Searching for the critical point of QCD: Theoretical benchmark calculations

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We present a comprehensive study of event-by-event multiplicity fluctuations in nucleon-nucleon and nucleus-nucleus interactions from the BNL Alternating Gradient Synchrotron/GSI Facility for Antiproton and Ion Research to BNL Relativistic Heavy Ion Collider energies within the ultrarelativistic quantum molecular dynamics transport approach. The scaled variances of negative, positive, and all charged hadrons are analyzed. The scaled variance in central Pb+Pb collisions increases with energy and behaves similar to inelastic $p+p$ interactions. We find a nontrivial dependence of multiplicity fluctuations on the rapidity and transverse-momentum interval used for the analysis and on the centrality selection procedure. Quantitative predictions for the NA49 experiment are given, taking into account the acceptance of the detector and the selection procedure of central events.

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I. INTRODUCTION

At high energy densities (\approx 1 GeV/fm³) a phase transition from a hadron gas to a quark-gluon plasma (QGP) is expected to occur. There are indications that at BNL Relativistic Heavy Ion Collider (RHIC) and top CERN Super Proton Synchrotron (SPS) energies a QGP is created in the early stages of heavy-ion collisions [\[1,2\]](#page-7-0). And indeed, the energy dependence of various observables show anomalies at low SPS energies that might be related to the onset of deconfinement [\[3,4\]](#page-7-0).

Although several observables [\[5\]](#page-7-0) have been proposed throughout the past decades to study the characteristics of the highly excited matter created in heavy-ion collisions the ones related to fluctuations and correlations seem to be the most prospective. Fluctuation probes might be more adequate for the exploration of heavy-ion reactions, because the distributions of energy density or initial temperature, isospin, and particle density have strong fluctuations from event to event [\[6–8\]](#page-7-0). On the theoretical side event-by-event fluctuations were suggested to study

- (i) kinetic and chemical equilibration in nuclear collisions $[9-17]$ $[9-17]$,
- (ii) the onset of the deconfinement phase $[4,18-25]$ $[4,18-25]$
- (iii) the location of the tricritical end point of the quantum chromodynamics (QCD) phase transition [\[26–28\]](#page-8-0) or
- (iv) the formation of exotic states, like disordered chiral condensates (DCCs) [\[29\]](#page-8-0).

On the experimental side, progress has been made by many experiments to extract momentum and particle number ratio fluctuations from heavy-ion reaction: Currently, event-by-event fluctuations are actively studied in the SPS energy regime (starting from 20*A* GeV) by the NA49 group [\[30–38\]](#page-8-0) and the CERES [\[39–42\]](#page-8-0) and WA98 Collaborations

[\[43\]](#page-8-0). At RHIC energies the PHENIX [\[44–46\]](#page-8-0) and STAR [\[47–49\]](#page-8-0) experiments are addressing the field of single-event physics.

In Ref. [\[23\]](#page-8-0) it was predicted that the onset of deconfinement should lead to a nonmonotonous behavior in multiplicity fluctuations ("shark fin"). Also droplet formation during the phase transition is expected to produce nonstatistical fluctuations 10–100 times the Poisson expectation [\[50\]](#page-8-0). Furthermore, lattice QCD calculations suggest the existence of a critical point in the phase diagram of strongly interacting matter that separates the first-order phase transition from a crossover. Thus, if the system passes the vicinity of the critical region during its evolution and remains there for long enough time one expects an increase of multiplicity fluctuations [\[27\]](#page-8-0).

The NA49 Collaboration is currently searching for such anomalies in the energy dependence of multiplicity fluctuations in Pb+Pb Collisions. A similar program to search for the critical point and signals for the onset of deconfinement will be undertaken by the RHIC experiments (the planned critRHIC program) with a lowering of the RHIC's beam energy toward the SPS energy regime and the NA61 (SHINE) experiment [\[51\]](#page-8-0) with the focus on light-ion collisions. For the present investigation, however, we will focus on the soon available data from the NA49 experiment on multiplicity fluctuations. Unfortunately both the geometrical acceptance of the detector and the centrality selection in the NA49 experiment is not trivial and have an influence on the multiplicity fluctuations. To observe an increase of fluctuations caused by one of the effects mentioned above, a systematic theoretical investigation within a transport approach is needed. Only with this baseline for the expected multiplicity fluctuations within the experimental acceptance a possible excess of fluctuations in data could be unambiguously interpreted as a signal for the critical point or the onset of deconfinement. The model predictions presented in this article are obtained using ultrarelativistic quantum molecular dynamics (UrQMD) model version 1.3 [\[52,53\]](#page-8-0). For a complementary transport theoretical study of multiplicity fluctuations, the reader is referred to Refs. [\[54–57\]](#page-8-0).

