

## Dipole excitation and geometry of Borromean nuclei

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We analyze the Coulomb breakup cross sections of  $^{11}\text{Li}$  and  $^6\text{He}$  nuclei using a three-body model with a density-dependent contact interaction. We show that the concentration of the  $B(E1)$  strength near the threshold can be well reproduced with this model. With the help of the calculated  $B(E1)$  value, we extract the root-mean-square (rms) distance between the core nucleus and the center-of-mass of two valence neutrons without resorting to the sum rule, which may suffer from unphysical Pauli forbidden transitions. Together with the empirical rms distance between the neutrons obtained from the matter radius study and also from the three-body correlation study in the breakup reaction, we convert these rms distances to the mean opening angle between the valence neutrons from the core nucleus. We find that the obtained mean opening angles in  $^{11}\text{Li}$  and  $^6\text{He}$  agree with the three-body model predictions.

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It has been well recognized by now that weakly bound nuclei exhibit a strong multipole strength that is concentrated near the continuum threshold, because of the optimal matching of wave functions between a weakly bound state and continuum states [1–5]. Recently, Nakamura *et al.* remeasured the low-lying dipole excitations in  $^{11}\text{Li}$  nucleus and confirmed for the first time the strong concentration of the dipole strength near the threshold in this two-neutron (2n) halo nucleus [6]. The low-lying dipole strength for another 2n halo nucleus,  $^6\text{He}$ , has also been measured by Aumann *et al.* [7].

As well as being of considerable interest in its own right, the  $B(E1)$  strength distribution of two-neutron halo nuclei is also important as it is intimately related to the root-mean-square (rms) distance,  $\sqrt{\langle r_{c-2n}^2 \rangle}$ , between the core nucleus and the center-of-mass of two valence neutrons [8–10]. Together with additional information for the rms distance between the two neutrons,  $\sqrt{\langle r_{nn}^2 \rangle}$ , one can then extract the geometry of 2n halo nuclei, such as the mean opening angle between the neutrons from the core [6,11]. This information is particularly important to extract the strength of dineutron correlations in halo nuclei.

In the previous applications, the rms distance  $\sqrt{\langle r_{c-2n}^2 \rangle}$  has been obtained from the measured  $B(E1)$  strength using the relation [8–10]

$$B(E1) = \frac{3}{\pi} \left( \frac{Ze}{A} \right)^2 \langle r_{c-2n}^2 \rangle. \quad (1)$$

This relation is obtained with closure, which includes unphysical Pauli forbidden transitions to the states with negative excitation energies. Although the effect of Pauli forbidden transitions is not large, it leads to a non-negligible correction. In Ref. [10], a better prescription was proposed recently, which uses a model calculation for the  $B(E1)$  value and  $\langle r_{c-2n}^2 \rangle$  to extract the “experimental value” for  $\langle r_{c-2n}^2 \rangle$  [See Eq. (6) in Ref. [10] and Eq. (2) below]. Although this prescription uses theoretical values, it has been shown that the model dependence is insignificant [10].

The aim of this article is to analyze the mean opening angle of valence neutrons in the 2n halo nuclei,  $^{11}\text{Li}$  and  $^6\text{He}$ , using several empirical information. To this end, we first discuss the new prescription for  $\langle r_{c-2n}^2 \rangle$  by analyzing the Coulomb dissociation cross sections of these nuclei with a three-body model. Assuming the three-body character, one can also extract the distance between two neutrons,  $\sqrt{\langle r_{nn}^2 \rangle}$ , from the empirical information of matter radii and  $\langle r_{c-2n}^2 \rangle$  [see Eq. (3)]. An alternative way to extract the value for  $\sqrt{\langle r_{nn}^2 \rangle}$  is the three-body correlation study in the dissociation of two neutrons in halo nuclei [12]. We discuss the two ways to determine the mean opening angle by using these empirical information.

The three-body model that we employ in this article is exactly the same as that used in Refs. [13–15]. The model adapts a density-dependent contact interaction between the valence neutrons [8,9]. The recoil kinetic energy of the core nucleus is taken into account as in Ref. [15]. Single-particle continuum states are discretized by putting a nucleus in a large box. The wave functions for the ground state with  $J^\pi = 0^+$  and for the excited  $J^\pi = 1^-$  states are then obtained by diagonalizing the three-body Hamiltonian within a large model space that is consistent with the  $nn$  interaction. We use the same values for the parameters as in Ref. [13].

