Effect of the tensor force in the exchange channel on the spin-orbit splitting in ^{23}F in the Hartree-Fock framework

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We study the spin-orbit splitting (ls splitting) for the proton d orbits in ^{23}F in the Hartree-Fock framework with the tensor force in the exchange channel. ^{23}F has one more proton around the neutron-rich nucleus ^{22}O. A recent experiment indicates that the ls splitting for the proton d orbits in ^{23}F is reduced from that in ^{17}F. Our calculation shows that the ls splitting in ^{23}F becomes smaller by a few MeV because of the tensor force. This effect comes from the interaction between the valence proton and occupied neutrons in the $0d_{5/2}$ orbit through the tensor force and makes the ls splitting in ^{23}F close to the experimental data.

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I. INTRODUCTION

Spin-orbit splitting (ls splitting) is important in the structure of nuclei. A large ls splitting between single-particle orbits with the same orbital angular momentum is responsible for the shell structure of nuclei [1]. Recently we have been obtaining much information about unstable nuclei from various experiments. Experimental evidence indicates that the shell structure in neutron-rich nuclei changes from that in stable nuclei. To confirm the change of the shell structure, information about single-particle orbits around closed-shell or closed-subshell nuclei is important. Michimasa and his collaborators experimentally studied the proton single-particle orbits in ^{23}F through the proton transfer reaction [2]. ^{23}F has one more proton around ^{22}O. They reported that ls splitting for the proton d orbits $(5/2^+ - 3/2^+)$ is 4.06 MeV, while ls splitting for the proton d orbits in ^{17}F is 5.00 MeV [3,4], which is similar to that for the neutron d orbits in ^{17}O (5.08 MeV) [3,4] due to the isospin symmetry. It indicates that there is a possibility that ls splitting is changed by excess neutrons around ^{16}O. Shell model calculations nicely reproduce the change of the ls splitting from ^{17}F to ^{23}F [2,5]. In the shell model calculation, the ls splitting in ^{17}F (^{17}O) is an input parameter. Hence, it is interesting to study ls splitting with a mean-field-type model, where ls splitting is obtained self-consistently.

Hartree-Fock and Hartree-Fock-Bogoliubov calculations can now be performed in the whole mass region over the nuclear chart. Such mean-field calculations can reproduce binding energies and radii of nuclei including unstable ones using effective forces with relatively simple forms like the Skyrme or Gogny forces [6,7]. In the mean-field calculations, ls splitting of single-particle orbits is produced mainly by the spin-orbit (LS) force. The ls splitting of single-particle orbits and the magic number for binding energies can be explained with the LS force having the same strength in almost the whole mass region at least near the stability line. Some studies show that ls splitting in neutron-rich nuclei becomes small because the diffuseness of the neutron density becomes large and the spin-orbit potential is weakened [8,9].

The tensor force acts on the spin of the nucleon directly and should affect ls splitting. Although the tensor force is not usually included in mean-field calculations, some Hartree-Fock calculations explicitly including the tensor force or the pion in the relativistic model showed that the tensor force affects ls splitting in spin-unsaturated nuclei [10–17]. Only one orbit of spin-orbit partners is occupied in a spin-unsaturated nucleus, while both the spin-orbit partners are fully occupied in a spin-saturated nucleus. For example, ^{48}Ca is a spin-unsaturated nucleus, where the neutron $0f_{7/2}$ orbit is a spin-unsaturated orbit and ^{40}Ca is a spin-saturated nucleus. Because the total spin coming from the intrinsic spin of a nucleon is zero in a spin-saturated nucleus if the wave functions of the spin-orbit partners have the same radial forms, the tensor force does not act between the spin-saturated core and a particle or a hole around the core. In a spin-unsaturated nucleus, the total intrinsic spin coming from the spin-unsaturated orbit has a finite value, and the tensor force becomes active. In fact, the sizes of the ls splitting for hole orbits change from ^{40}Ca to ^{48}Ca and ^{16}O to ^{22}O in the results of the Hartree-Fock calculations with the tensor force or the pion [11–15]. For the calcium isotopes, there is experimental evidence [18] that ls splitting becomes smaller from ^{40}Ca to ^{48}Ca and the order of the change is comparable to that induced by the tensor force or the pion [13,15]. It should be noted that in the Hartree-Fock approximation, the energy contribution from the tensor force or the pion from the direct channel becomes zero and only that

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from the exchange channel has a finite value in closed-shell nuclei.

