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Dependence of the wave function of a bound nucleon on its momentum and the EMC effect

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It is widely discussed in the literature that the wave function of the nucleon bound in a nucleus is modified by the interaction with the surrounding medium. We argue that the modification should strongly depend on the momentum of the nucleon. We study such an effect in the case of the pointlike configuration component of the wave function of a nucleon bound in a nucleus A, considering the case of arbitrary final states of the spectator $A-1$ system. We show that for nonrelativistic values of the nucleon momentum, the momentum dependence of the nucleon deformation appears to follow from rather general considerations and discuss the implications of our theoretical observation for two different phenomena: (i) the search for medium-induced modifications of the nucleon radius of a bound nucleon through the measurement of the electromagnetic nucleon form factors via the $A(e,e'p)X$ process and (ii) the A dependence of the EMC effect.

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I. INTRODUCTION

One of the main challenges in particle physics, nuclear physics, and astrophysics is the necessity to achieve unambiguous understanding of the physics of cold dense nuclear matter, the limiting mass and radius of neutron stars, and many other questions that deeply interconnect astrophysics with the physics of particles and nuclei. To this end, it is important to reliably evaluate the modifications of the wave function of a nucleon embedded in dense nuclear matter. The laboratory investigation of the quark-gluon structure of a nucleon bound within a nucleus may help to quantify this important phenomenon. Twenty-five years after the discovery of the suppression of the nucleus structure function as compared to that for a free nucleon at moderate value of the Bjorken scaling variable x—the EMC effect (see, e.g., Ref. [1])—its origin remains a matter of considerable discussion. Various effects have been advocated to explain it, including (a) the modification of the bound nucleon structure function owing to (i) possible change of nonperturbative QCD scale in nuclei [2], (ii) meson-nucleon interactions [3], (iii) and the dependence of the strength of the nucleon interaction upon the size of the quark-gluon configuration, which leads to oscillations of this effect as the function of nuclear density [4]; (b) the presence of non-nucleonic dynamic degrees of freedom in the nucleus (see, e.g., Ref. [5]), which carry a fraction of the total nucleus momentum, leading to depletion from one of the fraction of nucleus momentum carried by nucleons; and (c) relativistic effects resulting from nucleon binding and Fermi motion [6,7] and models using the Bethe-Salpeter vertex function as nucleus wave function [8]. Experiments at Jlab, especially after the 12-GeV upgrade, will be able to break the deadlock through a series of dedicated experiments (see, e.g., Ref. [9]).

A certain restriction on the models follows from the investigation of the Drell-Yan process [10], which found no enhancement of the antiquark distribution in nuclei. It appears difficult to explain this fact within models where the non-nucleonic degrees of freedom are mesons. The conclusion, which follows solely from the requirements of baryon charge and momentum conservation, is that the EMC effect signals the presence of non-nucleonic degrees of freedom in nuclei, though it is not yet clear which are the most relevant ones.

A possibility, which is discussed in a number of models of the nucleon, is that the bound nucleon wave function is deformed by the presence of nearby nucleons. A distinctive property of QCD, which is a consequence of color gauge invariance, is that different components of the hadron wave function interact with different strengths. The extreme example is the pointlike configurations (PLCs) in hadrons, which have interaction strength much smaller than average, leading to the phenomenon of color transparency (for a recent review see, e.g., Ref. [11]). This phenomenon has been recently observed for the process of coherent high-energy pion dissociation into two jets [12], with various characteristics of the process consistent with the original QCD predictions [13]. There is also evidence for color transparent (CT) effects in quasielastic production of ρ mesons off nuclei, investigated first by the E665 experiment at FNAL [14] and, recently, by the HERMES experiment at DESY [15]. The data of HERMES are well described by a model [16] that takes into account the squeezing of the $q\bar{q}$ state in the production vertex and its expansion while propagating through the nucleus.

We will consider in this paper the effect of the suppression of PLCs in bound nucleons, extending the analysis to the case of a nucleon bound in ^{3}He and in complex nuclei, with the spectator system being in specific energy states. Similar to Refs. [4,17] we find that the effect of suppression of PLCs strongly depends on the momentum of the nucleon. Moreover, by taking into account the energy state of the spectator $A-1$ system, we find a new effect, namely a strong dependence of the effect on the excitation energy of the residual system. Overall the analysis of the derived formula allows us to establish the connection with another language, which explores the concept of off-mass-shell particles. In fact, we find that the effect depends on the virtuality of the interacting nucleon defined via the kinematics of the spectator system.

More generally, we will argue that a strong dependence of the deformation of the nucleon wave function upon the momentum is a general phenomenon in the lowest order over p^2/μ^2, where p is the nucleon momentum and $\mu \sim 0.5$–1 GeV is the strong interaction scale. We also discuss two implications of this argument; the first one is the need to look for the deviations of the bound nucleon electromagnetic (e.m.) form factors from the free ones as a function of the struck nucleon momentum; the second one is the estimate of the A dependence of the EMC effect through the energy binding and the mean excitation energies of nuclei. Our paper is organized as follows: In Sec. II we briefly review the arguments concerning the reduction of PLCs in the nuclear medium and extend the analysis of Refs. [4,17] to three- and many-body nuclei; the connection between nucleon virtuality and the suppression of PLCs is presented in Sec. III; the effects of the reduction of PLCs in quasielastic and deep inelastic scattering is discussed in Sec. IV; in Sec. V the results of our calculations of the EMC effect in nuclei, including the deuteron, are presented; conclusions are given in Sec. VI.

II. THE SUPPRESSION OF POINTLIKE CONFIGURATIONS IN NUCLEI

We will consider how the wave function of a bound nucleon is modified by medium effects. Our approach implements a well-understood and established property of perturbative quantum chromodynamics (pQCD): If the collision energy is not too large, the interaction between hadrons is proportional to their size. This property relies on the nonrelativistic Schrödinger equation for the nucleus wave function, which describes the motion of centers of mass of the nucleons. Some notations are therefore in order. The Schrödinger equation for a nucleus composed of A nucleons interacting via two-body interactions is

$$H_A \Psi_A^f = \left[\sum_i \frac{\mathbf{p_i^2}}{2m_N} + \sum_{i<j} V_{ij} \right] \Psi_A^f = E_A^f \Psi_A^f, \qquad (1)$$

where the index $f \equiv \{0, 1, 2, \ldots\}$ denotes the excitation spectrum of the system. (Note that from now on the ground-state energy and wave functions will be simply denoted by E_A and Ψ_A, instead of $E_A^{(0)}$ and $\Psi_A^{(0)}$; moreover, in case of the deuteron, instead of $A = 2$ we will simply use the notation D). We will consider a nucleon with four-momentum $p \equiv (E_p, \mathbf{p})$ and denote the center-of-mass four-momentum of the *spectator* $(A-1)$ nucleons as p_s or P_{A-1}. The mass of the nucleon will be denoted by m_N. We will also need to define the energy necessary to remove a nucleon from a nucleus A, leaving the residual system in a state with intrinsic (positive) excitation energy E_{A-1}^f; such a quantity is the (positive) *removal energy* defined as $E = E_{\rm min} + E_{A-1}^f$ with $E_{\rm min} = |E_A| - |E_{A-1}|$, with E_A and E_{A-1} being the (negative) ground-state energies of A and $A-1$ systems, respectively. Eventually, the ground-state energy per particle will be denoted by $\epsilon_A = E_A/A$.

A. General considerations

In QCD the Fock space decomposition of the hadron wave function contains components of the size much smaller than the average size of the hadron; these components determine, at $Q^2 \to \infty$, the asymptotic behavior of the elastic hadron form factors and, for a pion, they were explicitly observed in the exclusive dijet production [12].

It was argued in Refs. [4,17] that because the small size configurations of the bound nucleon experience a smaller nucleon attraction, their probability should be smaller in the bound state since such a reduction would lead back to an increase of the nuclear binding. The discussed effect was formally described by an expression obtained within the closure approximation [4,17,18].

The reduction of PLCs might be relevant for the explanation of the EMC effect, though only in a restricted region of the Bjorken scaling variable $x = \frac{Q^2}{2m_N\nu}$. Indeed, it has been predicted by several models (see, e.g., Refs. [4,19]) that the behavior of the structure functions at $x \to 1$ should be sensitive to the small-size quark gluon configurations. One general argument is that at moderate values of the four-momentum transfer Q^2, PLCs compete in elastic form factors with the end-point contribution, which is also but gradually sqized with an increase of Q^2 as a consequence of Sudakov-type form factors, which, on the other side, are connected to the inclusive structure functions at $x \to 1$ via the Drell-Yan-West relation (see the discussion in Refs. [20,21]). Another argument, mostly relevant for the nucleon parton density, is that large size configurations with the pion cloud do not contribute at $x \simeq 1$, since in these configurations pions carry a significant part of the total light-cone momentum of the nucleon.