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II. MEASURE OF MULTIPLICITY FLUCTUATIONS

The probability to have in an event a given number of particles n in the acceptance is denoted as $P(n)$, with the normalization $\sum_{n} P(n) = 1$.

n The measure of multiplicity fluctuations used in this article is the scaled variance *ω* defined as

$$
\omega = \frac{\text{Var}(n)}{\langle n \rangle} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle},\tag{1}
$$

where $\text{Var}(n) = \sum_{n} (n - \langle n \rangle)^2 P(n)$ and $\langle n \rangle = \sum_{n} n \times P(n)$ are the variance and the mean of the multiplicity distributions, respectively.

This measure is used because of its two properties. First, in a grand canonical statistical model neglecting quantum effects the multiplicity is a Poisson distribution:

$$
P(n) = \frac{\langle n \rangle^n}{n!} \times \exp(-\langle n \rangle).
$$
 (2)

The variance of a Poisson distribution is equal to its mean; the scaled variance is therefore $\omega = 1$, independent of mean multiplicity.

Second, in a wounded nucleon model [\[58\]](#page-8-0), the scaled variance in $A + A$ collisions is the same as in proton-proton collisions provided the number of wounded nucleons is fixed. If the particles are produced independently in momentum space, the scaled variance in a limited acceptance is related to the scaled variance in full phase space (4π) :

$$
\omega_{\text{acc}} = p(\omega_{4\pi} - 1) + 1,\tag{3}
$$

where *p* is the fraction of tracks that are in the corresponding acceptance. For a small acceptance p the scaled variance approaches 1. Note that effects like resonance decays, quantum statistics and energy-momentum conservation introduce correlations in momentum space and therefore a scaling according to Eq. (3) is generally not valid.

In the following the scaled variance of positively, negatively, and all charged hadrons are denoted as $\omega(h^+)$, $\omega(h^-)$, and $\omega(h^{\pm})$, respectively.

III. THE URQMD MODEL

For our investigation, we apply the UrQMD (version 1.3) [\[52,53\]](#page-8-0) to heavy-ion reactions from $E_{\text{beam}} = 20A \text{ GeV}$ to $E_{\text{beam}} = 158A \text{ GeV}$. This microscopic transport approach is based on the covariant propagation of constituent quarks and diquarks accompanied by mesonic and baryonic degrees of freedom. It simulates multiple interactions of in-going and newly produced particles, the excitation and fragmentation of color strings and the formation and decay of hadronic resonances. Toward higher energies, the treatment of subhadronic degrees of freedom is of major importance. In the present model, these degrees of freedom enter via the introduction of a formation time for hadrons produced in the fragmentation of strings [\[59–61\]](#page-8-0). A phase transition to a quark-gluon state is not incorporated explicitly into the model dynamics. However, a detailed analysis of the model in equilibrium, yields an effective equation of state of Hagedorn type [\[62,63\]](#page-8-0).

This model has been used before to study event-by-event fluctuations rather successfully [\[8,](#page-7-0)[11,20,25,29,57,64,65\]](#page-8-0) and yields a reasonable description of inclusive particle distributions. For a complete review of the model, the reader is referred to Refs. [\[52,53\]](#page-8-0).

IV. ENERGY DEPENDENCE OF MULTIPLICITY FLUCTUATIONS

The energy dependence of the mean multiplicity, normalized by the number of nucleons of one projectile $(A = 1$ for $p+p$, $p+n$, $A = 208$ for Pb+Pb) of positively, negatively, and all charged particles in $p+p$, $p+n$, and Pb+Pb collisions is shown in Fig. 1. For $p+p$ and $p+n$ interactions all inelastic collisions are selected. For Pb+Pb the impact parameter of the collisions are set to $b = 0$. The calculations were performed

FIG. 1. (Color online) Mean multiplicity in 4π of inelastic $p+p$, $p+n$, and central Pb+Pb collisions as a function of collision energy. Top: positively, middle: negatively, bottom: all charged hadrons.

for BNL Alternating Gradient Synchrotron (AGS) (*E*lab = 6*.*87*A* GeV), SPS (*E*lab = 20*A,* 30*A,* 40*A,* 80*A*, and 158*A* GeV), and RHIC ($\sqrt{s_{NN}}$ = 62.4 and 200 GeV) energies. In the UrQMD 1.3 model the mean multiplicity per number of projectile nucleons is significantly larger in Pb+Pb collisions compared to $p+n$ interactions.