The dipole strength distributions for the  $^6\text{He}$  and  $^{11}\text{Li}$  nuclei obtained with this model are shown in Fig. 1.<sup>1</sup> Also

<sup>1</sup>We have found that the matrix elements for the off-diagonal part of the recoil kinetic energy were not properly evaluated for the  $J^\pi = 1^-$  states in Fig. 5 of Ref. [13]. This technical problem has been cured in Fig. 1 of the present article (it had been cured in the  $B(E2)$  calculations of  $^{16}\text{C}$  in Ref. [16]). Although this error did not cause any substantial change in  $^{11}\text{Li}$ , the  $B(E1)$  distribution for  $^6\text{He}$  is considerably different from the previous calculation. The energy of the first peak in the  $B(E1)$  distribution is now at 1.75 and 0.66 MeV for  $^6\text{He}$  and  $^{11}\text{Li}$ , respectively (previously, it was at 1.55 and 0.66 MeV for  $^6\text{He}$  and  $^{11}\text{Li}$ , respectively). These values are still close to  $1.6S_{2n}$ , where  $S_{2n}$  is the 2n separation energy, and our conclusion that the dineutron correlation plays an important role in these nuclei remains unchanged.

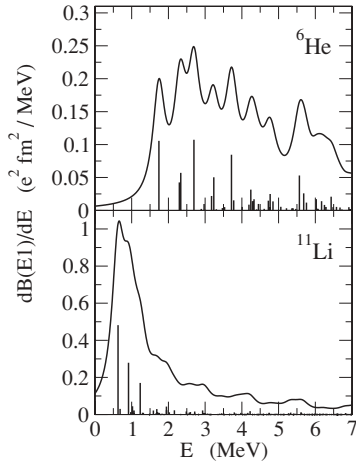


FIG. 1. The  $B(E1)$  distribution for the  ${}^6\text{He}$  and  ${}^{11}\text{Li}$  nuclei. The solid curve is obtained with a smearing procedure with  $\Gamma = 0.2$  MeV.

shown by the solid curves are the  $B(E1)$  distributions smeared with the Lorentzian function with the width of  $\Gamma = 0.2$  MeV. For the  ${}^6\text{He}$  nucleus, we obtain the total  $B(E1)$  strength of  $0.660 e^2\text{fm}^2$  up to  $E \leq 5$  MeV and  $1.053 e^2\text{fm}^2$  up to  $E \leq 10$  MeV. These are in good agreement with the experimental values,  $B(E; E \leq 5 \text{ MeV}) = 0.59 \pm 0.12 e^2\text{fm}^2$  and  $B(E; E \leq 10 \text{ MeV}) = 1.2 \pm 0.2 e^2\text{fm}^2$  [7]. For the  ${}^{11}\text{Li}$  nucleus, we obtain the total  $B(E1)$  strength of  $1.405 e^2\text{fm}^2$  up to  $E_{\text{rel}} = E - S_{2n} \leq 3$  MeV, which is compared to the experimental value,  $B(E1; E_{\text{rel}} \leq 3 \text{ MeV}) = 1.42 \pm 0.18 e^2\text{fm}^2$  [6]. Again, the experimental data is well reproduced within the present model. In Ref. [10], it is proposed to estimate the experimental value for  $\langle r_{c-2n}^2 \rangle$  using the relation

$$\langle r_{c-2n}^2 \rangle_{\text{exp}} = \frac{B(E1; E \leq E_{\text{max}})_{\text{exp}}}{B(E1; E \leq E_{\text{max}})_{\text{cal}}} \cdot \langle r_{c-2n}^2 \rangle_{\text{cal}}. \quad (2)$$

From the calculated values for  $\langle r_{c-2n}^2 \rangle_{\text{cal}}$ , that is, 13.2 and 26.3  $\text{fm}^2$  for  ${}^6\text{He}$  and  ${}^{11}\text{Li}$ , respectively, we thus obtain  $\sqrt{\langle r_{c-2n}^2 \rangle_{\text{exp}}} = 3.878 \pm 0.324$  fm and  $5.15 \pm 0.327$  fm for  ${}^6\text{He}$  and  ${}^{11}\text{Li}$ , respectively. Notice that the value for the  ${}^6\text{He}$  nucleus is somewhat larger than the one estimated in Ref. [7], that is,  $3.36 \pm 0.39$  fm.