The key characteristic of PLCs, which allowed one to derive a compact expression for their modification in the bound nucleon, is that the potential energy associated with the interaction of a PLC is much smaller than V_{ij}, the NN potential averaged over *all* configurations. Using the decomposition of the PLC over the hadronic states and the closure approximation one finds for the nuclear wave function, which includes the

PLC in the nucleon i, the following expression [4,17]:

$$\tilde{\psi}_A(i) \approx \left(1 + \sum_{j\neq i} \frac{V_{ij}}{\Delta E^{(N/A)}}\right) \psi_A(i), \tag{2}$$

where $\psi_A(i)$ is the usual wave function with all nucleons, including i, in average configurations, and $\Delta E^{(N/A)} \sim m_{N^*} - m_N \sim 600$–$800$ MeV parametrizes the energy denominator depending upon the average virtual excitation of a nucleon N in the nucleus A.

B. The deuteron

By using the equations of motion for ψ_A, the momentum dependence of the probability to find a bound nucleon with momentum p in a PLC was obtained in Refs. [4,17] within the mean-field and two-nucleon correlation approximations. In particular, for the deuteron the Shrödinger equation in momentum representation leads to

$$V_{12} = -2\frac{\mathbf{p}^2}{2m_N} + E_D, \tag{3}$$

where E_D is the (negative) binding energy of the deuteron. Using the same closure approximation and equation of motions for the higher order terms in $\frac{V_{ij}}{\Delta E}$ (assuming that ΔE is approximately the same for the higher order terms) one obtains the suppression of the probability of PLCs in the deuteron for a nucleon with momentum $\mathbf{p}$:

$$\delta_D(\mathbf{p}) = \left(1 + \frac{2\frac{\mathbf{p}^2}{2m_N} - E_D}{\Delta E^{(N/D)}}\right)^{-2}. \tag{4}$$

Thus, if for a given x PLCs dominate in the nucleon parton distribution functions, the structure function of the bound nucleon would be suppressed by a factor given by Eq. (4), that is,

$$\begin{aligned} F_D(x, \mathbf{p}, Q^2) &\simeq \delta_D(\mathbf{p}) F_{2N}(x, Q^2) \\ &\simeq \left(1 + \frac{2\frac{\mathbf{p}^2}{2m_N} - E_D}{\Delta E^{(N/D)}}\right)^{-2} F_{2N}(x, Q^2). \end{aligned} \tag{5}$$

Note that Eq. (5) can equally well be applied to the semi-inclusive process when the transition to one particular final state of the spectator is considered. In the derivation of the previous formulas, it has been assumed that for a PLC $|V_{ij}^{(\mathrm{PLC})}(x)|/|V_{ij}| \ll 1$. If one probes large values of x, for which $|V_{ij}^{(\mathrm{PLC})}(x)|/|V_{ij}| = \lambda(x) < 1$, the suppression factor will be obviously smaller and, in the lowest order in $p^2/2m_N\Delta E$, Eq. (4) will be modified as follows:

$$\delta_D(\mathbf{p}) = \left(1 + [1-\lambda(x)]\frac{2\frac{\mathbf{p}^2}{2m_N} - E_D}{\Delta E^{(N/D)}}\right)^{-2}. \tag{6}$$

If the dominant nucleon configurations in F_{2N} interact with a strength substantially smaller than the average (say, $\lambda \leqslant 0.5$) for $x \geqslant 0.5$–0.6, the PLC suppression may help explain the EMC effect. Because we are interested in this paper in the A dependence of the deviation of the EMC ratio from one, for ease of presentation, we will simply use $\lambda(x) = 0$ in what follows, though in the comparison with experimental data, to be presented in Sec. V, a value of $\lambda(x) \neq 0$ has been used.

A key test of the PLC suppression is the study of the tagged structure functions [4,22–26].

C. The three-body nuclei

Let us consider now three-body nuclei. In this case we face a more complicated situation owing to several possible final states of the two-body spectator system. For this reason, the suppression of PLCs will depend upon the transition densities between the wave function (2) and the final-state wave functions. Therefore the full nuclear spectral function of ^{3}He is required to evaluate $\delta_{^3\mathrm{He}}$. Let us discuss this point in detail. To this end, we first introduce the wave function ϕ_2, the solution of the two-body Schrödinger equation for nucleons 2 and 3:

$$(\hat{T}_2 + \hat{T}_3 + V_{2,3})\phi_2^f(2,3) = E_2^f \phi_2^f(2,3), \tag{7}$$

where T is the operator for the kinetic energy and f labels the quantum numbers of the state, which can be either the ground (D) or the continuum (pn) states of a neutron-proton pair or the continuum (pp) state of a neutron-neutron pair. [Note that Eq. (7) has the same spectrum as the final two-nucleon state in the case of deep inelastic scattering on ^{3}He (i.e., D along with pn and pp in the continuum).] Then the relevant quantities are the following densities:

$$\begin{aligned} &\phi_1^\dagger(1)\phi_2^{f\dagger}(2,3)\tilde{\psi}_3(1,2,3) \\ &\quad = \phi_1^\dagger(1)\phi_2^{f\dagger}(2,3)\left(1 + \frac{V_{1,2}+V_{1,3}}{\Delta E^{(N/3)}}\right)\psi_3(1,2,3), \end{aligned} \tag{8}$$

where $\phi_1(1)$ is the wave function of the struck nucleon. By considering the full three-nucleon Shrödinger equation

$$\begin{aligned} &(\hat{T}_1 + \hat{T}_2 + \hat{T}_3 + V_{2,3} + V_{1,3} + V_{1,2})\psi_3(1,2,3) \\ &\quad = E_3\psi_3(1,2,3), \end{aligned} \tag{9}$$

we obtain

$$\begin{aligned} &\phi_1^\dagger(1)\phi_2^{f\dagger}(2,3)(V_{1,3}+V_{1,2})\psi_3(1,2,3) \\ &\quad = \left(E_3 - E_2^f - T_1\right)\phi_1^\dagger(1)\phi_2^{f\dagger}(2,3)\psi_3(1,2,3). \end{aligned} \tag{10}$$

The density on the right-hand side (r.h.s.) of this equation defines the channels f of the spectral function of ^{3}He, to be denoted $P_3^{(f)}(|\mathbf{p}|, E)$ [27]. For the three different channels we obtain (and for the inclusive process we sum over the states of the spectator, f)

$$\delta_3^{(D)}(\mathbf{p}) \simeq \left(1 + \frac{|E_3| - |E_D| + \frac{3\mathbf{p}^2}{4m_N}}{\Delta E^{(N/3)}}\right)^{-2}, \tag{11}$$

$$\delta_3^{(pn)}(\mathbf{p},\mathbf{k}) \simeq \left(1 + \frac{|E_3| + \frac{\mathbf{k}^2}{m_N} + \frac{3\mathbf{p}^2}{4m_N}}{\Delta E^{(N/3)}}\right)^{-2}, \tag{12}$$

$$\delta_3^{(pp)}(\mathbf{p},\mathbf{k}) \simeq \left(1 + \frac{|E_3| + \frac{\mathbf{k}^2}{m_N} + \frac{3\mathbf{p}^2}{m_N}}{\Delta E^{(N/3)}}\right)^{-2}, \quad (13)$$

where we neglected the difference of the proton and neutron masses and also a possible isospin dependence of $\Delta E^{(N/3)}$. Here $\mathbf{k}$ is the momentum of the nucleon in the spectator pair, in the pair's c.m. frame. In terms of *removal energies* we have $E_{\rm min} = |E_3| - |E_2|$ for the process ^{3}He $\to p + D$, and $E_{\rm min} = |E_3|$ for the process ^{3}He $\to p + (pn)$; the corresponding excitation energies are $E_{A-1}^f = 0$ and $E_{A-1}^f = \mathbf{k}^2/m_N$, respectively. Thus Eqs. (11), (12), and (13) can be unified as

$$\delta_3^{(f)}(\mathbf{p},E) \simeq \left(1 + \frac{E + \frac{3\mathbf{p}^2}{4m_N}}{\Delta E^{(N/3)}}\right)^{-2}, \quad (14)$$

where $E = E_{\rm min} + E_2^f$ generates the dependence upon f of the r.h.s.. We will need in what follows the average value of $\delta_3^{(f)}(\mathbf{p},E)$ with respect to $\mathbf{p}$ and E. This can be obtained, provided the nucleon spectral function in channel f, $P_3^{(f)}(|\mathbf{p}|,E)$, is known; in such a case one has

$$\langle\delta_3^{(f)}(\mathbf{p},E)\rangle = \int \delta_3^{(f)}(\mathbf{p},E)P_3^{(f)}(|\mathbf{p}|,E)\,dE\,d\mathbf{p}. \quad (15)$$

Following Ref. [27], we will label various quantities pertaining to the channel ^{3}He $\to D + p$ with the superscript (gr) and quantities pertaining to the channels ^{3}He $\to p + (np)$ and ^{3}He $\to n + (pp)$ with the superscript ex. The corresponding spectral functions will be denoted $P_3^{(D)}(\mathbf{p},E) \equiv P_3^{(\rm gr)}(\mathbf{p},E)$ and $P_3^{(NN)}(\mathbf{p},E) \equiv P_3^{(\rm ex)}(\mathbf{p},E)$.

