Λ-hypernuclei single-particle energies with the Nijmegen ESC04 baryon-baryon interaction

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Λ-hypernuclei single-particle energies are calculated for the new Nijmegen ESC04 baryon-baryon interaction. A Brueckner-Hartree approach is employed. Whereas good agreement with experimental results is obtained for the $A = 3$, 4, and 5 systems, the heavier systems are considerably overbound. A possible source for some of the overbinding is the spin-orbit odd interaction and its effect on the $^3P_2$ channel.

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Several versions of Nijmegen baryon-baryon (BB) potentials have become available in the past several years. The series began with models D and F [1], the letters corresponding to the dominant invariant coupling employed. These potentials were derived from one boson exchange (OBE) with SU(3) symmetry in the coupling constants and required hard cores to fit the scattering data. The soft-core potential appeared in 1989 [2]. Although essentially a OBE potential, this version allowed for the exchange of Regge trajectories. Form factors replaced the hard cores, and nonlocal contributions to the potentials were retained. The NSC97 potential [3] followed. This version included some two-meson exchange contributions and employed the $^3P_0$ quark-antiquark pair creation model as a means of breaking SU(3) symmetry. The ESC04 [4,5] models improved upon NSC97 by introducing a zero in the form factor of the scalar mesons and including exchange of the axial-vector mesons. The zero in the form factor removes the unwanted bound state in the ΛN system.

Reliable forms for BB potentials are required for studies of hypernuclear structure, hypernuclear formation and decay rates, and the effect of strangeness on dense neutron matter. Therefore, fits to the YN scattering data must be tested in bound systems. The (π^+, K^+) reaction has provided valuable data for making these tests. The large momentum transfer in this reaction has allowed investigators to identify the Λ single-particle energies. These energies can be compared with the corresponding Brueckner-Hartree calculations. The procedure for these calculations is described in Refs. [6] and [7]. Briefly, the four-component Bethe-Goldstone equation is solved to obtain reference spectrum g-matrix elements. These are then $Q - 1$ corrected to the QTQ propagator, $Q/[\omega - Q(T_Y + T_N + \Delta mc^2\delta_{Y\Sigma})Q]$. The single-particle energies are calculated according to

$$\varepsilon_\Lambda = \langle\Lambda\,|T|\,\Lambda\rangle + \sum_N \langle\Lambda N\,|g(\omega = \varepsilon_\Lambda + \varepsilon_N)|\,\Lambda N\rangle, \qquad (1)$$

all with harmonic oscillator wave functions. A coordinate space effective interaction is then fit to the g-matrix elements and a Hartree correction determined as the difference between a calculation with oscillator wave functions and a calculation where the Λ wave function is expanded as a sum of oscillator wave functions, measured from the center of mass of the core nucleus. The expansion coefficients are chosen to minimize the energy. This Hartree correction is added to Eq. (1). For $^5_\Lambda$He the process must be iterated in ω.

The results for the open shell systems, $^3_\Lambda$H, $^4_\Lambda$H(0^+), and $^4_\Lambda$H(1^+), do not include a Hartree correction but are calculated with the approximate equations given in Ref. [8]. Although the $^3_\Lambda$H, $^4_\Lambda$H(0^+), and $^4_\Lambda$H(1^+) results in Table I should only be accurate to ±300 keV, they, along with the more accurate $^5_\Lambda$He calculation, demonstrate that the ESC04a interaction does well for $0s$-shell Λ hypernuclei.

Of the four interactions available in the package on the Web site www.nn-online.org, interaction ESC04a gives the best results for the single-particle energies. It should be noted that this version of the ESC04 turns off the nonlocal tensor interaction, presumably because it is small. The results for $^5_\Lambda$He, $^{13}_\Lambda$C, $^{17}_\Lambda$O, $^{41}_\Lambda$Ca, $^{49}_\Lambda$Ca, $^{91}_\Lambda$Zr, and $^{209}_\Lambda$Pb are shown in Fig. 1 as solid and dashed lines along with the data of Refs. [9–14]. The oscillator constants used are $b = 1.394$, 1.664, 1.793, 1.970, 1.970, 2.205, and 2.516 fm, respectively. The solid lines connect the $j = l + 1/2$ energies and the dashed lines the $j = l - 1/2$. The data points corresponding to a particular value of l are connected with a dotted line. The highest dotted line should be compared with the highest solid line, and so forth. One can see that the calculations are providing considerably more binding than that observed in the heavier systems. Similar calculations for model D gave underbinding for the light systems and overbinding for the heavier systems [15]. The soft core underbound all systems [15]. A refit of the soft core parameters gave reasonable fits to all but the inner shell of the heavier systems [7].