We next evaluate the Coulomb breakup cross sections, especially paying attention to the recoil effect of the core nucleus. Based on the relativistic Coulomb excitation theory [17,18], the cross sections are obtained by multiplying the virtual photon number  $N_{E1}(E)$  to the  $B(E1)$  distribution shown in Fig. 1. The solid curves in Figs. 2 and 3 show the Coulomb breakup cross sections thus obtained for the  ${}^6\text{He} + \text{Pb}$  reaction at 240 MeV/nucleon [7] and the  ${}^{11}\text{Li} + \text{Pb}$  reaction at 70 MeV/nucleon [6], respectively. To facilitate the comparison with the experimental data, we smear the discretized cross sections with the Lorentzian function with an energy-dependent width,  $\Gamma = \alpha \cdot \sqrt{E_{\text{rel}}}$ . We take  $\alpha = 0.15$  and  $0.25 \text{ MeV}^{1/2}$  for  ${}^6\text{He}$  and  ${}^{11}\text{Li}$ , respectively. We see that the experimental breakup cross sections are reproduced remarkably well within the present three-body model, especially for the  ${}^{11}\text{Li}$  nucleus.

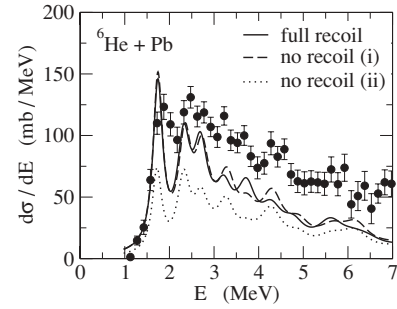


FIG. 2. Coulomb breakup cross sections for  ${}^6\text{He} + \text{Pb}$  at 240 MeV/nucleon. The solid line is the result of the full three-body calculations, while the dashed line is obtained by neglecting the off-diagonal component of the recoil kinetic energy in the excited states. The dotted line is obtained by neglecting the off-diagonal recoil term both in the ground and the excited states. These results are smeared with an energy-dependent width of  $\Gamma = 0.15 \cdot \sqrt{E_{\text{rel}}}$  MeV. The experimental data are taken from Ref. [7].

One of the advantages of using the contact interaction is that the continuum response can be calculated relatively easily [9]. In the presence of the recoil kinetic energy of the core nucleus, however, this advantage disappears because the off-diagonal part of the recoil energy,  $\mathbf{p}_1 \cdot \mathbf{p}_2 / (A_c m)$  (the last term in Eq. (1) of Ref. [13]), is a finite range two-body term, although the diagonal part,  $(\mathbf{p}_1^2 + \mathbf{p}_2^2) / (2A_c m)$ , can be easily included through the reduced mass. To examine the effect of the recoil term, Figs. 2 and 3 compare the Coulomb breakup cross sections calculated by taking into account the recoil term exactly (the solid curves) with those calculated approximately (the dashed and dotted curves). For the dashed curves, the off-diagonal component of the recoil kinetic energy is neglected in the excited  $J^\pi = 1^-$  states, while it is fully taken into account in the ground state. It is interesting to notice that these calculations lead to results similar to the one in which the recoil term is treated exactly (the solid curves). The continuum response was obtained in this way in Ref. [10] for the  ${}^{11}\text{Li}$  nucleus. The dotted curves, on the other hand, are obtained by neglecting the off-diagonal part of the recoil term both for the ground and the  $J^\pi = 1^-$  states. For this calculation, we slightly

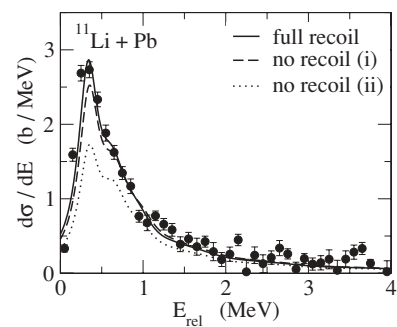


FIG. 3. Coulomb breakup cross sections for the  ${}^{11}\text{Li} + \text{Pb}$  at 70 MeV/nucleon. The meaning of each line is the same as described in Fig. 2. The calculated results are smeared with an energy-dependent width of  $\Gamma = 0.25 \cdot \sqrt{E_{\text{rel}}}$  MeV. The experimental data are taken from Ref. [6].