Otsuka and his collaborators discussed the effect of the tensor force on single-particle energy in other mass regions. They nicely reproduced the change of the splitting between $\pi 0h_{11/2}$ and $\pi 0g_{7/2}$ in the Sb isotopes with neutron number [19] by the monopole shift induced by the tensor force [15]. They also suggested the effect of the tensor force on the shell evolution in the neutron-rich sd- and pf-shell regions [15,16]. They discussed that the neutron shell structure changes with proton number because of the monopole interaction between proton and neutron orbits and explained the appearance of the magic number 16 and the disappearance of the magic number 20 in the neutron-rich sd-shell region [15,20,21]. They claimed that the monopole interaction is caused by the tensor force [15,16]. To confirm such a discussion, the direct information about a single-particle state is essential.

In this paper, we perform the Hartree-Fock calculation for ^{22}O and ^{23}F. We include the tensor force and study its effect on ls splitting. We also calculate ^{15,16,17}O to see the effect of excess neutrons on ls splitting and its relation to the tensor force by comparing with ^{22}O and ^{23}F. The formulation is given in Sec. II and the results are given in Sec. III. Section IV is devoted to the summary of the paper.

II. FORMULATION

In the present paper, we adopt two types of Hamiltonian. One includes the three-body force in addition to the kinetic term and the two-body force. The other includes the density-dependent force instead of the three-body force. The Hamiltonian with the three-body force H^{3B} and that with the density-dependent force H^{DD} have the following forms:

$$H^{3B} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2M} + \sum_{i<j}^{A} v(r_i, r_j) + \sum_{i<j<k}^{A} v^{(3)}(r_i, r_j, r_k) - E_{\text{c.m.}}, \tag{1}$$

$$H^{DD} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2M} + \sum_{i<j}^{A} v(r_i, r_j) + \sum_{i<j}^{A} v^{(DD)}(\rho; r_i, r_j) - E_{\text{c.m.}}. \tag{2}$$

In these expressions, $\boldsymbol{p}$, r, and M are the momentum, coordinate including spin and isospin, and nucleon mass, respectively. A is a mass number. v and $v^{(3)}$ are the two-body and three-body potentials, respectively. $v^{(DD)}$ is the density-dependent potential with the one-body density ρ. We subtract the energy of the center-of-mass motion $E_{\text{c.m.}} = (\sum_i^A \boldsymbol{p}_i)^2/2AM$.

In the Hartree-Fock calculation, we assume the wave function of the nucleus has the form

$$\Psi = \mathcal{A} \prod_{\alpha} \psi_\alpha(r_\alpha), \tag{3}$$

with the antisymmetrization operator $\mathcal{A}$ for nucleon coordinates. α labels each single-particle state and runs over all occupied states. With the wave function, the total energies become

$$E^{3B} = \sum_{\alpha} \langle \psi_\alpha | \frac{\boldsymbol{p}^2}{2M} | \psi_\alpha \rangle + \sum_{\alpha<\beta} \langle \psi_\alpha \psi_\beta | v | \widetilde{\psi_\alpha \psi_\beta} \rangle + \sum_{\alpha<\beta<\gamma} \langle \psi_\alpha \psi_\beta \psi_\gamma | v^{(3)} | \widetilde{\psi_\alpha \psi_\beta} \psi_\gamma \rangle \tag{4}$$

for H^{3B} and

$$E^{DD} = \sum_{\alpha} \langle \psi_\alpha | \frac{\boldsymbol{p}^2}{2M} | \psi_\alpha \rangle + \sum_{\alpha<\beta} \langle \psi_\alpha \psi_\beta | v | \widetilde{\psi_\alpha \psi_\beta} \rangle + \sum_{\alpha<\beta} \langle \psi_\alpha \psi_\beta | v^{(DD)}(\rho) | \widetilde{\psi_\alpha \psi_\beta} \rangle \tag{5}$$