One difference between the ESC04a and the refit soft core (RSC) is the strength of the attractive $^3P_2$ channel. In Table II shows the relative reference g-matrix elements, $\langle n^3P_2(\Lambda)|V|n'^3P_2(\Lambda)\rangle$, for ESC04a, RSC, and ESC04a with the spin-orbit potential set to zero. The matrix elements are for $b = 1.970$ fm, $N_{CM} = L_{CM} = 0$, and $\omega = -30$ MeV. The ESC04 matrix elements are about twice the size of the RSC matrix elements. Also, the ESC04 matrix elements continue to increase in magnitude from $(n, n') = (3, 0)$ to (4,0). When the spin-orbit interaction is turned off, the ESC04 matrix elements become small and decrease in magnitude from $(n, n') = (3, 0)$ to (4,0).

TABLE I. $-B_\Lambda$ (MeV).

	ESC04a	Exp.
${}^{3}_{\Lambda}$H	0.14	−0.13
${}^{4}_{\Lambda}$H(1^+)	−1.33	−1.15
${}^{4}_{\Lambda}$H(0^+)	−2.04	−2.20
${}^{5}_{\Lambda}$He	−3.00	−3.10

TABLE II. The relative reference matrix elements $\langle n^3P_2(\Lambda)|V|n'^3P_2(\Lambda)\rangle$ (MeV).

$n\backslash n'$	0	1	2	3	4
0 ESC04a	−0.50	−0.67	−0.75	−0.79	−0.81
1	−0.67	−0.91	−1.05	−1.13	−1.17
2	−0.75	−1.05	−1.23	−1.34	−1.40
3	−0.79	−1.13	−1.34	−1.48	−1.56
4	−0.81	−1.17	−1.40	−1.56	−1.67
0 RSC	−0.34	−0.43	−0.46	−0.47	−0.45
1	−0.43	−0.57	−0.62	−0.64	−0.64
2	−0.46	−0.62	−0.71	−0.74	−0.76
3	−0.47	−0.64	−0.74	−0.80	−0.83
4	−0.45	−0.64	−0.76	−0.83	−0.87
0 ESC04a No S-O	−0.20	−0.25	−0.26	−0.25	−0.24
1	−0.25	−0.32	−0.34	−0.35	−0.34
2	−0.26	−0.34	−0.39	−0.41	−0.41
3	−0.25	−0.35	−0.41	−0.44	−0.44
4	−0.24	−0.34	−0.41	−0.44	−0.46

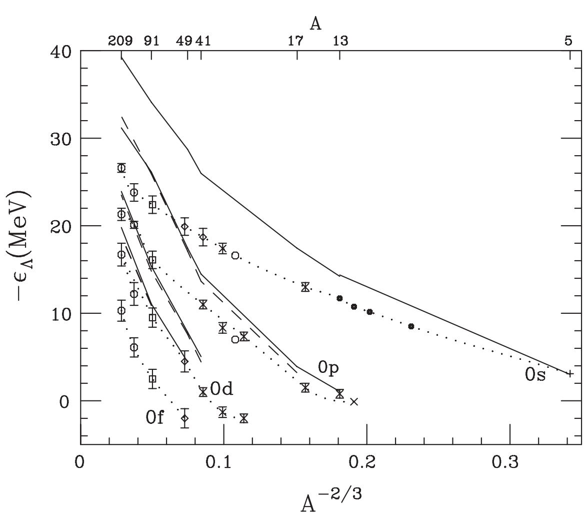

FIG. 1. Calculated single-particle energies including the Hartree correction are connected with straight lines. The $j = l - 1/2$ states are connected with dashed lines and the $j = l - 1/2$ with solid lines. Data are from Refs. [9–14]. Data points with the same l are connected with dotted lines.

