction fully derived from quark-quark interactions. We use as input the NN $^1S_0$ and $^3S_1$-$^3D_1$ partial waves. Our result for the quartet scattering length agrees well with the experimental value but the result for the doublet scattering length does not. However, if we take the result for the doublet scattering length together with the one for the triton binding energy they agree well with the so-called Phillips line.

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In a previous paper [1] the properties of the bound state of three nucleons, the triton, were studied using a nonlocal nucleon-nucleon (NN) interaction obtained from the chiral quark model [2]. In this Brief Report we would like to complete that study by calculating within the same framework the neutron-deuteron doublet and quartet scattering lengths.

The nonlocal nucleon-nucleon interaction, obtained from the chiral quark model, is based on a Lippmann-Schwinger formulation of the resonating group method in momentum space. A detailed description of the method and a summary of the parameters of the model is given in Ref. [1]. In this reference it was shown that this NN interaction describes very well the properties of the deuteron as well as the $^1S_0$ and $^3S_1$-$^3D_1$ phase shifts. In addition, it was found that the nonlocal NN interaction obtained from the chiral quark model predicts a triton binding energy of 7.72 MeV, comparable to the predictions by conventional meson-exchange models. Thus, it is important to see whether that result extends also to the neutron-deuteron scattering lengths.

We will write down the integral equations that determine the *nd* scattering lengths using the partial-wave basis states

$$|p_iq_i;\rho_i\rangle \equiv |p_iq_i;\ell_is_ij_ii_i\lambda_iJ_i\rangle, \tag{1}$$

where if σ_i and τ_i stand for the spin and isospin of particle i then ℓ_i, s_i, j_i, i_i, λ_i, and J_i are, respectively, the orbital angular momentum, spin, total angular momentum, and isospin of the pair jk, λ_i is the orbital angular momentum between particle i and the pair jk, and J_i is the result of coupling λ_i and σ_i. The conserved quantum numbers are J, the total angular momentum, and I, the total isospin.

The Faddeev equations that determine the *nd* scattering lengths in the special case when one considers only S wave configurations were written down in Ref. [3]. The generalization of these equations to the case when one includes arbitrary orbital angular momenta are

$$\begin{aligned}\langle p_iq_i;\rho_i|T_i^{JI}|\phi_0\rangle &= 2\delta_{s_i1}\delta_{j_i1}\delta_{i_i0}\delta_{\lambda_i0}\frac{1}{q_i^2}\delta(q_i)G_0^{-1}(E;p_iq_i)\phi_{\ell_i}(p_i)\\ &+\sum_{\ell_i'\rho_j}\int_0^\infty q_j^2dq_j\int_{-1}^1 d\cos\theta\, t_{\ell_i\ell_i's_ij_ii_i}\left(p_i,p_i';E-\frac{3q_i^2}{4M}\right)\\ &\times G_0(E;p_jq_j)D_{ij;JI}^{\rho_i'\rho_j}(q_i,q_j,\cos\theta)\langle p_jq_j;\rho_j|T_j^{JI}|\phi_0\rangle,\end{aligned} \tag{2}$$

where $t_{\ell_i\ell_i's_ij_ii_i}(p_i,p_i';e)$ are the nucleon-nucleon t matrices, M is the mass of the nucleon, and $E=-B_d$, where B_d is the binding energy of the deuteron. The function $\phi_{\ell_i}(p_i)$ is the deuteron wave function with orbital angular momentum ℓ_i, and

$$G_0(E;p_iq_i)=\frac{1}{E-p_i^2/M-3q_i^2/4M+i\epsilon}, \tag{3}$$

$$p_i'=\sqrt{q_j^2+q_i^2/4+q_iq_j\cos\theta}, \tag{4}$$

$$p_j=\sqrt{q_i^2+q_j^2/4+q_iq_j\cos\theta}, \tag{5}$$

$$\rho_i'\equiv\{\ell_i's_ij_ii_i\lambda_iJ_i\}, \tag{6}$$

and $D_{ij;JI}^{\rho_i'\rho_j}(q_i,q_j,\cos\theta)$ are the angular momentum-spin-isospin recoupling coefficients, which are defined by Eqs. (21)–(26) of Ref. [4].

After obtaining the solution of Eqs. (2) the neutron-deuteron scattering lengths are calculated as

$$\begin{aligned}a_J &= \frac{2M\pi}{3}\sum_{\rho_j}\sum_{\rho_i}\delta_{s_j1}\delta_{j_j1}\delta_{i_j0}\delta_{\lambda_j0}\int_0^\infty q_i^2dq_i\phi_{\ell_j}(q_i)\\ &\times D_{ji;J\frac{1}{2}}^{\rho_j\rho_i}(0,q_i,0)\langle q_i/2,q_i;\rho_i|T_i^{J\frac{1}{2}}|\phi_0\rangle.\end{aligned} \tag{7}$$

Since in Ref. [1] the triton binding energy was calculated using as input the NN $^1S_0$ and $^3S_1$-$^3D_1$ partial waves we will use the same prescription in our calculation of the neutron-deuteron scattering lengths. This implies a five-channel Faddeev calculation for the doublet scattering length $a_{1/2}$ and a seven-channel calculation for the quartet scattering length $a_{3/2}$. We give these channels in Table I. Our method [4] to solve the Faddeev equations consists in transforming them from being integral equations in two continuous variables into integral equations in just one continuous variable. This is achieved by expanding the two-body t matrices in terms of Legendre

TABLE I. Three-body channels that contribute to a given NNN state with total isospin I and total angular momentum J.