for H^{DD}, where the tildes represent the antisymmetrization. In the above equations, $E_{\text{c.m.}}$ is dropped for simplicity. By taking a variation of the total energy with respect to a single-particle wave function ψ_α, we obtain the Hartree-Fock equation for each case:

$$\begin{aligned} &\frac{\boldsymbol{p}^2}{2M}\psi_\alpha(x) + \sum_\beta \int dy \psi_\beta^\dagger(y) v(x,y) [\psi_\beta(y)\psi_\alpha(x) - \psi_\alpha(y)\psi_\beta(x)] \\ &+ \frac{1}{2}\sum_{\beta,\gamma} \int dy \int dz \psi_\beta^\dagger(y)\psi_\gamma^\dagger(z) v^{(3)}(x,y,z) \\ &\times [\{\psi_\beta(y)\psi_\gamma(z) - \psi_\gamma(y)\psi_\beta(z)\}\psi_\alpha(x) \\ &+ \{\psi_\gamma(y)\psi_\alpha(z) - \psi_\alpha(y)\psi_\gamma(z)\}\psi_\beta(x) \\ &+ \{\psi_\alpha(y)\psi_\beta(z) - \psi_\beta(y)\psi_\alpha(z)\}\psi_\gamma(x)] = \varepsilon_\alpha \psi_\alpha(x) \end{aligned} \tag{6}$$

for the three-body force case, and

$$\begin{aligned} &\frac{\boldsymbol{p}^2}{2M}\psi_\alpha(x) + \sum_\beta \int dy \psi_\beta^\dagger(y) v(x,y) \\ &\times [\psi_\beta(y)\psi_\alpha(x) - \psi_\alpha(y)\psi_\beta(x)] \\ &+ \sum_\beta \int dy \psi_\beta^\dagger(y) v^{(DD)}(\rho; x, y) [\psi_\beta(y)\psi_\alpha(x) \\ &- \psi_\alpha(y)\psi_\beta(x)] \\ &+ \sum_{\beta<\gamma} \int dy \int dz \psi_\beta^\dagger(y)\psi_\gamma^\dagger(z) \frac{\delta v^{(DD)}}{\delta \rho}(\rho; y, z) \frac{\delta \rho}{\delta \psi_\alpha^\dagger}(x) \\ &\times [\psi_\beta(y)\psi_\gamma(z) - \psi_\gamma(y)\psi_\beta(z)] = \varepsilon_\alpha \psi_\alpha(x) \end{aligned} \tag{7}$$

for the density-dependent force case. In the above expression, the integrations over y and z include the summation over the spin and isospin indices.

In the present study, we assume each single-particle state as an eigenfunction of total spin $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$. With this assumption, a single-particle wave function can be expressed as

$$\psi_\alpha(\boldsymbol{r}) = R_\alpha(r) \mathcal{Y}_{l_\alpha j_\alpha m_\alpha}(\Omega) \zeta(\mu_\alpha), \tag{8}$$

where R is a radial wave function, $\mathcal{Y}$ is an eigenfunction of $\boldsymbol{j}$, and ζ is an isospin wave function. α stands for node n_α, total spin j_α, its projection on the z axis m_α, and isospin μ_α.

We do not assume the degeneracy for the orbits with the same n_α, j_α, and μ_α because the spherical symmetry of a mean field is broken in odd nuclei. It means that the states with the same n, j, and μ but different m's are allowed to have different radial wave functions. In such a case, we need to perform an angular momentum projection to obtain a wave function with a good total angular momentum. The expectation value for the total angular momentum J^2 with the wave function obtained in the Hartree-Fock calculation for a one-particle or one-hole state does not deviate from $j_\nu(j_\nu+1)$ largely (less than 1%), where j_ν is the total spin of the particle or hole orbit. It indicates the obtained wave function is almost an eigenstate of angular momentum. Hence, we do not perform the angular momentum projection.

We approximate the density in a density-dependent force as

$$\rho(\boldsymbol{r}) \approx \frac{1}{4\pi}\sum_\alpha R_\alpha^\dagger(r)R_\alpha(r) \tag{9}$$

for calculational convenience. This expression is exact for a closed-shell nucleus with the spherical symmetry and should be a good approximation for a one-particle or one-hole nucleus with almost a spherical core.