The mean multiplicity of all charged hadrons in $p+p$ interactions obtained by various experiments is parametrized in Ref. [\[66\]](#page-8-0) as

$$
\langle n^{\pm} \rangle \approx -4.2 + 4.69 \cdot (\sqrt{s_{NN}}/\text{GeV})^{0.31}.
$$
 (4)

Except for top RHIC energies the parametrization of the experimental data is in agreement with the UrQMD result.

The energy dependence of scaled variance in full phase space is shown in Figure 2.

An increase of scaled variance with increasing collision energy is observed for $p+p$, $p+n$, and Pb+Pb collisions. For AGS and low SPS energies the scaled variance is smaller than 1 and the multiplicity distributions are narrower than the corresponding Poisson distributions. For higher energies the scaled variance is larger than 1. A similar behavior of $p+p$ and $p+n$ collisions is observed, the small difference is probably caused by the additional proton in $p+p$ collisions, which does not fluctuate. Therefore the scaled variance for positively and all charged particles is a bit lower in $p + p$ than in $p+n$ collisions. The scaled variance in Pb+Pb collisions behaves similarly as in $p+p$ interactions. The HSD model yields similar results [\[55\]](#page-8-0).

For positively and negatively charged hadrons the scaled variance is similar, where the values are about twice as high for all charged hadrons. This is partly due to resonances decaying

FIG. 2. (Color online) Scaled variance in 4*π* of inelastic $p+p$, $p+n$, and central Pb+Pb collisions as a function of collision energy in comparison to hadron gas model predictions [\[67\]](#page-8-0) for Pb+Pb collisions. Top: positively, middle: negatively, bottom: all charged hadrons.

FIG. 3. (Color online) Scaled variance of positively charged hadrons produced in inelastic $p+p$, $p+n$ and central Pb+Pb collisions as a function of collision energy. (Top) $0 < y < y_{\text{beam}}$; (middle) $0 < y < 1$; (bottom) $1 < y < y$ _{beam}.

into two oppositely charged particles. Such a resonance is detected as two charged particles, therefore the fluctuations are increased.

The experimental data on the energy dependence of scaled variance of all charged hadrons in $p+p$ interactions is parametrized in Ref. [\[66\]](#page-8-0) as

$$
\omega(h^{\pm}) \approx 0.35 \cdot \frac{(\langle h^{\pm} \rangle - 1)^2}{\langle h^{\pm} \rangle}.
$$
 (5)

The UrQMD results are in agreement with the data at AGS and SPS energies, but the fluctuations are slightly overpredicted at RHIC energies.

Figure [2](#page-2-0) shows that the energy dependence of scaled variance predicted by the UrQMD model is totally different to the predictions of a hadron gas model [\[67\]](#page-8-0) (for further details

the reader is referred also to $[68–72]$). In the grand canonical (GCE), canonical (CE), and microcanonical (MCE) ensemble the scaled variance stays constant for high energies, where in the UrQMD model it strongly increases with energy. Therefore experimental data on multiplicity fluctuations, preferably at high (RHIC, LHC) energies, should be able to distinguish between hadron gas and string-hadronic models [\[55\]](#page-8-0).

For a more differential study of fluctuations and for a better comparison to experimental results, three different rapidity intervals, one at midrapidity $0 < y < 1$, one at forward rapidity $1 < y < y_{\text{beam}}$ and a combination of both $0 < y < y_{\text{beam}}$, covering most of the forward hemisphere, were taken. For $p+n$ collisions the forward hemisphere includes the rapidity of the projectile neutron. The scaled variance for these intervals for positively, negatively, and all charged particles are shown in Figs. [3–](#page-2-0)5.

FIG. 4. (Color online) Scaled variance of negatively charged hadrons produced in inelastic $p+p$, $p+n$ and central Pb+Pb collisions as a function of collision energy. (Top) $0 < y < y_{\text{beam}}$; (middle) $0 < y < 1$; (bottom) $1 < y < y_{\text{beam}}$.

FIG. 5. (Color online) Scaled variance of all charged hadrons produced in inelastic $p+p$, $p+n$ and central Pb+Pb collisions as a function of collision energy. (Top) $0 < y < y_{\text{beam}}$; (middle) $0 < y <$ 1; (bottom) $1 < y < y_{beam}$.

As in full phase space, in the three different rapidity intervals a similar behavior of scaled variance of $p+p$, $p+n$, and Pb+Pb collisions was observed.

The energy dependence of fluctuations in the forward hemisphere $(0 < y < y_{\text{beam}})$ looks similar to the one in the full phase space, the absolute number of scaled variance is similar to the result expected when applying the acceptance extrapolation according to formula [\(3\)](#page-1-0) (shown as stars in Figs. [3–](#page-2-0)[5\)](#page-3-0).