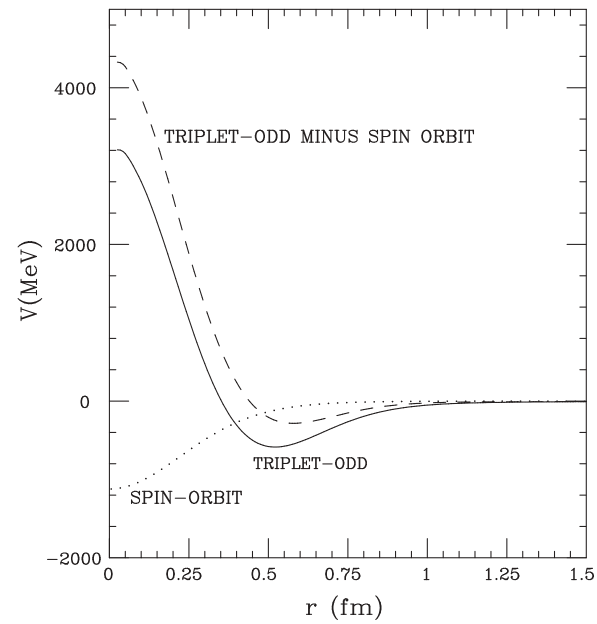

FIG. 2. The triplet-odd local potential. Solid line is the potential; dotted line is the spin-orbit contribution; dashed line is the potential minus the spin-orbit contribution.

Figure 2 shows the triplet-odd local potential, the local spin-orbit odd potential contribution, and the local triplet-odd without the spin-orbit potential. One can see that the spin-orbit contribution is substantial. Shown in Fig. 3 are the single-particle energies with no Hartree correction for the ESC04a as solid lines and the ESC04a with no Hartree correction when the relative ${}^3P_2$ matrix elements are set to zero as dashed lines.

The ${}^3P_2$ matrix elements do not contribute to the binding of the 0s-shell hypernuclei in lowest order, but outside the 0s-shell, they begin to contribute. Also, the higher the mass, the more the matrix elements with higher n contribute. Figure 2

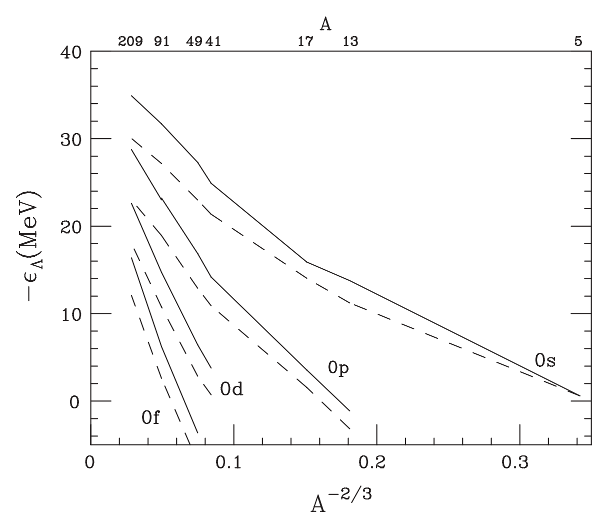

FIG. 3. Single-particle energies with no Hartree correction and for $j = l + 1/2$. Solid lines are with full potential; dashed lines are with ${}^3P_2$ relative matrix elements set to zero.

demonstrates that about one-half of the overbinding comes from these matrix elements. The RSC, with its smaller triplet odd contribution, is less affected. In addition, the RSC has a larger contribution to the binding from the $\Lambda\Sigma$ coupling, and the contribution is reduced in heavier systems by Pauli blocking. Increasing the $\Lambda\Sigma$ coupling may be one mechanism for additional reduction of the ESC04 binding energies.

The influence of the $^3P_2$ matrix elements would be anticipated from Table XIX of Ref. [5], where the zero-momentum Λ potential energies at normal density are listed. Here one sees a substantial contribution from the $^3P_2$ channel, which pushes the potential below those from NSC97e-f. The NSC97e-f potentials have already been shown to overbind $^{16}_{\Lambda}$O [16].

One cannot simply create a new potential by reducing the spin-orbit interaction or reducing the triplet-odd interaction. Both are a result of many different meson exchanges. However, an attempt has been made to refit the parameters of the ESC04 YN interaction by including bound-state data as well as the scattering data as in Ref. [6]. The parameters FSB, $a_{PV}, \alpha_{PV}, \alpha_V^e, \alpha_V^m, \alpha_A, \alpha_S, \theta_S$ were varied. Unfortunately, no suitable solution was found.

In conclusion, it was found that the ESC04a interaction does well on the 0s-shell Λ-hypernuclei, but it tends to overbind the Λ in heavier systems. At least part of this overbinding can be traced to the triplet-odd interaction and the large spin-orbit contribution to this channel. The ESC04 would best be used in light systems and some caution should be exercised if it is used in heavy systems or infinite neutron matter.

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