We expand a radial wave function $R_\alpha(r)$ by Gaussian functions with widths of a geometric series [22]. We take 11 Gaussian functions with the minimum width 0.5 fm and the maximum width 7 fm for each single-particle state. The Hartree-Fock equation is solved by the gradient or damped-gradient method [23].

III. RESULT

In this section, we apply the Hartree-Fock method to ^{15,16,17,22}O and ^{23}F. We assume ^{16}O as a closed-shell nucleus up to the $0p$ shell and ^{22}O as a closed-subshell nucleus where the neutron $0d_{5/2}$ orbit is fully occupied in addition to the occupied orbits in ^{16}O. For ^{22}O, experimental evidence suggests it has the closed-subshell structure of the neutron $0d_{5/2}$ orbit [24,25]. In the ^{15}O case, one neutron is subtracted from the neutron $0p_{1/2}$ orbit or the neutron $0p_{3/2}$ orbit in ^{16}O. In the ^{17}O case, we add one neutron in the $0d_{5/2}$ orbit around ^{16}O. We do not put a neutron in the $0d_{3/2}$ orbit in ^{17}O because there are no bound states in this configuration. In the ^{23}F case, we add a proton in the $0d_{5/2}$, $0d_{3/2}$, or $1s_{1/2}$ orbit around ^{22}O.

As for the effective interaction, we adopt the modified Volkov force No. 1 (MV1) [26] for the central part and the G3RS force [27] for the tensor part. We also include the Coulomb force. The G3RS force is determined to reproduce the nucleon-nucleon scattering data and, therefore, the tensor force in the G3RS force is the one in the free space. For the strength of the tensor force in the nuclear medium, we do not have a definite guideline at present. The effective interaction obtained from the G-matrix theory has a tensor part with a strength comparable to the tensor force in the free space [15,28–30] at least in the region where the relative distance is greater than about 0.8 fm. We use the tensor force in the free space in the present calculation, but we need a further investigation to determine the strength of the tensor force to be used in a mean-field calculation. It should be noted that the difference in the short range ($r < 0.8$ fm) does not influence the tensor force matrix elements significantly [30]. As for the LS force, we take the δ-type LS force [6,7]:

$$iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\overleftarrow{\boldsymbol{k}}\times\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\overrightarrow{\boldsymbol{k}}. \tag{10}$$

The Majorana parameter in the MV1 force is fixed to 0.59, which is determined to reproduce the binding energy of ^{16}O. W_0 in the LS force is taken as 115 MeV fm^5, which is the same as in the Gogny D1 force and is determined to reproduce the ls splitting for the $0p$ orbits in ^{15}O [7].

In Table I, the results for ^{16}O, ^{17}O, and ^{15}O are summarized. The experimental data are also given in parentheses if available. The potential energy from the tensor force becomes quite small, because ^{16}O is an LS-closed-shell nucleus. In the LS-closed-shell nucleus, both spin-orbit partners are completely occupied. Hence, the LS-closed-shell nucleus is a spin-saturated nucleus. The LS-closed-shell nucleus does not have a finite total orbital angular momentum and a finite total spin angular momentum, if the radial forms of the wave functions for the spin-orbit partners are the same. The tensor force consists of the rank 2 tensors of the orbital and spin angular momenta. Thus, the tensor force does not work between the LS-closed-shell nucleus and a particle or a hole around it, because a particle or hole has a spin angular momentum 1/2. In the last row, the energy differences between ^{15}O $(0p_{3/2}^{-1})$ and ^{15}O $(0p_{1/2}^{-1})$ are shown. It corresponds to the ls splitting for the $0p$ orbits and is about 10% smaller than the experimental value. The contribution from the LS force is 5.8 MeV and is almost the same as the total ls splitting. This indicates that ls splitting is mainly produced by the LS force. The large contribution from the kinetic energy is almost canceled out with the contributions from the central and three-body forces. In ^{15}O, the effect of the tensor force on ls splitting is negligible.