For low energies a large fraction of particles is in the midrapidity interval $(0 < y < 1)$, whereas a very small amount of particles is in the forward rapidity interval $(1 < y < y_{\text{beam}})$. With increasing energy both the width and the number of particles in the forward rapidity interval increases strongly, whereas the number of particles in the forward rapidity interval increases only weakly.

At midrapidity $(0 < y < 1)$ the scaled variance is in the same order of magnitude as in the rapidity interval $0 < y <$ *y*beam, but the mean multiplicity is much lower. The acceptance extrapolation formula [\(3\)](#page-1-0) strongly underpredicts fluctuations in this rapidity region. At forward rapidity $(1 < y < y_{beam})$ the fluctuations are much smaller than predicted by the acceptance extrapolation formula. For lower energies the scaled variance decreases with energy, for higher energies it increases. This can be qualitatively understood by the interplay of an increasing fraction of particles in this rapidity interval and an increasing scaled variance in 4π , which is smaller than 1 for lower and larger than 1 for higher energies.

V. RAPIDITY AND TRANSVERSE-MOMENTUM DEPENDENCE

As already shown in Sec. IV , the scaled variance is a nontrivial function of the selected phase space. To study the dependence of scaled variance on rapidity, 12 different rapidity intervals are constructed in such a way that the mean multiplicity in each interval is the same. If the scaled variance would follow the acceptance scaling formula (3) , the scaled variance would be the same in each interval. In Fig. 6 it is shown that this is not the case. The scaled variance is much higher near midrapidity than in forward and backward rapidities.

FIG. 6. Rapidity dependence of scaled variance in UrQMD simulation performed in full acceptance of positive (top left), negative (top right), and all charged (bottom) hadrons in central Pb+Pb collisions at 158*A* GeV. The rapidity bins are constructed in such a way that the mean multiplicity in each bin is the same.

FIG. 7. (Color online) Transverse-momentum dependence of multiplicity fluctuations of positively (top left), negatively (top right), and all charged hadrons (bottom) for all rapidities, $0 < y < 0.5$ and $1.25 < y < 1.75$. The transverse-momentum bins are constructed in such a way that the mean multiplicity in each bin is the same.

The transverse-momentum dependence of scaled variance is shown in Fig. 7 for the full longitudinal phase space and for a midrapidity and a forward rapidity interval. The scaled variance decreases with increasing transverse momentum for the full acceptance and at forward rapidity. At midrapidity it stays approximately constant. The decrease of scaled variance is stronger for positively charged hadrons than for negatively charged ones because the protons, which have smaller relative fluctuations due to the large number of protons that enter the collision, have a larger mean transverse momentum.

A similar effect of decreasing fluctuations for larger rapidities and transverse momenta is observed as a result of energy and momentum conservation in a hadron gas model using the microcanonical ensemble [\[73\]](#page-8-0). It costs more energy to create a particle with high momentum, therefore their number is expected to fluctuate less.

VI. PREDICTIONS FOR THE NA49 EXPERIMENT

Preliminary data of the NA49 experiment on the energy dependence of multiplicity fluctuations in very central Pb+Pb collisions was shown in Refs. [\[74,75\]](#page-8-0). Final data obtained in a larger geometrical acceptance will be published soon.

To compare the experimental data with model calculations, both the geometrical acceptance of the detector and the centrality selection have to be implemented in the model calculation.

The geometrical acceptance of the NA49 experiment [\[76\]](#page-8-0) is located mostly in the forward hemisphere. The acceptance defined by the detector geometry and the track selection criteria is different for each collision energy and a complicated function of the particle momentum \vec{p} . Acceptance tables in $y(\pi)$, p_T and ϕ can be obtained at the author's Web site (http://www.ikf.physik.uni-frankfurt. de/users/lungwitz/acceptance/). In the NA49 experiment it is not possible to identify a particle on the track-by-track basis, therefore for the calculation of rapidity in the fixed target laboratory system pion mass is assumed. The rapidity is then transformed into the center of mass system of the collision.

For the UrQMD model predictions showed in this section both the geometrical acceptance defined by the acceptance tables and the assumption of pion mass when calculating rapidity and transforming into the center-of-mass system are taken into account.

In the NA49 experiment the centrality of a collision can be measured by the energy of projectile spectators, which is registered by a calorimeter. This veto calorimeter is adjusted in such a way that it registers all spectator protons, neutrons, and fragments of the projectile. The lower the energy in the veto calorimeter the more central is the collision. For the multiplicity fluctuation analysis the 1% most central collisions are selected. For this selection, the fluctuations in the number of target participants is also minimized [\[57\]](#page-8-0). Remarks on the contribution of target participant fluctuations to multiplicity fluctuations are presented in Ref. [\[77\]](#page-8-0).