The spectral functions in different channels f are normalized as follows:

$$\int P_3^{(f)}(|\mathbf{p}|,E)d\,\mathbf{p}\,dE = S_f, \quad (16)$$

where $f = \{\rm gr, ex\}$.

The spectral function of ^{3}He is described in detail in the Appendix and the mean values of various quantities calculated with a realistic spectral function of ^{3}He [28] are listed in Table I.

D. General case

Equation (14) can be readily generalized to the case of an arbitrary nucleus A, giving

$$\delta_A^{(f)}(\mathbf{p},E) \simeq \left(1 + \frac{E + \frac{A}{A-1}\frac{\mathbf{p}^2}{2M}}{\Delta E^{(N/A)}}\right)^{-2}. \quad (17)$$

It is also trivial to modify Eq. (17) to account for the case of small but finite size configurations by introducing a factor $1 - \lambda(x)$ as in Eq. (6). The evaluation of the average values of Eq. (17),

$$\langle\delta_A^{(f)}(\mathbf{p},E)\rangle = \int \delta_A^{(f)}(\mathbf{p},E)P_A^{(f)}(|\mathbf{p}|,E)\,dE\,d\mathbf{p}, \quad (18)$$

requires knowledge of the spectral function of the nucleus A in channel f, $P_A^{(f)}(|\mathbf{p}|,E)$. As illustrated in Refs. [7,29], the spectral function of a complex nucleus can be written in the form

$$P_A(|\mathbf{p}|,E) = P_0(|\mathbf{p}|,E) + P_1(|\mathbf{p}|,E), \quad (19)$$

where $P_0(|\mathbf{p}|,E)$ describes the transition to the ground state and to the discrete shell-model statesof the nucleus $A-1$, whereas $P_1(|\mathbf{p}|,E)$ is responsible for the transitions to the whole of the continuum states generated by short-range nucleon-nucleon correlations. For complex nuclei we will consider two average values for the suppression of PLCs, namely

$$\langle\delta_A^{(0)}(\mathbf{p},E)\rangle \simeq \int \delta_A^{(0)}(\mathbf{p},E)P_0(|\mathbf{p}|,E)\,dE\,d\mathbf{p} \quad (20)$$

and

$$\langle\delta_A^{(1)}(\mathbf{p},E)\rangle \simeq \int \delta_A^{(1)}(\mathbf{p},E)P_1(|\mathbf{p}|,E)\,dE\,d\mathbf{p}, \quad (21)$$

where $f = \{0, 1\}$ plays, in a sense, the role of $f = $ gr, ex in the case of ^{3}He. The mean values of various quantities pertaining to complex nuclei calculated with the spectral function of Ref. [29] are reported in Table II.

TABLE I. The normalization factors S_f [Eq. (16)], the mean kinetic $\langle T\rangle$ and removal $\langle E\rangle$ energies, and the energy per nucleon $|\epsilon_3|$ for helium-3, calculated with the spectral function of Ref. [28] and the Pisa Group wave function [39] corresponding to the AV18 interaction [40]. The state $f = $ gr corresponds to the spectator proton-neutron system in the ground state (a deuteron), whereas the state $f = $ ex corresponds to the proton-neutron or proton-proton systems in the continuum.

	Norm, S			$\langle T\rangle$ (MeV)			$\langle E\rangle$ (MeV)			$\lvert\epsilon_3\rvert$ (MeV)		
	gr	ex	total[a]	gr	ex	total[a]	gr	ex	total[a]	gr	ex	total[a]
Proton	0.65	0.35	1	4.67	8.60	13.27	3.72	6.81	10.53	0.69	1.26	1.95
Neutron	0	1	1	0	17.69	17.69	0	16.33	16.33	0	3.74	3.74
Per nucleon[b]	–	–	1	3.11	11.63	14.74	2.48	9.99	12.47	0.46	2.09	2.55[c]

[a]Total = gr + ex.
[b]Per nucleon = (2 protons + neutron)/3.
[c]$3 \times \epsilon_3 = E_3 \approx 7.7$ MeV, the value computed in Ref. [39].

TABLE II. The same as in Table I but for the deuteron and complex nuclei. The results for $A = 2$ correspond to the AV18 interaction and the ones for $4 \leqslant A \leqslant 208$ to the spectral function of Ref. [29].

A	S_0	S_1	$\langle T\rangle_0$ (MeV)	$\langle T\rangle_1$ (MeV)	$\langle T\rangle$ (MeV)	$\langle E\rangle_0$ (MeV)	$\langle E\rangle_1$ (MeV)	$\langle E\rangle$ (MeV)	$\|\epsilon_A\|$ (MeV)
D	1.0	–	–		11.07			2.226	1.113
^{4}He	0.8	0.2	8.23	17.55	25.78	15.85	19.20	35.05	8.93
^{12}C	0.8	0.2	13.54	18.93	32.47	18.40	26.55	44.95	7.72
^{16}O	0.8	0.2	11.22	19.73	30.95	19.42	27.20	46.62	8.87
^{40}Ca	0.8	0.2	13.39	20.45	33.84	21.28	28.57	49.85	8.44
^{56}Fe	0.8	0.2	11.45	21.26	32.71	20.00	29.06	49.06	8.47
^{208}Pb	0.8	0.2	14.72	24.40	39.12	18.53	34.79	53.32	7.19

III. NUCLEON VIRTUALITY, THE SUPPRESSION OF PLCs, AND THE VARIATION OF NUCLEON PROPERTIES IN THE MEDIUM

A. Nucleon virtuality

Let us now consider the interaction of a bound nucleon with a virtual photon. The virtuality of the interacting nucleon v is as follows

$$v = p^2 - m_N^2 = (P_A - P_{A-1})^2 - m_N^2. \quad (22)$$

In impulse approximation ($\mathbf{p} = -\mathbf{P}_{A-1}$) we have

$$\begin{aligned} v(|\mathbf{p}|, E) &= (P_A - P_{A-1})^2 - m_N^2 \\ &= \left(M_A - P_{A-1}^{(0)}\right)^2 - \mathbf{p}^2 - m_N^2 \\ &= \left(M_A - \sqrt{(M_A - m_N + E)^2 + \mathbf{p}^2}\right)^2 - \mathbf{p}^2 - m_N^2. \end{aligned} \quad (23)$$

The nonrelativistic reduction of Eq. (23) in the rest frame of the nucleus A, which corresponds to neglecting higher order terms in $\sim\frac{E}{m_N}$ and $\sim\frac{T_{A-1}}{m_N}$, yields

$$v_{\rm NR}(|\mathbf{p}|, E) \approx -2m_N \left(\frac{A}{A-1}\frac{\mathbf{p}^2}{2m_N} + E\right). \quad (24)$$