Table II summarizes the results for ^{22}O and ^{23}F. The binding energy of ^{23}F $(0d_{5/2})$ (the ground state) is about 5 MeV smaller than the experimental value. The dependence of ls splitting on the separation energy of a valence proton will be discussed later. In ^{22}O, the neutron $0d_{5/2}$ orbit around the ^{16}O core is fully occupied. Because the spin-orbit partner, the neutron $0d_{3/2}$ orbit, is empty, ^{22}O is a spin-unsaturated nucleus. Hence, ^{22}O has a finite total orbital angular momentum and a finite total spin angular momentum, and the expectation value for the tensor potential energy in ^{22}O becomes finite. In ^{22}O, the energy contributions from the LS force and the tensor force are -20.8 and 1.9 MeV, respectively. In ^{23}F, a proton is added to ^{22}O. If the proton is put in the $0d_{3/2}$ orbit, the absolute value of the LS potential energy becomes small by 4.5 MeV; and if the proton is put in the $0d_{5/2}$ orbit, that of the LS potential energy becomes large by 3.3 MeV. In contrast, the tensor potential energy becomes small by 1.8 MeV when the proton is in the $0d_{3/2}$ orbit and becomes large by 1.3 MeV when the proton is in the $0d_{5/2}$ orbit. As a result, the contribution to the ls splitting for the proton $0d$ orbits in ^{23}F from the LS force is 7.8 MeV and that from the tensor force is -3.1 MeV. The sum of them is 4.5 MeV. The relatively small ls splitting of 4.2 MeV after adding the contributions from the kinetic and other potential

TABLE I. Total energy ($E_{\rm TOT}$), kinetic energy (T), and potential energy (V) of ^{16}O, ^{17}O, and ^{15}O. $V_{\rm LS}$ and V_T are the contributions from the LS and tensor forces to the potential energy. Those are give in MeV. R_c and R_m are the charge and matter radii in fm. The last row shows the differences of energies between ^{15}O ($0p_{3/2}^{-1}$) and ^{15}O ($0p_{1/2}^{-1}$). In the parentheses, the experimental data are given.

	$E_{\rm TOT}$		T	V	$V_{\rm LS}$	V_T	R_c		R_m	
^{16}O	−128.3	(−127.6[a])	233.8	−362.0	−1.0	0.0	2.71	(2.730(25)[b])	2.58	(2.54(02)[c])
^{17}O ($0d_{5/2}$)	−132.3	(−131.8[a])	254.7	−387.0	−4.1	0.0	2.72	(2.662(26)[b])	2.64	(2.59(05)[c])
^{15}O ($0p_{1/2}^{-1}$)	−110.2	(−112.0[a])	219.6	−329.7	−4.9	−0.1	2.70		2.55	(2.44(04)[c])
^{15}O ($0p_{3/2}^{-1}$)	−104.5		212.4	−316.9	0.9	0.0	2.74		2.59	
$\Delta(0p_{3/2}^{-1} - 0p_{1/2}^{-1})$	5.7	(6.18[d])	−7.2	12.8	5.8	0.1				

[a]Reference [31].
[b]Reference [32].
[c]Refefence [33].
[d]References [3,34].

energies, which is close to the experimental value, is realized by the cancellation between the contributions from the LS and tensor forces.

Our result for the proton $1s_{1/2}$ orbit fails to reproduce the experimental value for the splitting between the $0d_{5/2}$ and $1s_{1/2}$ orbits. In fact, it is located slightly above the proton $0d_{3/2}$ orbit in our calculation. It should be noted that the tensor force does not affect the single-particle energy of the proton $1s_{1/2}$ orbit. The position of the neutron $1s_{1/2}$ orbit in ^{17}O in our calculation is also higher than that in the experimental data by about 2.5 MeV. The fact that we fail to reproduce the experimental data for the splitting between the $0d_{5/2}$ and $1s_{1/2}$ orbits in both ^{17}O and ^{23}F and the binding energy of ^{23}F as shown in Table II indicates that we need further refinement of the effective interaction used in the Hartree-Fock calculation. In ^{23}F, our assumption that ^{22}O has a closed-shell structure is not good, because there is a low-lying state at 3.2 MeV in ^{22}O [24,25]. It may affect single-particle states through particle-vibration coupling. For these points we need further study.