readjust the parameters of the pairing interaction so that the ground state energy remains the same. By neglecting the recoil term in the ground state, the value for  $\langle r_{c-2n}^2 \rangle$  decreases, from 13.2 to 9.46 fm<sup>2</sup> for <sup>6</sup>He and from 26.3 to 20.58 fm<sup>2</sup> for <sup>11</sup>Li. Consequently, the  $B(E1)$  distribution as well as the breakup cross sections are largely underestimated [15]. The fraction of the main components in the ground state wave function is also altered by neglecting the recoil term: for <sup>6</sup>He, the fraction of the  $(p_{3/2})^2$  component changes from 83.0 to 90.8%; and for <sup>11</sup>Li, the fraction of the  $(s_{1/2})^2$  component changes from 22.6 to 17.1%, and the fraction of the  $(p_{1/2})^2$  component from 59.1 to 65.7%. These results clearly indicate that the recoil term is important for the ground state, while it has a rather small effect on the excited states.

Let us now discuss the geometry of the <sup>6</sup>He and <sup>11</sup>Li nuclei. Using the experimental value for  $\langle r_{c-2n}^2 \rangle$  obtained from the  $B(E1)$  distribution, one can extract the mean opening angle between the valence neutrons once additional information is available. In Ref. [6], Nakamura *et al.* used the noncorrelated calculation for the  $B(E1)$  distribution given in Ref. [9] to estimate the rms distance between the core nucleus and one of the valence neutrons and obtained the mean opening angle of  $\langle \theta_{nn} \rangle = 48_{-18}^{+14}$  degrees. However, this method is highly model dependent and also it is not obvious whether the noncorrelated calculation is reasonable to estimate the rms distance. The mean opening angle can be extracted more directly when the rms distance between the valence neutrons,  $\langle r_{nn}^2 \rangle$ , is available (see Fig. 4). This quantity is related to the matter radius and  $\langle r_{c-2n}^2 \rangle$  in the three-body model [8,10,15,19],

$$\langle r_m^2 \rangle = \frac{A_c}{A} \langle r_m^2 \rangle_{A_c} + \frac{2A_c}{A^2} \langle r_{c-2n}^2 \rangle + \frac{1}{2A} \langle r_{nn}^2 \rangle, \quad (3)$$

where  $A_c = A - 2$  is the mass number of the core nucleus. The matter radii  $\langle r_m^2 \rangle$  can be estimated from interaction cross sections. Employing the Glauber theory in the optical limit, Tanihata *et al.* obtained  $\sqrt{\langle r_m^2 \rangle} = 1.57 \pm 0.04, 2.48 \pm 0.03, 2.32 \pm 0.02,$  and  $3.12 \pm 0.16$  fm for <sup>4</sup>He, <sup>6</sup>He, <sup>9</sup>Li, and <sup>11</sup>Li, respectively [20]. Using these values, we obtain the rms neutron-neutron distance of  $\sqrt{\langle r_{nn}^2 \rangle} = 3.75 \pm 0.93$  and  $5.50 \pm 2.24$  fm for <sup>6</sup>He and <sup>11</sup>Li, respectively. Combining these values with the rms core-dineutron distance,  $\sqrt{\langle r_{c-2n}^2 \rangle}$ , obtained with Eq. (2), we obtain the mean opening angle of  $\langle \theta_{nn} \rangle = 51.56_{-12.4}^{+11.2}$  and  $56.2_{-21.3}^{+17.8}$  degrees for <sup>6</sup>He and <sup>11</sup>Li, respectively. These values are comparable to the result of

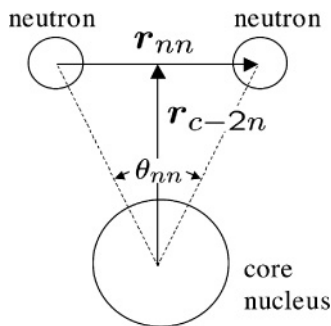


FIG. 4. Geometry of a 2n halo nucleus consisting of a core nucleus and two valence neutrons.

the three-body model calculation,  $\langle \theta_{nn} \rangle = 66.33$  and  $65.29$  degrees for <sup>6</sup>He and <sup>11</sup>Li, respectively [13], although the experimental values are somewhat smaller. We should remark here that it is misleading to say that the two neutrons are mostly sitting with an opening angle obtained in this way. Instead, the mean opening angle is most probably an average of a smaller and a larger correlation angle in the density distribution as has been suggested in Ref. [13].