I	J	ℓ_i	s_i	j_i	i_i	λ_i	J_i
0	1/2	0	0	0	1/2	0	1/2
		0	1	1	1/2	0	1/2
		2	1	1	1/2	0	1/2
		0	1	1	1/2	2	3/2
		2	1	1	1/2	2	3/2
0	3/2	0	0	0	1/2	2	3/2
		0	1	1	1/2	0	1/2
		2	1	1	1/2	0	1/2
		0	1	1	1/2	2	3/2
		0	1	1	1/2	2	5/2
		2	1	1	1/2	2	3/2
		2	1	1	1/2	2	5/2

polynomials as

$$t_i(p_i, p_i'; e) = \sum_{nr} P_n(x_i)\tau_i^{nr}(e)P_r(x_i'), \tag{8}$$

where P_n and P_r are Legendre polynomials,

$$x_i = \frac{p_i - b}{p_i + b}, \tag{9}$$

$$x_i' = \frac{p_i' - b}{p_i' + b}, \tag{10}$$

and p_i and p_i' are the initial and final relative momenta of the pair jk, with b a scale parameter on which the results do not depend. We found that using $b = 3$ fm^{-1} leads to very stable results and for the expansion (8) we found convergence with twelve Legendre polynomials (i.e., $0 \leqslant n \leqslant 11$).

We have also calculated the triton binding energy to check that our calculation reproduces the value obtained in Ref. [1]. We give in Table II our results for the triton binding energy and the two neutron-deuteron scattering lengths as well as the corresponding experimental results [5]. As pointed out in Ref. [1], the result $B = 7.72$ MeV for the triton binding energy predicted by the chiral quark model is comparable to the values obtained by conventional meson-exchange models such as Nijmegen or Bonn [6] since the theoretical value differs by less than 1 MeV from the experimental result. The situation in the case of the doublet scattering length $a_{1/2}$ appears to be somewhat worse because the theoretical value 1.13 fm is almost a factor of 2 larger than the experimental result. However, as we will see next, that is not the case. As is well known [7], the results obtained from a given theoretical model for the triton binding energy B and the doublet scattering length $a_{1/2}$ are strongly correlated. The results obtained from different models follow what is known as the Phillips line, which is a straight line relating B versus $a_{1/2}$. For example, in Ref. [8] a five-channel calculation similar to ours was performed using three local potentials [9–11]; therefore, we made a minimum-square fit of their results for B and $a_{1/2}$ to obtain the Phillips line $a_{1/2} = 6.352 - 0.677B$, which for $B = 7.72$ MeV gives $a_{1/2} = 1.13$ fm, in agreement with our result. The most complete calculations of B and $a_{1/2}$ have been performed in Ref. [12] for a variety of modern nucleon-nucleon force models [13–15] and where the higher angular momentum two-body channels as well as three-body forces have been included [16–19]. Using the values of B and $a_{1/2}$ for the 48 different models considered in Ref. [12] we obtained the Phillips line $a_{1/2} = 7.028 - 0.756B$, which for $B = 7.72$ MeV will give a doublet scattering length $a_{1/2} = 1.19$ fm, which is also quite close to the result of our calculation. Our result for the quartet scattering length $a_{3/2} = 6.40$ fm agrees well with experiment and with Refs. [8,12].

TABLE II. Triton binding energy B and scattering lengths $a_{1/2}$ and $a_{3/2}$ compared with the corresponding experimental values.

Quantity	Theory	Experiment
B (MeV)	7.72	8.48
$a_{1/2}$ (fm)	1.13	0.65 ± 0.04
$a_{3/2}$ (fm)	6.40	6.35 ± 0.02

The reason why our calculation agrees well with experiment in the case of the $J = 3/2$ channel but not in the case of the $J = 1/2$ channel is that the former is determined by the pure S-wave configuration $\ell = \lambda = 0$ whereas the latter has important contributions from higher angular momentum two-body channels and from three-body forces, both of which are lacking into our model. This is very similar to the situation encountered in effective field theory where the $J = 3/2$ channel is very well explained by S-wave models without three-body forces [20,21] whereas for the $J = 1/2$ channel the theory can produce sensible results only when a three-body force is included [22,23].

In summary, we conclude that the nonlocal NN interaction obtained from the chiral quark model gives results for the neutron-deuteron scattering lengths that are comparable to those obtained from conventional meson-exchange models.

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