It can therefore be seen that in the nonrelativistic limit the argument of $\delta_A(|\mathbf{p}|, E)$, for any A, is the same as the nonrelativistic reduction of the virtuality $v_{\rm NR}$, so that the suppression of PLCs can be expressed in terms of the nucleon virtuality as

$$\delta_A(|\mathbf{p}|, E) = \left(1 - \frac{v_{\rm NR}(|\mathbf{p}|, E)}{2\,m_N \Delta E^{(N/A)}}\right)^{-2}, \quad (25)$$

with $v_{\rm NR}(|\mathbf{p}|, E)$ given by Eq. (23). Note that using the Koltun sum rule [30] corresponding to a Hamiltonian containing only two-body forces, that is,

$$2|\epsilon_A| = \langle E\rangle - \langle T\rangle \frac{A-2}{A-1}, \quad (26)$$

where $\langle T\rangle$ and $\langle E\rangle$ are the average kinetic and removal energies per particle, respectively, one gets

$$\langle v_{\rm NR}\rangle = -2m_N \left(\frac{A}{A-1}\frac{\langle \mathbf{p}^2\rangle}{2m_N} + \langle E\rangle\right) = -4m_N[\langle T\rangle + |\epsilon_A|], \quad (27)$$

so that the average value of Eq. (17) [or Eq. (25)] can be written as follows:

$$\langle \delta_A(|\mathbf{p}|, E)\rangle = \left\langle \left(1 + \frac{E + \frac{A}{A-1}\frac{\mathbf{p}^2}{2m_N}}{\Delta E^{(N/A)}}\right)^{-2} \right\rangle. \quad (28)$$

Since our derivation was nonrelativistic one cannot distinguish the cases when $v_{\rm NR}$ or v are used. Hence, to check the sensitivity to the higher order terms, we will also consider an expression for δ_A in which v is used instead of $v_{\rm NR}$:

$$\langle \delta_A(|\mathbf{p}|, E)\rangle_v = \left\langle \left(1 - \frac{v(|\mathbf{p}|, E)}{2m_N \Delta E^{(N/A)}}\right)^{-2} \right\rangle, \quad (29)$$

with $v(|\mathbf{p}|, E)$ given by Eq. (23).

Eventually, we will also consider the partial virtualities (i.e., the virtuality in a given state f), defined as

$$\left\langle v_{\rm NR}^{(f)}\right\rangle = -2m_N \left(\frac{A}{A-1}\langle T\rangle_f + \langle E\rangle_f\right), \quad (30)$$

and the corresponding partial coefficient of suppression of PLCs, that is,

$$\left\langle \delta_A^{(f)}(|\mathbf{p}|, E)\right\rangle_f = \left\langle \left(1 + \frac{E + \frac{A}{A-1}\frac{\mathbf{p}^2}{2m_N}}{\Delta E^{(N/A)}}\right)^{-2} \right\rangle_f, \quad (31)$$

satisfying

$$\langle \delta_A(|\mathbf{p}|, E)\rangle = \sum_f \left\langle \delta_A^{(f)}(|\mathbf{p}|, E)\right\rangle, \quad (32)$$

where the average in Eq. (31) has to be taken with the proper partial spectral function.

If we expand Eq. (28), we get its lowest order (LO) approximation in $(\mathbf{p}/m_N)^2$:

$$\langle \delta_A(|\mathbf{p}|, E)\rangle_{\rm LO} \approx \left(1 - \frac{4\left(\langle T\rangle + |\epsilon_A|\right)}{\Delta E^{(N/A)}}\right). \quad (33)$$

Let us now address the physical reasons for the derived structure of Eq. (25).

It is often discussed in the literature that various properties of a nucleon bound in the nucleus should be modified because of the interactions with the surrounding nucleons (usually referred to as *medium modifications*). Such a possibility has been considered for the case of the electromagnetic nucleon form factors, the parton densities, and other quantities (see, e.g., Refs. [4,31,32]). Medium modifications are theoretically often considered within the mean-field approximation and are assumed to depend on the mean nuclear density, with an implicit assumption that the modification does not depend on the momentum of the nucleon. However, we have just shown that the contribution of PLCs exhibit a strong momentum dependence arising naturally from the reduction of the interaction strength. Accordingly, one expects that in this model the modification of, for example, the radius of a bound nucleon, may also depend upon the nucleon momentum. One intuitively expects that possible modifications of the properties of a bound nucleon should depend upon its off-shellness, which can be expressed in terms of the nucleon virtuality as defined by Eq. (22).

To elucidate this point, let us consider the electrodisintegration of the deuteron, $e\,D \to e\,pn$, as a function of the momentum of the spectator nucleon p_s. (Another option would be to consider deep inelastic scattering (DIS) off the deuteron in a tagged mode, that is, when the spectator momentum is detected.) The amplitude $\mathcal{A}(\gamma^* + D \to pn)$ is an analytic function of the Mandelstam variables, for example,

$$t = (p_D - p_s)^2, \tag{34}$$

that is, the square of the momentum transfer, and therefore it can be expressed as a series in terms of the variable $m_N^2 - t$. The continuation to the pole $t = m_N^2$ of the propagator of the interacting nucleon would correspond to the interaction between γ^* and a free nucleon (analogous to the case of the Chew-Low theorem relating the amplitude of the process $\pi + N \to \pi\pi N$ to the $\pi - \pi$ scattering amplitude [33]). Hence, for small enough values of $m_N^2 - t$, the effect of medium modifications are expected to be proportional to

$$m_N^2 - t = m_N^2 - (p_d - p_s)^2 = (\mathbf{p^2} - m_N\,E_D) + O(\mathbf{p}^4, \mathbf{p}^2\,E_D, E_D\,\mathbf{p}^2), \tag{35}$$

which is exactly the functional dependence of Eq. (4). Our reasoning is heavily based upon the analyticity of the amplitude in the t variable, which justifies the validity of the Taylor expansion near the nucleon pole in terms of powers of $(t - m_N^2)$. Obviously, our argument can be applied to the scattering off heavier nuclei, provided the residual $(A-1)$ nucleon system has small enough momentum and excitation energy. In this case the relevant perturbation parameter γ_A is given by Eq. (25), that is,

$$\gamma_A(|\mathbf{p}|, E) = -(P_A - p_s)^2 + m_N^2 \approx 2m\left(\frac{A}{A-1}\frac{\mathbf{p}^2}{2m_N} + E\right) = -v(|\mathbf{p}|, E). \tag{36}$$

In the leading order of perturbation theory over binding effects Eq. (28) for the average value of δ_A is recovered by using the Koltun sum rule

$$\langle\delta_A(|\mathbf{p}|, E)\rangle = \left\langle\left(1 + \frac{\gamma_A(|\mathbf{p}|, E)}{2m_N\Delta E^{(N/A)}}\right)^{-2}\right\rangle \approx \left(1 - 4\,\frac{\langle T\rangle + |\epsilon_A|}{\Delta E^{(N/A)}}\right). \tag{37}$$

B. PLCs and the variation of nucleon properties in the medium

Denoting by $\kappa(|\mathbf{p}|, E) - 1$ the deviation from one of the ratio of a certain characteristic (say, the structure functions or the radii) of the bound nucleon to its vacuum value, we can expect that it will be given by

$$\kappa(|\mathbf{p}|, E) - 1 = \frac{\gamma(|\mathbf{p}|, E)}{m_N^2}. \tag{38}$$

Therefore we can write

$$\frac{\kappa(|\mathbf{p}|, E) - 1}{\langle\kappa(|\mathbf{p}|, E)\rangle - 1} = \frac{\gamma(|\mathbf{p}|, E)}{\langle\gamma(|\mathbf{p}|, E)\rangle} = \frac{E + \frac{A}{A-1}\frac{\mathbf{p}^2}{2m}}{2[\langle T\rangle + |\epsilon_A|]} = \frac{2E\,m_N + \mathbf{p}^2\frac{A}{A-1}}{4m_N[\langle T\rangle + |\epsilon_A|]}, \tag{39}$$

which does not depend on the value of ΔE or on the strength of the interaction for the probed property. It follows, from this relation, that the region of small nucleon momenta and small excitation energies is the least sensitive to the effects of possible modifications of the nucleon properties. Hence such a region is suitable for the extraction of the properties of the free neutron from scattering processes off the deuteron and ^{3}He using the analog of the Chew-Low procedure (see, e.g., the discussion in Ref. [9]). For the same reason, the 3% upper limit for the change of the magnetic nucleon radius obtained from the analysis of the Q^2 dependence of the inclusive (e, e') cross section near the quasielastic peak [34] implies a much weaker limit on the average change of the nucleon radius in nuclei. In fact, in inclusive (e, e') scattering, the cross section at the quasielastic peak ($x \simeq 1$) is proportional to $\int d(|\mathbf{p}|)|\mathbf{p}|n_A(|\mathbf{p}|)= \langle\frac{1}{|\mathbf{p}|}\rangle$, giving $\langle\mathbf{p}^2\rangle \simeq \langle|\mathbf{p}|\rangle\langle 1/|\mathbf{p}|\rangle$, whereas in DIS ($x < 1$), it is directly proportional to $\langle|\mathbf{p}|^2\rangle$; therefore, in quasielastic scattering the average value of the probed $\langle|\mathbf{p}|^2\rangle$ is significantly smaller than the corresponding quantity in DIS, roughly by the factor

$$C = \frac{\langle|\mathbf{p}|\rangle}{\langle|\mathbf{p}|^2\rangle\langle 1/|\mathbf{p}|\rangle}. \tag{40}$$

As we have demonstrated, the study of the momentum dependence of the properties of a bound nucleon may be a better tool for the investigation of modification effects in nuclei. This can be achieved, for example, by means of semi-inclusive processes (tagged structure functions) and by the measurement of the momentum dependence of the ratio of electric to magnetic nucleon form factors.