The energy differences between one-particle states and their corresponding cores are shown in Table III. The LS potential energies from the cores for the $0d_{5/2}$ orbit are −3.0 MeV in ^{17}O and −3.3 MeV in ^{23}F. The LS potential energy from the core for the $0d_{3/2}$ orbit in ^{23}F is smaller than the value which is expected from that for the $0d_{5/2}$ orbit [$3.3 \times (2+1)/2 \approx 5.0$ MeV]. It is probably due to a weak binding of the $0d_{3/2}$ orbit compared to the $0d_{5/2}$ one. The contribution from the tensor force to the ls splitting for the $0d$ orbits in ^{23}F is about a half that from the LS force with the opposite sign, as discussed in the previous paragraph. The results for ^{17}O and ^{23}F in Table III indicate that the contribution to the ls splitting from the LS force mainly comes from the ^{16}O core and that from the tensor force comes from the excess neutron orbit (the neutron $0d_{5/2}$ orbit).

In Table IV, the contributions to the tensor potential energy from the triplet-even and triplet-odd parts are shown separately. In ^{22}O, the tensor potential energy mainly comes from the triplet-odd part, which is natural because only the neutron $0d_{5/2}$ orbit is occupied and there are no protons around the ^{16}O core. In ^{23}F, the contribution from the triplet-even part is comparable to that from the triplet-odd part for the $0d_{3/2}$ orbit, and they have opposite signs. For the $0d_{5/2}$ orbit, the contribution from the triplet-even part is smaller than that from the triplet-odd part, and they have the same sign. To see the effect of the tensor force on the valence proton, the energy differences between ^{23}F and ^{22}O are shown in the table. The differences are dominated by the triplet-even part, which means that the contribution to the ls splitting from the tensor force mainly comes from the triplet-even tensor force.

As indicated in Table II, the binding energy for ^{23}F in our calculation is smaller than the experimental value by about 5 MeV. Because the effect of the tensor force on the ls splitting mainly comes from neutrons around ^{16}O, it will be affectedby a change of the separation energy for a valence proton orbit. To see the separation energy dependence of the ls splitting, we

TABLE II. Same as Table I, but for ^{22}O and ^{23}F.

	$E_{\rm TOT}$		T	V	$V_{\rm LS}$	V_T	$R_{\rm c}$	R_m	
^{22}O	−161.8	(−162.0[a])	361.4	−523.2	−20.8	1.9	2.74	2.85	(2.88(06)[b])
^{23}F ($0d_{5/2}$)	−170.7	(−175.3[a])	383.9	−554.5	−24.1	3.2	2.84	2.87	(2.79(04)[b])
^{23}F ($0d_{3/2}$)	−166.5		376.4	−542.8	−16.3	0.1	2.89	2.90	
^{23}F ($1s_{1/2}$)	−166.3		375.9	−542.3	−20.3	1.8	2.91	2.91	
$\Delta(0d_{3/2} - 0d_{5/2})$	4.2	(4.06[c])	−7.5	11.7	7.8	−3.1			
$\Delta(1s_{1/2} - 0d_{5/2})$	4.3	(2.27[c])	−7.9	12.3	3.8	−1.4			

[a]Reference [31].
[b]Reference [33].
[c]Reference [2].

TABLE III. Differences of the LS potential energy [$\Delta(V_{\rm LS})$] and the tensor potential energy [$\Delta(V_T)$] between one-particle nuclei and their core nuclei. They are given in MeV.