A small fraction of the produced particles in a collision is also entering the calorimeter and introducing a small bias on centrality measurement. Acceptance tables of the veto calorimeter as a function of p, p_T , and ϕ can be obtained on the author's Web site (http://www.ikf.physik.unifrankfurt.de/users/lungwitz/acceptance/) and are used for the UrQMD predictions of multiplicity fluctuations.

FIG. 8. UrQMD predictions for scaled variance of positively charged hadrons in NA49 acceptance produced in central Pb+Pb collisions as a function of collision energy. (Top) $0 < y < y_{\text{beam}}$, (middle) $0 < y < 1$; (bottom) $1 < y < y$ _{beam}.

FIG. 9. UrQMD predictions for scaled variance of negatively charged hadrons in NA49 acceptance produced in central Pb+Pb collisions as a function of collision energy. (Top) $0 < y < y_{\text{beam}}$; (middle) $0 < y < 1$; (bottom) $1 < y < y$ _{beam}.

The predictions for the energy dependence of scaled variance measured in the NA49 experiment are shown in Figs. 8[–10.](#page-7-0) To study the influence of centrality selection the scaled variance was also calculated for Pb+Pb collisions with a zero impact parameter *b*.

The UrQMD model predicts a weak energy dependence of scaled variance for positively and negatively charged hadrons in forward acceptance. At midrapidity, at full experimental acceptance and for all charged hadrons at all acceptances an increase of scaled variance with collision energy is predicted.

In forward acceptance a UrQMD simulation for events with zero impact parameter $(b = 0)$ gives similar results to the simulation for events selected according to their energy in the veto calorimeter. In midrapidity and in full experimental acceptance the scaled variance for events selected by their veto energy is larger, probably due to target participant fluctuations.

FIG. 10. UrQMD predictions for scaled variance of all charged hadrons in NA49 acceptance produced in central Pb+Pb collisions as a function of collision energy. (Top) $0 < y < y_{\text{beam}}$; (middle) $0 <$ $y < 1$; (bottom) $1 < y < y$ _{beam}.

VII. SUMMARY

We present predictions for event-by-event multiplicity fluctuations for very central Pb+Pb and nucleon + nucleon interactions from $E_{\text{lab}} = 20A$ GeV to $E_{\text{lab}} = 158A$ GeV within a hadron-string transport approach. We find that the fluctuations do generally increase strongly toward higher beam energies, both for elementary and massive nuclear

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reactions. This can be used to distinguish the present model from a hadron gas model, which predicts a rather weak dependence of the fluctuations on energy. The amount of fluctuations in full phase space is generally non-Poissonian (smaller at low energies, higher at high energies), crossing the Poissonian value in the SPS energy regime. Applying a forward rapidity cut yields a nonmonotonous behavior as a function of energy, with a local minimum at SPS energies. We further predict the rapidity dependence of the scaled variance at the highest SPS energy and find a strong rapidity dependence of the fluctuations, even if trivial multiplicity effects are scaled out. This might render the procedure to simply scale thermal model predictions for fluctuations to the experimentally covered phase space questionable. The transverse-momentum dependence of the fluctuations does generally tend to decrease toward higher transverse momenta. This effect is related to energy conservation, allowing stronger fluctuations for low energetic particles, while constraining the high energetic particles. Finally, we analyze the influence of the veto trigger used by the NA49 experiment, compared to simple zero impact parameter reactions. Here we observe a systematic deviation for the midrapidity results on the order of 10%, at forward rapidities, the veto trigger can be well approximated with a the zero impact parameter interaction.

The present study therefore provides a detailed baseline calculation for the search of critical phenomena in event-byevent multiplicity fluctuations. If nonmonotonous deviations from these predictions, as, e.g., expected by droplet formation, are observed, these enhanced fluctuations might indicate the onset of deconfinement and/or the critical point.

The NA49 Collaboration is currently studying the energy dependence of multiplicity fluctuations from 20*A* GeV to 158*A* GeV. Further detailed exploration is planned for the NA61 (SHINE) experiment [\[51\]](#page-8-0) at the CERN SPS. In addition the critRHIC experiment and the CBM experiment [\[78\]](#page-8-0) at the GSI Facility for Antiproton and Ion Research facility as well as and the MPD experiment at the Joint Institute for Nuclear Research in Dubna will be able to explore this energy region in the near future.

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