IV. SUPPRESSION OF PLCS IN INCLUSIVE SCATTERING: THE *A* DEPENDENCE OF THE EMC EFFECT

Let us now illustrate how the derived equations allow one to improve previous estimates of the A dependence of the EMC effect in a particular region of x. To this end let us consider the well-known EMC ratio for an isoscalar nucleus,

$$R_A(x, Q^2) \equiv \frac{A F_{2A}(x, Q^2)}{Z F_{2p}(x, Q^2) + N F_{2n}(x, Q^2)}, \tag{41}$$

where F_{2A} and F_{2N} are the nuclear and nucleon structure functions, respectively. We will consider in the Bjorken limit two models for the nuclear structure function F_{2A}, namely the light cone (LC) and the virtual nucleon convolution models. In both cases the proton and neutron spectral functions and momentum distributions are considered to be the same.

A. The light-cone quantum mechanical model

Light-cone quantum mechanics of nuclei is based on the following assumptions: (i) bound nucleons are on-shell; (ii) closure over final states is performed; and (iii) the light-cone momentum of the nucleus is entirely carried by nucleons. Within this model the nuclear structure function F_{2A} reads as follows:

$$F_{2A}^{\mathrm{LC}}(x, Q^2) = A \int_x^A \frac{d\alpha}{\alpha} d^2\mathbf{p}_\perp F_{2N}(x/\alpha, Q^2)\rho^{\mathrm{LC}}(\alpha, \mathbf{p}_\perp), \tag{42}$$

where $\alpha = \frac{A}{M_A} p_-$ ($d^4 p = \frac{M_A}{A} d\alpha d p_+ d^2\mathbf{p}_\perp$ with $p_\pm$ as the corresponding light-cone variables defined relative to the direction of the momentum transfer) is the light-cone fraction carried by the interacting nucleon scaled to vary between zero and A, and $\rho^{\mathrm{LC}}(\alpha, \mathbf{p}_\perp)$ is the nucleon LC density matrix normalized according to the baryon charge sum rule

$$\int_0^A \frac{d\alpha}{\alpha} d^2\mathbf{p}_\perp \rho^{\mathrm{LC}}(\alpha, \mathbf{p}_\perp) = 1 \tag{43}$$

and automatically satisfying the momentum sum rule

$$\int \alpha \frac{d\alpha}{\alpha} d^2\mathbf{p}_\perp \rho^{\mathrm{LC}}(\alpha, \mathbf{p}_\perp) = 1, \tag{44}$$

corresponding to $\langle\alpha\rangle = 1$. In the nonrelativistic approximation for the nucleon motion within a nucleus, LC quantum mechanics coincides with the conventional nuclear theory based on the nonrelativistic Schrödinger equation. An evident advantage of the LC mechanics is the accurate account of relativistic effects, including those related to pair production off vacuum resulting from the Lorentz transformation.

To calculate the effect of the suppression of PLCs in bound nucleons we have to substitute in the convolution formula F_{2N} by $F_{2N}^{\mathrm{bound}} = F_{2N}\,\delta_A(|\mathbf{p}|, E)$, (see Eq. (5)) obtaining

$$\frac{F_{2A}^{\mathrm{LC}(\delta)}(x, Q^2)}{A} = \int_x^A \frac{d\alpha}{\alpha} d^2\mathbf{p}_\perp \delta_A(|\mathbf{p}|, E) F_{2N}(x/\alpha, Q^2)\rho^{\mathrm{LC}}(\alpha, \mathbf{p}_\perp), \tag{45}$$

where $\delta_A(|\mathbf{p}|, E)$ is given by Eq. (17) [or Eq. (25)]. By expanding $F_{2N}(x/\alpha, Q^2)$ in Eq. (45) in a power series about $\alpha = 1$ one obtains

$$\frac{F_{2A}^{\mathrm{LC}}(x, Q^2)}{A} \simeq \langle\delta_A(|\mathbf{p}|, E)\rangle F_{2N}(x, Q^2) + x F'_{2N}(x, Q^2)\langle(\alpha-1)\rangle + \left[x F'_{2N}(x, Q^2) + \frac{x^2}{2} F''_{2N}(x, Q^2)\right]\langle(\alpha-1)^2\rangle, \tag{46}$$

where the averages have to be evaluated with the light-cone density $\rho^{\mathrm{LC}}(\alpha, \mathbf{p}_\perp)$. By considering that $\langle(\alpha-1)\rangle = 0$ [by Eqs. (43) and (44)] and taking the nonrelativistic limit to order $\frac{\langle\mathbf{p}^2\rangle}{m_N^2}$ of the third term, the average values can be evaluated with the nonrelativistic momentum distributions, obtaining $\langle(\alpha-1)^2\rangle = \langle\mathbf{p}^2\rangle/3m_N^2$, so that

$$\frac{F_{2A}^{\mathrm{LC}(\delta)}(x, Q^2)}{A} \simeq \langle\delta_A(|\mathbf{p}|, E)\rangle F_{2N}(x, Q^2) + \left[x F'_{2N}(x, Q^2) + \frac{x^2}{2} F''_{2N}(x, Q^2)\right] \frac{2\langle T\rangle}{3m_N}. \tag{47}$$

Supposing that the behavior of $F_2(x)$ at $x \leqslant 0.5$–0.7 is governed solely by u quarks [$u(x) \sim (1-x)^n$ with $n \sim 3$], one gets

$$R_A^{\mathrm{LC}(\delta)}(x, Q^2) = \frac{F_{2A}^{\mathrm{LC}(\delta)}(x, Q^2)}{A\, F_{2N}(x, Q^2)} \simeq \langle\delta_A(|\mathbf{p}|, E)\rangle + nx \frac{x(n+1)-2}{6(1-x)^2} \frac{2\langle T\rangle}{m_N}, \tag{48}$$

yielding the result of Ref. [35] when $\delta_A = 1$. For $n = 3$ we obtain

$$R_A^{\mathrm{LC}}(x, Q^2) = \frac{F_{2A}^{\mathrm{LC}}(x, Q^2)}{A\, F_{2N}(x, Q^2)} \simeq 1 + x \frac{(2x-1)}{(1-x)^2} \frac{2\langle T\rangle}{m_N}, \tag{49}$$

leading to a cancelation of the Fermi motion effects for $x = 1/2$. Hence $x \sim 0.5$ is especially convenient for the analysis, for one has, provided $\mathbf{p}$ is not very large,

$$R_A^{\mathrm{LC}(\delta)}(x \sim 0.5) \propto \langle\delta_A(|\mathbf{p}|, E)\rangle \simeq \left\langle \left(1 + \frac{\gamma_A(p)}{2\, m_N \Delta E^{(N/A)}}\right)^{-2} \right\rangle \simeq 1 - 4\frac{|\epsilon_A| + \langle T\rangle}{\Delta E^{(N/A)}}. \tag{50}$$

When $\delta_A = 1$, one has, obviously,

$$R_A^{\mathrm{LC}}(x \sim 0.5) \simeq 1. \tag{51}$$

We remind the reader that, to simplify the discussion, we have up to now placed $\lambda(x) = 0$ on the r.h.s. of Eq. (6). Now, to compare our calculations with the experimental data, we necessitate an explicit consideration of the value $\lambda(x)$, so we will use the following expression for the ratio $R_A^{\mathrm{LC}(\delta)}$:

$$R_A^{\mathrm{LC}(\delta)}(x \sim 0.5) \simeq 1 - [1 - \lambda(0.5)]\, 4\frac{\langle T\rangle + |\epsilon_A|}{\Delta E^{(N/A)}}. \tag{52}$$