An alternative way to extract the value for  $\sqrt{\langle r_{nn}^2 \rangle}$  has been proposed that uses the three-body correlation study in the dissociation of two neutrons in halo nuclei [12]. The two-neutron correlation function provides the experimental values for  $\sqrt{\langle r_{nn}^2 \rangle}$  to be  $5.9 \pm 1.2$  and  $6.6 \pm 1.5$  fm for <sup>6</sup>He and <sup>11</sup>Li, respectively [12]. Very recently, Bertulani and Hussein used these values to estimate the mean opening angles and obtained  $\langle \theta_{nn} \rangle = 83_{-10}^{+20}$  and  $66_{-18}^{+22}$  degrees for <sup>6</sup>He and <sup>11</sup>Li, respectively [11]. When one adopts the presently obtained value for  $\sqrt{\langle r_{c-2n}^2 \rangle}$  with Eq. (2) instead of those in Refs. [6,7], one obtains  $\langle \theta_{nn} \rangle = 74.5_{-13.1}^{+11.2}$  and  $65.2_{-13.0}^{+11.4}$  degrees for <sup>6</sup>He and <sup>11</sup>Li, respectively. Notice that these values are in better agreement with the results of the three-body calculation [13], especially for the <sup>6</sup>He nucleus, as compared to the values obtained by Bertulani and Hussein [11]. We summarize our results in Table I. One should notice that there are still large uncertainties in the empirical values of  $\sqrt{\langle r_{nn}^2 \rangle}$  and, consequently, in the average opening angles  $\langle \theta_{nn} \rangle$  as listed in Table I. It is still an open challenging problem to determine experimentally the values for  $\sqrt{\langle r_{nn}^2 \rangle}$  with higher precision.

In summary, we used the three-body model with a density-dependent contact interaction to analyze the  $B(E1)$  distributions as well as the Coulomb breakup cross sections of the <sup>6</sup>He and <sup>11</sup>Li nuclei. We showed that the strong concentration of the  $B(E1)$  strength near the continuum threshold can be well reproduced with the present model for both the nuclei. We also examined the recoil effect of the core nucleus on the Coulomb breakup cross sections. It is shown that the recoil effect plays an important role in the ground state while it may be neglected in the excited states. Using the calculated  $B(E1)$  strength, we extracted the experimental value for the rms distance between the core and the center of two neutrons, which was then converted to the mean opening angle of the two valence neutrons from the core nucleus. We found that

TABLE I. The geometry of the <sup>6</sup>He and <sup>11</sup>Li nuclei extracted from various experimental data. The mean opening angles calculated with the three-body model [13] are given in the last line for each nucleus in the table.

Nucleus	$\sqrt{\langle r_{c-2n}^2 \rangle}$ (fm)	$\sqrt{\langle r_{nn}^2 \rangle}$ (fm)	Method	$\langle \theta_{nn} \rangle$ (degree)
<sup>6</sup> He	$3.88 \pm 0.32$	$3.75 \pm 0.93$	Matter radii	$51.6_{-12.4}^{+11.2}$
		$5.9 \pm 1.2$	2n correlations	$74.5_{-13.1}^{+11.2}$
				66.33 [13]
<sup>11</sup> Li	$5.15 \pm 0.33$	$5.50 \pm 2.24$	Matter radii	$56.2_{-21.3}^{+17.8}$
		$6.6 \pm 1.5$	2n correlations	$65.2_{-13.0}^{+11.4}$
				65.29 [13]

the mean opening angles thus obtained are in good agreement with the results of the three-body model calculation.

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in the  ${}^6\text{He}$  and  ${}^{11}\text{Li}$  nuclei. This work was supported by the Japanese Ministry of Education, Culture, Sports, Science and Technology by a Grant-in-Aid for Scientific Research under Program Number 19740115.

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