	$\Delta(V_{\rm LS})$	$\Delta(V_T)$
^{17}O$(0d_{5/2})-{}^{16}$O	−3.0	0.0
^{23}F$(0d_{3/2})-{}^{22}$O	4.5	−1.8
^{23}F$(0d_{5/2})-{}^{22}$O	−3.3	1.3

solve a potential problem using the Hartree-Fock potential calculated with the wave function of ^{22}O obtained in the Hartree-Fock calculation. We add to the Hartree-Fock potential a Woods-Saxon potential with the diffuseness parameter $a = 0.67$ fm and the radius parameter R with the same value as the matter radius obtained in the Hartree-Fock calculation for ^{22}O. We change the potential depth of the Woods-Saxon potential to adjust the separation energy for the proton $0d_{5/2}$ orbit. The ls splitting $\Delta(\pi 0d_{3/2}-\pi 0d_{5/2})$ between the proton $0d_{3/2}$ and $0d_{5/2}$ orbits is shown in Fig. 1 as a function of the separation energy for a proton in the $0d_{5/2}$ orbit [$S_p(\pi 0d_{5/2})$]. $S_p(\pi 0d_{5/2})$ in Table II is 8.9 MeV and the experimental value is 13.3 MeV. $\Delta(\pi 0d_{3/2}-\pi 0d_{5/2})$ becomes larger when $S_p(\pi 0d_{5/2})$ increases. At $S_p(\pi 0d_{5/2}) = 13.3$ MeV, $\Delta(\pi 0d_{3/2}-\pi 0d_{5/2})$ is 4.8 MeV. It deviates from the experimental value a little bit more than the one in Table II. In Fig. 1, we also plot $\Delta(\pi 0d_{3/2}-\pi 0d_{5/2})$ calculated without the tensor force, in which case, $\Delta(\pi 0d_{3/2}-\pi 0d_{5/2})$ becomes larger by about 3 MeV for all $S_p(\pi 0d_{5/2})$ shown in Fig. 1. This result indicates the importance of the effect of the tensor force on the ls splitting in ^{23}F, even if the separation energy of the valence proton becomes larger than the one in Table II.

To see whether our choices of the strengths of the LS and tensor forces are reasonable in a higher sd-shell region, we calculate the ls splitting for the proton $0d$ orbits in ^{39}K, which is a one-hole state of a spin-saturated nucleus ^{40}Ca. We obtain 6.3 MeV for the ls splitting, although we need to change the Majorana parameter in the MV1 force from 0.59 to 0.608 to reproduce the binding energy of ^{40}Ca. This value is consistent with recent experimental data, as shown in Fig. 1 of Ref. [14].

Finally, we compare the ls splitting calculated with other effective interactions with our result discussed above (MV1) in Table V. We also show the result without the tensor force (MV1 without V_T). The Gogny D1S force [35] does not have a tensor part and has a stronger LS part ($W_0 = 130$ MeV fm^5) than the one we adopted above. The M3Y-P2 force [36] has a weak tensor part and an LS part comparable to the Gogny D1S force. The GT2 [16,37] force has a tensor part comparable to that in the free space and a strong LS part ($W_0 = 160$ MeV fm^5). While a rather schematic form of the tensor force is adopted in Ref. [16], we replace the tensor part of the GT2 force with the G3RS force we used above. The sizes of the ls splitting for the MV1 force without the tensor force, the Gogny force, and the M3Y-P2 force are larger than the experimental value. This indicates that a relatively strong tensor force comparable to that in the free space is needed to reproduce the ls splitting in ^{23}F. Although the GT2 force has the strong tensor part, it gives quite a large ls splitting, because of the strong LS part of the GT2 force. The contribution from the LS force to the ls splitting is much larger than those with other effective interactions. In fact, the ls splitting for the $0p$ orbits in ^{15}O with the GT2 force is 8.3 MeV. It is much larger than the experimental value. This indicates that a proper strength of the LS force that gives reasonable ls splitting in ^{15}O is needed to reproduce the ls splitting in ^{23}F.

The tensor force also induces the two-particle-two-hole (2p2h) correlation, which cannot be treated in a usual mean-field calculation. The 2p2h correlation by the tensor force produces large attractive energy in nuclei [38,39]. Recently, we developed a mean-field framework which can treat the

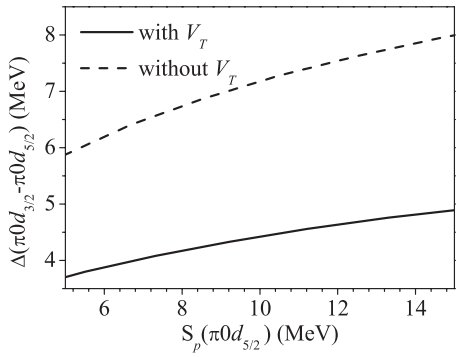

FIG. 1. ls splitting between the $\pi 0d_{3/2}$ and $\pi 0d_{5/2}$ orbits in ^{23}F as a function of the separation energy for a proton in the $\pi 0d_{5/2}$ orbit, with and without the tensor force.