B. The virtual nucleon convolution model

Another approach to the description of the DIS is the approach in which the role of the nuclear wave function is played by the covariant vertex function described by appropriate Feynman diagrams. In the case of DIS this approximation is usually referred to as the virtual nucleon convolution (VNC) model. In this approximation the interacting nucleon is off-shell and the light-cone fraction carried by the interacting nucleon can be expressed through the laboratory-frame momentum and energy of the residual system as

$$z = \frac{A}{M_A}(p_0 - p_3), \tag{53}$$

where

$$p_0 = M_A - \sqrt{(M_A - m_N + E)^2 + \mathbf{p}^2}. \quad (54)$$

The nuclear structure function has the following form:

$$F_{2A}^{\rm VNC}(x, Q^2)/A = \int F_{2N}(x/z,\ Q^2) f_A(z) dz, \quad (55)$$

with the longitudinal momentum distributions f_A^N given by

$$f_A(z) = \int d^4 p S_A(p) z\delta\left(z - \frac{A}{M_A}[p_0 - p_3]\right). \quad (56)$$

The relation between the relativistic, $S_A(p)$, and nonrelativistic, $P_A(|\mathbf{p}|, E)$, spectral functions is, to order $(|\mathbf{p}|/m_N)^2$,

$$S_A(p) = P_A(|\mathbf{p}|, E)\left[1 + \mathcal{O}\left(\frac{|\mathbf{p}|}{m_N}\right)^2 + \cdots\right], \quad (57)$$

and baryon charge conservation is enforced by properly normalizing $f_A(z)$, that is [7],

$$\int f_A(z) dz = C_A \int dE\, d\mathbf{p} P_A(|\mathbf{p}|, E)\, z\, dz \times \delta\left(z - \frac{A}{M_A}[p_o - |\mathbf{p}|\cos\theta_{\widehat{\mathbf{pq}}}]\right) = 1. \quad (58)$$

The light-cone fraction carried by the nucleons in this model is less than one [36]:

$$\langle z \rangle = \int z\, f_A(z) = 1 - \frac{\langle E \rangle - |\epsilon_A| + \langle T_R \rangle}{m_N} \equiv \eta < 1, \quad (59)$$

where $\langle T_R \rangle \simeq \langle \mathbf{p}^2 \rangle / 2(A-1)m_N$ and η is the total light-cone momentum carried by nucleons. The momentum sum rule is restored by assuming that non-nucleonic components carry the fraction $1 - \eta$ of the missing momentum. These components should be added explicitly to satisfy the momentum sum rule.

By expanding $F_{2N}(x/z,\ Q^2)$ in Eq. (55) about $z \sim 1$, we obtain, to order $\frac{\langle \mathbf{p}^2 \rangle}{m_N^2}$,

$$F_{2A}^{\rm VNC}(x, Q^2)/A \simeq F_{2N}(x, Q^2) + x F'_{2N}(x, Q^2)\frac{\langle E \rangle - |\epsilon_A| - \frac{2}{3}\langle T \rangle + \langle T_R \rangle}{m_N} + \left[x F'_{2N}(x, Q^2) + \frac{x^2}{2} F''_{2N}(x, Q^2)\right]\frac{2\langle T \rangle}{3m_N}. \quad (60)$$

Choosing again $x = 0.5$ we obtain

$$R_A^{\rm VNC}(x \sim 0.5) = 1 - 3\,\frac{|\epsilon_A| + \frac{1}{3}\langle T \rangle}{m_N}. \quad (61)$$

We notice that the A dependence of all terms contributing to R is very similar since all coefficients are dominated by the contribution of the kinetic energy term. Hence, independent of the details one expects an approximate factorization of

$$R_A(x, Q^2) - 1 = \frac{\phi(x, Q^2)}{f(A)}, \quad (62)$$

which works well experimentally.

An important quantity considered in the literature is the relation between the EMC effect ratio in the deuteron, $R_D(x, Q^2)$, given by Eq. (41), and the value of $R_A(x, Q^2)/R_D(x, Q^2)$ measured experimentally. Such a relation has been used to extract the neutron to proton ratio F_{2n}/F_{2p}. Previous estimates gave for ^{56}Fe [4,17]

$$R_D(x, Q^2) - 1 = c\left(\frac{R_A(x, Q^2)}{R_D(x, Q^2)} - 1\right), \quad c = \frac{1}{4}. \quad (63)$$

This relation is referred to by Yang and Bodek [37] as a *density model*, since in the mean-field approximation the average kinetic energy is proportional to the average nuclear density. [Note, however, that the average nuclear density is hardly defined for light nuclei whereas expressions (50) and (61) are well defined even for $A = 2$.] It is also worth emphasizing that $\langle \mathbf{p}^4 \rangle / \langle \mathbf{p}^2 \rangle$ is significantly smaller in the deuteron than in heavy nuclei, so that as soon as the terms proportional to $\langle \mathbf{p}^4 \rangle$ become important, Eq. (63) breaks down, which occurs at $x \sim 0.7$–0.8. At $x = 1$ Eq. (63) is badly violated, since the r.h.s. remains finite in this limit, whereas the left-hand side tends to infinity.

Relation (63) has been used in several papers (see, e.g., Ref. [37]) to extract the neutron to proton ratio F_{2n}/F_{2p}. We will show in the next section that by using realistic nuclear spectral functions a different relation will be obtained.

We will explore in the next section the sensitivity of the A dependence of the EMC effect predicted by our models.

V. RESULTS OF CALCULATIONS

In this section the results of our calculations based upon realistic spectral functions for few-nucleon systems and complex nuclei are presented. We have calculated the following quantities:

(i) The normalization (S) and the average values of the kinetic ($\langle T \rangle$) and removal ($\langle E \rangle$) energies in ^{3}He corresponding to various states of the spectator two-nucleon system presented in Table I.

(ii) The same quantities as in Table I but for $A = 2$ and $4 \leqslant A \leqslant 208$ reported in Table II.

(iii) Various average values of powers of the nucleon momentum $\mathbf{p}$ listed in Table III.

(iv) The average values (divided by $2m_N$) of the virtuality in the states $f\langle v_{\rm NR}^{(f)} \rangle$ [Eq. (30)] and their sum [Eq. (27)] listed in Table IV.

(v) The average value of the coefficient $\delta(\mathbf{p}, E)$ of the suppression of PLCs in various configurations [Eq. (31)] and their sum [Eq. (32)], together with $\delta^{(v)}$ [Eq. (29)], presented in Table V. In the case of ^{3}He, $\langle \delta^{(0)} \rangle \equiv \langle \delta^{\rm gr} \rangle$ and $\langle \delta^{(1)} \rangle \equiv \langle \delta^{\rm ex} \rangle$ [cf. the sentences after Eq. (14)].

(vi) The EMC ratio given by

$$R_A(x, Q^2) = \frac{F_{2A}(x, Q^2)}{F_{2D}(x, Q^2)}, \quad (64)$$

calculated at $x = 0.5$ with Eqs. (50), (51), and (61) [all multiplied by AF_{2N}/F_{2D}, to be consistent with the experimentally measured $R_A(x, Q^2)$] and compared with SLAC experimental data fitted by $R^{\rm exp} = 1.009\,A^{-0.0234}$ [38]. [Note that Eq. (64) differs from Eq. (41) in that the denominator represents the deuteron structure function

TABLE III. Various average values of the nucleon momentum **p**. Wave functions and spectral functions are as in Tables I and II.

A	$\langle \mathbf{p}^2\rangle_0$ (fm^{-2})	$\langle \mathbf{p}^2\rangle_1$ (fm^{-2})	$\langle \mathbf{p}^2\rangle$ (fm^{-2})	$\langle\|\mathbf{p}\|\rangle_0$ (fm^{-1})	$\langle\|\mathbf{p}\|\rangle_1$ (fm^{-1})	$\langle\|\mathbf{p}\|\rangle$ (fm^{-1})	$\langle\|\mathbf{p}\|\rangle_0^{-1}$ (fm)	$\langle\|\mathbf{p}\|\rangle_1^{-1}$ (fm)	$\langle\|\mathbf{p}\|\rangle^{-1}$ (fm)	$\frac{\langle\|\mathbf{p}\|\rangle}{\langle\mathbf{p}^2\rangle\,\langle 1/\langle\|\mathbf{p}\|\rangle\rangle}$
D	0.533	–	0.533	0.502	–	0.502	3.74	–	3.74	0.25
^{3}He	0.150	0.5651	0.71	0.224	0.415	0.64	1.31	0.548	1.86	0.49
^{4}He	0.396	0.845	1.24	0.517	0.346	0.86	1.59	0.19	1.78	0.39
^{12}C	0.652	0.912	1.56	0.6777	0.369	1.05	1.13	0.17	1.30	0.51
^{16}O	0.540	0.951	1.49	0.618	0.380	0.998	1.24	0.157	1.40	0.48
^{40}Ca	0.645	0.985	1.63	0.679	0.385	1.06	1.13	0.16	1.29	0.51
^{56}Fe	0.552	1.024	1.58	0.631	0.400	1.03	1.20	0.14	1.34	0.49
^{208}Pb	0.709	1.176	1.88	0.719	0.480	1.20	1.03	0.18	1.21	0.53

and not the sum of the nucleon structure functions F_{2N}.] In the case of the LC with suppression of PLCs we have used both $\Delta E^{(N/D)} = 800$ MeV and $\Delta E^{(N/A)} \sim 500$ MeV and $\Delta E^{(N/D)} = \Delta E^{(N/A)} = 800$ MeV. Wave functions and spectral functions are as in Tables I and II.

In our calculations we have employed the spectral function of ^{3}He given in Ref. [28] obtained using the ground-state three-body wave function from the Pisa Group [39] corresponding to the AV18 interaction [40]. For complex nuclei, calculations have been performed with the model spectral function of Ref. [29], which correctly reproduces the momentum and energy distributions as obtained from realistic calculations on complex nuclei.

The following comments concerning the obtained results are in order:

(i) *Tables I and II*. The average kinetic and removal energies in channels ex(f) are much larger than the corresponding quantities in channels gr(0) and the high momentum components are linked to high removal energies, which is a well-known result demonstrated long ago [27].

(ii) *Table III*. The effects of correlations on the high momentum components is clearly seen. The value of the quantity C [Eq. (40)] indicates that the probed value of $\langle \mathbf{p}^2\rangle$ in deep inelastic scattering is larger by a factor 2 than in quasielastic scattering.

(iii) *Tables IV and V*. The nucleon virtuality in states ex(f) is much higher than in states gr(0) owing to the higher average values of the removal energy and momentum. Consequently, the suppression of PLCs in states ex(f) is expected to be higher than in states gr(0). Semi-inclusive processes with the spectator $A-1$ nucleus in high excited states should be a very effective tool to investigate the suppression of PLCs.

(iv) *Table VI*. Both the LC model with suppression of PLCs and the VNC model predict almost no A dependence of the EMC effect in the range $4 \leqslant A \leqslant 56$. The results pertaining to the former model do depend upon the value of $\Delta E^{(N/D)}$. We have tried both $\Delta E^{(N/D)} = 800$ MeV and $\Delta E^{(N/A)} \sim 500$ MeV and $\Delta E^{(N/D)} = \Delta E^{(N/A)} = 800$ MeV; in the former case to reproduce the magnitude of the EMC effect for iron at $x = 0.5$ one needs $\lambda \sim 0.4$, which is similar to the value used in Ref. [17].

To better illustrate the A dependence of the EMC effect, we show in Fig. 1 the results presented in Table VI normalized to the SLAC experimental value of R for carbon. It can be clearly seen that, at variance with the trend of the SLAC data, the VNC model does not predict, in the interval $4 \leqslant A \leqslant 208$, any A dependence. A flattening of the A dependence of R is also predicted by the LC approach with suppression of PLCs, which appears to be sensitive to the value of $\Delta E^{(N/A)}$. Note that our results seem to agree with recent experimental data

TABLE IV. The quantities $\langle v_{\rm NR}^{(f)}\rangle/2m_N$, where $\langle v_{\rm NR}^{(f)}\rangle$ is the average value of the virtuality in various states [Eq. (30)] and their sum $\langle v_{\rm NR}\rangle/2m_N = -2(\langle T\rangle + |\epsilon_A|)$ [cf. the discussion after Eq. (27)]. In the case of ^{3}He, $\langle v_{\rm NR}^{(0)}\rangle \equiv \langle v_{\rm NR}^{(\rm gr)}\rangle$ and $\langle v_{\rm NR}^{(1)}\rangle \equiv \langle v_{\rm NR}^{(\rm ex)}\rangle$ [cf. Eq. (14)]. Wave functions and spectral functions are as in Tables I and II. All quantities are in MeV.

A	$\langle v_{\rm NR}^{(0)}\rangle$	$\langle v_{\rm NR}^{(1)}\rangle$	$\langle v_{\rm NR}\rangle$
^{3}He	−7.15	−27.44	−34.59
^{4}He	−26.82	−42.58	−69.40
^{12}C	−33.17	−49.11	−82.28
^{16}O	−31.40	−48.28	−79.68
^{40}Ca	−35.00	−49.54	−84.54
^{56}Fe	−31.66	−50.76	−82.44
^{208}Pb	−32.87	−59.33	−92.20

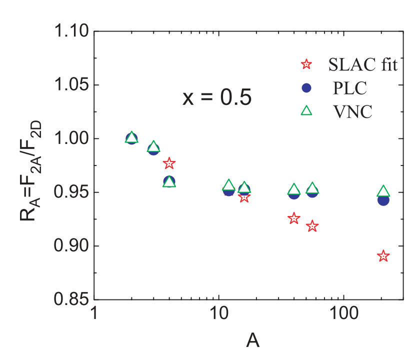

FIG. 1. (Color online) The EMC ratio F_{2A}/F_{2D} [Eq. (64)] at $x = 0.5$ corresponding to the values given in Table VI. Note that the SLAC fit [38] to the experimental data $R^{\rm exp} = 1.009\,A^{-0.0234}$ does not include systematic and statistic errors and has a tendency to underestimate the effect for ^{4}He.

TABLE V. The average value of the coefficient of the suppression of PLCs in various configurations [Eq. (31)] and their sum [Eq. (32)]. The latter is compared with the results of Eq. (29), where the nonrelativistic reduction of the virtuality [Eq. (23)] has not been performed. The last column exhibits the lowest order value [in $(\mathbf{p}/m_N)^2$] given by Eq. (33). In the case of ^{3}He, $\langle\delta^{(0)}\rangle \equiv \langle\delta^{(gr)}\rangle$ and $\langle\delta^{(1)}\rangle \equiv \langle\delta^{(ex)}\rangle$ [cf. the discussion after Eq. (14)]. Wave functions and spectral functions are as in Tables I and II.

A	$\langle\delta^{(0)}(\|\mathbf{p}\|, E)\rangle$	$\langle\delta^{(1)}(\|\mathbf{p}\|, E)\rangle$	$\langle\delta(\|\mathbf{p}\|, E)\rangle$	$\langle\delta(\|\mathbf{p}\|, E)\rangle_v$	$\langle\delta(\|\mathbf{p}\|, E)\rangle_{LO}$
D	0.95	–	0.95	0.96	0.94
^{3}He	0.43	0.48	0.91	0.92	0.90
^{4}He	0.70	0.12	0.82	0.82	0.72
^{12}C	0.68	0.11	0.79	0.81	0.68
^{16}O	0.69	0.11	0.80	0.83	0.68
^{40}Ca	0.68	0.10	0.78	0.83	0.66
^{56}Fe	0.69	0.10	0.79	0.83	0.61
^{208}Pb	0.68	0.13	0.82	0.86	0.64

from JLab [41], which exhibit practically the same EMC effect for ^{4}He and ^{12}C. It should also be pointed out, in this respect, that for heavy nuclei ($A \geqslant 50$) the Coulomb effect, which we will discuss in a separate publication, leads to an additional suppression of the EMC ratio that increases with A (for a brief discussion see Ref. [42]).

Our studies of the A dependence of the EMC effect allow us to make predictions for the EMC effect in the deuteron that will be useful for a comparison with the forthcoming JLab data aiming at determining $F_{2n}(x, Q^2)$ from the measurements of the deuteron tagged structure function. Our results suggest that

$$R_D(x, Q^2) - 1 = c_A(x)\left(\frac{R_A}{R_D} - 1\right), \tag{65}$$

with $c_A(x)$ practically independent of x for $0.3 \leqslant x \leqslant 0.6$. Within the VNC model we obtain $c_{12}(x) \simeq 0.34$ and $c_{56}(x) = 0.33$, whereas in the LC model with suppression of PLCs we have $c_{12}(x) \simeq 0.23$ and $c_{56}(x) = 0.22$ in the case of $\Delta E^{N/A} \neq \Delta E^{N/D}$ and $c_{12}(x) \simeq 0.43$ and $c_{56}(x) = 0.41$ in the case of $\Delta E^{N/A} = \Delta E_{N/D} = 800$ MeV. Our results, which are somewhat different from the estimate $c_{56}(x) = 1/4$, might have consequences on the extraction of the ratio F_{2n}/F_{2p}.