TABLE IV. Potential energy contributions from the triplet-even tensor force ($V_T^{\rm 3E}$) and the triplet-odd tensor force ($V_T^{\rm 3O}$) in MeV. In the last two rows, the differences between ^{23}F ($0d_{3/2}$ or $0d_{5/2}$) and ^{22}O are given.

	$V_T^{\rm 3E}$	$V_T^{\rm 3O}$
^{22}O	0.1	1.8
^{23}F $(0d_{3/2})$	−1.3	1.4
^{23}F $(0d_{5/2})$	1.0	2.1
$\Delta(^{23}{\rm F}(0d_{3/2})-{}^{22}{\rm O})$	−1.4	−0.4
$\Delta(^{23}{\rm F}(0d_{5/2})-{}^{22}{\rm O})$	1.0	0.3

TABLE V. ls splitting for the proton $0d$ orbits in ^{23}F with various effective interactions (see the text). $\Delta(V_{\rm LS})$, $\Delta(V_T)$, and Δ(others) are the contributions to the ls splitting from the LS force, the tensor force, and the other forces including the kinetic term, respectively. They are given in MeV. The experimental value for $\Delta(0d_{3/2}-0d_{5/2}) = 4.06$ MeV.

	$\Delta(0d_{3/2}-0d_{5/2})$	$\Delta(V_{\rm LS})$	$\Delta(V_T)$	Δ(others)
MV1	4.2	7.8	−3.1	−0.5
MV1 without V_T	7.2	8.3	0.0	−1.1
Gogny D1S	8.5	9.4	0.0	−0.9
M3Y-P2	7.6	9.2	−0.4	−1.2
GT2	8.2	12.2	−3.3	−0.7

2p2h tensor correlation by introducing single-particle states with charge and parity mixing [40–43]. We applied the extended mean-field model to closed-subshell oxygen isotopes including neutron-rich ones [43] and found that the potential energy from the tensor force is comparable to that from the LS force. The importance of the 2p2h tensor correlation for ls splitting is indicated in some studies [42,44–46]. It is interesting to study the effect of the 2p2h tensor correlation on the ls splitting with our extended mean-field model. Because our calculation showed that the excess neutrons around ^{16}O do not contribute strongly to the 2p2h tensor correlation [43], the Hartree-Fock calculation seems to be sufficient as the first step.

IV. SUMMARY

We have performed the Hartree-Fock calculation with the tensor force for ^{15}O, ^{16}O, ^{17}O, ^{22}O, and ^{23}F to study the effect of the tensor force on ls splitting.

The tensor force does not affect the ls splitting for the $0p$ orbits in ^{15}O, because ^{16}O is an LS-closed-shell nucleus. The ls splitting is almost produced by the LS force in ^{15}O.

In ^{22}O, the neutron $0d_{5/2}$ orbit is fully occupied. It gives the finite expectation value for the tensor force in ^{22}O. In ^{23}F, a proton is added to ^{22}O. The LS force works to provide the ls splitting for the proton $0d$ orbits in ^{23}F by 7.8 MeV. In contrast, the tensor force reduces the ls splitting by 3.1 MeV. The effect of the tensor force mainly comes from the occupied neutron $0d_{5/2}$ orbit. The resulting ls splitting of 4.2 MeV close to the experimental data is realized by the cancellation between the effects of the LS and tensor forces. The contribution from the tensor force to the ls splitting in ^{23}F mainly comes from the triplet-even part of the tensor force.

We have compared the results with various effective interactions with and without the tensor force. The effective interactions without the tensor force or with the weak tensor force do not explain the experimental value for the ls splitting for the proton $0d$ orbits in ^{23}F. Our study indicates that LS and tensor forces with reasonable strengths are needed to reproduce ls splitting in ^{15}O and ^{23}F simultaneously.

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