TABLE VI. The EMC ratio [Eq. (64)] given by Eqs. (50), (52), and (61) multiplied by $A\,F_{2N}/F_{2D}$ (see text), calculated at $x = 0.5$ with the value of λ fixed to reproduce the experimental data of ^{12}C [$\lambda(0.5) \approx 0.82$ for the case $\Delta E^{N/A} \neq \Delta E^{N/D}$, and $\lambda(0.5) \approx 0.66$ for $\Delta E_{N/A} = \Delta E^{N/D} = 800$ MeV, respectively]. The theoretical results are compared with the SLAC experimental data fitted by $R^{\rm exp} = 1.009\,A^{-0.0234}$ [38]. Wave functions and spectral functions are as in Tables I and II.

A	$R_A^{LC(\delta)}$ $\Delta E^{N/A} \neq \Delta E^{N/D}$	$R_A^{LC(\delta)}$ $\Delta E^{N/A} = \Delta E^{N/D}$	R_A^{LC}	R_A^{VNC}	R_A^{exp}
D	1.0	1.0	1.0	1.0	
^{3}He	0.99	0.99	1.0	0.99	0.980
^{4}He	0.96	0.962	1.0	0.96	0.980
^{12}C	0.95	0.95	1.0	0.955	0.950
^{16}O	0.95	0.953	1.0	0.953	0.946
^{40}Ca	0.949	0.948	1.0	0.952	0.930
^{56}Fe	0.951	0.950	1.0	0.953	0.920
^{208}Pb	0.943	0.941	1.0	0.950	0.890

VI. CONCLUSIONS

We have provided a derivation of the suppression of the PLCs of a nucleon bound in the nucleus A with the spectator nucleus $A - 1$ being in a particular energy configuration. We have pointed out that the result we have obtained can be interpreted as a specific dependence of the nucleon deformation upon the nucleon virtuality and argued that such a pattern is of quite general nature for small values of excitation energy and the momentum of the nucleon. Within such a framework, we have discussed the effects of the nucleon virtuality on the investigation of the modification of the radius and form factor of nucleons embedded in the nuclear medium, illustrating that deep inelastic processes might be more effective than quasielastic processes, in that in the former higher momentum components are probed. Eventually, we have illustrated the implications of our approach for the A dependence of the EMC effect, obtaining very similar effects for ^{4}He and ^{12}C, which agree well with the preliminary JLab data [41].

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APPENDIX: THE NONRELATIVISTIC SPECTRAL FUNCTIONS OF ^{3}He

For ^{3}He one has to define the *proton* and *neutron* spectral functions formed by the three different channels $^3\text{He} \to n(pp), ^3\text{He} \to p(D)$, and $^3\text{He} \to p(np)$:

$$P_p(|\mathbf{p}|, E) = P_{p(D)}(|\mathbf{p}|, E) + P_{p(pn)}(|\mathbf{p}|, E), \quad \text{(A1)}$$

$$P_n(|\mathbf{p}|, E) = P_{n(pp)}(|\mathbf{p}|, E), \quad \text{(A2)}$$

where $P_{p(D)}$ is usually referred to as the "ground" spectral function, P_p^{gr}, and $P_{p(pn)}$ and $P_{n(pp)}$ as the "excited" spectral functions, P_p^{ex} and P_n^{ex}, respectively [27]. This terminology refers to the final spectator state or, in other words, to the state of the residual $A-1$ system. The spectral functions are normalized in the following way:

$$\int P_p(|\mathbf{p}|, E) dE |\mathbf{p}|^2 d|\mathbf{p}| = 1, \qquad \int P_n(|\mathbf{p}|, E) dE |\mathbf{p}|^2 d|\mathbf{p}| = 1; \quad \text{(A3)}$$

therefore the isotopic factors and angular factors have to be explicitly taken into account. The mean values of the kinetic and separation energies are then given by

$$\langle E_p \rangle = \int P_p(|\mathbf{p}|, E) E dE |\mathbf{p}|^2 d|\mathbf{p}|, \qquad \langle E_n \rangle = \int P_n(|\mathbf{p}|, E) E dE |\mathbf{p}|^2 d|\mathbf{p}|, \quad \text{(A4)}$$

$$\langle T_p \rangle = \int P_p(|\mathbf{p}|, E) \frac{|\mathbf{p}|^2}{2m} dE |\mathbf{p}|^2 d|\mathbf{p}|, \qquad \langle T_n \rangle = \int P_n(|\mathbf{p}|, E) \frac{|\mathbf{p}|^2}{2m} dE |\mathbf{p}|^2 d|\mathbf{p}|. \quad \text{(A5)}$$

and the corresponding averages for the nucleon are

$$\langle E_N \rangle = \tfrac{2}{3} \langle E_p \rangle + \tfrac{1}{3} \langle E_n \rangle, \quad \text{(A6)}$$

$$\langle T_N \rangle = \tfrac{2}{3} \langle T_p \rangle + \tfrac{1}{3} \langle T_n \rangle. \quad \text{(A7)}$$

Using the Koltun sum rule [30]

$$\langle E \rangle = 2|\epsilon_A| + \frac{A-2}{A-1} \langle T \rangle \quad \text{(A8)}$$

one can define the effective binding energies per nucleon for all components. The corresponding averages for the nucleon are

$$\epsilon_N = \tfrac{1}{2} \left(\langle E_N \rangle - \tfrac{1}{2} \langle T_N \rangle \right), \quad \text{(A9)}$$

$$\epsilon_p = \tfrac{1}{2} \left(\langle E_p \rangle - \tfrac{1}{2} \langle T_p \rangle \right), \quad \text{(A10)}$$

$$\epsilon_n = \tfrac{1}{2} \left(\langle E_n \rangle - \tfrac{1}{2} \langle T_n \rangle \right), \quad \text{(A11)}$$

$$\epsilon_N = \tfrac{2}{3} \epsilon_p + \tfrac{1}{3} \epsilon_n, \quad \text{(A12)}$$

with the binding energy per particle of ^{3}He, ϵ_3, given by $3 \times \epsilon_N$.

The mean values associated with P^{gr} and P^{ex} are given by Eqs. (A1) and (A2) [29]:

$$\langle P^{\text{gr}} \rangle = \int P^{\text{gr}}(|\mathbf{p}|, E) dE |\mathbf{p}|^2 d|\mathbf{p}|, \qquad \langle P^{\text{ex}} \rangle = \int P^{\text{ex}}(|\mathbf{p}|, E) dE |\mathbf{p}|^2 d|\mathbf{p}|, \quad \text{(A13)}$$

$$\langle E^{\text{gr}} \rangle = \int P^{\text{gr}}(|\mathbf{p}|, E) E dE |\mathbf{p}|^2 d|\mathbf{p}|, \qquad \langle E^{\text{ex}} \rangle = \int P^{\text{ex}}(|\mathbf{p}|, E) E dE |\mathbf{p}|^2 d|\mathbf{p}|, \quad \text{(A14)}$$

$$\langle T^{\text{gr}} \rangle = \int P^{\text{gr}}(|\mathbf{p}|, E) \frac{|\mathbf{p}|^2}{2m} dE |\mathbf{p}|^2 d|\mathbf{p}|, \qquad \langle T^{\text{ex}} \rangle = \int P^{\text{ex}}(|\mathbf{p}|, E) \frac{|\mathbf{p}|^2}{2m} dE |\mathbf{p}|^2 d|\mathbf{p}|, \quad \text{(A15)}$$

$$\epsilon^{\text{gr}} = \tfrac{1}{2} \left(\langle E^{\text{gr}} \rangle - \tfrac{1}{2} \langle T^{\text{gr}} \rangle \right), \qquad \epsilon^{\text{ex}} = \tfrac{1}{2} \left(\langle E^{\text{ex}} \rangle - \tfrac{1}{2} \langle T^{\text{ex}} \rangle \right). \quad \text{(A16)}$$

The values of the various quantities that will be used in this paper are listed in Table I. They have been obtained with the spectral function of Ref. [28] and correspond to the wave function of the Pisa Group [39] obtained variationally using the AV18 interaction [40].

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