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Evolution of the proton *sd* states in neutron-rich Ca isotopes

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We analyze the evolution with increasing isospin asymmetry of the proton single-particle states 2s1/2 and 1d3/2 in Ca isotopes, using nonrelativistic and relativistic mean-field approaches. Both models give similar trends and it is shown that this evolution is sensitive to the neutron shell structure, the two states becoming more or less close depending on the neutron orbitals that are filled. In the regions where the states get closer some parametrizations lead to an inversion between them. This inversion occurs near ⁴⁸Ca as well as very far from stability where the two states systematically cross each other if the drip line predicted in the model is located far enough. We study in detail the modification of the two single-particle energies by using the equivalent potential in the Schroedinger-like Skyrme-Hartree-Fock equations. The role played by central, kinetic, and spin-orbit contributions is discussed. We finally show that the effect of a tensor component in the effective interaction considerably favors the inversion of the two proton states in ⁴⁸Ca.

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I. INTRODUCTION

Novel properties and new scenarios are expected for nuclei situated far from stability. The new generation of radioactive beam facilities will allow us to answer many open questions about the pecularities of these unstable systems. One of the major issues in the physics of exotic nuclei is the study of shell structure and magicity evolution when approaching the drip lines [1,2]. From a theoretical point of view, two aspects have been underlined as mainly responsible for the evolution of single-particle energies far from stability, the one-body spin-orbit potential that is strongly modified when the surface becomes more diffuse [1] and the tensor force between neutrons and protons in valence subshells [3].

Recently, the N = 28 shell closure has been experimentally analyzed in the 46 Ar $(d, p){}^{47}$ Ar transfer reaction [4]. A strong reduction of the neutron p spin-orbit splitting has been observed in ⁴⁷Ar with respect to the isotone ⁴⁹Ca. Because *p* states are mainly localized in the interior of the nucleus, this strong reduction cannot be justified by the presence of a diffuse surface that would affect only high-*l* states mainly concentrated at the surface. A theoretical analysis based on the relativistic mean field (RMF) approach has been proposed by Todd-Rutel et al. [5]. A strong reduction of the spin-orbit splitting for neutron 2p states is found in ⁴⁶Ar as compared to ⁴⁸Ca. At Z = 20, the state 2s1/2 is usually located between 1d5/2 and 1d3/2. In the RMF calculations of Ref. [5], however, 2s1/2 is less bound than 1d3/2 in both ⁴⁶Ar and ⁴⁸Ca (2s1/2-1d3/2 inversion). In this scenario, 2s1/2 is empty in ⁴⁶Ar and occupied in ⁴⁸Ca: thus, the proton density profile in ⁴⁶Ar presents a strong depletion in the interior of the

nucleus. This reduction of the charge density in the center would be responsible for the modification of the spin-orbit in the nuclear interior and, hence, for the reduction of the neutron 2p splitting.

This problem of 2s1/2-1d3/2 inversion of the proton states has been already analyzed by Campi and Sprung [6] within the Hartree-Fock (HF) + BCS model with an interaction derived from a *G* matrix [7]. ³⁶Ar was found as a candidate for this inversion. Skyrme forces do not lead to any inversion in this nucleus. It is thus worthwhile to revisit the problem for other nuclei in this region of the nuclear chart in the framework of the Skyrme-HF model.

In this work, we analyze the evolution of the s-d proton single-particle states in Ca isotopes and the possible 2s1/2-1d3/2 inversions. We also present some comparisons with the corresponding results obtained within RMF. We neglect pairing in our treatment because Ca isotopes are proton closed-shell nuclei. We have checked that, within RMF the inclusion of neutron pairing does not modify in a significant way the evolution of the proton states we are interested in. The only important effect due to pairing is the shift of the drip line toward heavier isotopes (for example, the drip line is shifted from ⁶⁰Ca to ⁷⁶Ca with the parametrization NL3 [8]). However, this aspect is not relevant in the present analysis, which is not intended to make any prediction on the drip line position. We choose the Ca isotopes because experimental signals for the inversion phenomena have been found at least in one of these isotopes, ⁴⁸Ca: the ground state of ⁴⁷K (one proton less than 48 Ca) is $1/2^+$ with a large spectroscopic factor [9] and the single-particle spectrum of 48 Ca has been measured, the proton state 1d3/2 being more bound than 2s1/2 by about 300 keV [10]. We mention that proton centroids extracted from (d, 3H) reactions show that the two states are almost degenerate [11]. In our analysis, we explore all the contributions, kinetic, central, spin-orbit, and tensor, which can modify the single-particle energies with increasing A and we show that not only the spin-orbit and tensor terms are determinant. The role of the central mean-field term is in particular discussed. Within the models that lead to the crossing between the two states, we show that this inversion occurs near ⁴⁸Ca as well as in very neutron-rich nuclei close to the drip line.

The article is organized as follows. In Sec. II we study the evolution with increasing A of the difference $\Delta \epsilon$ between the 2s1/2 and 1d3/2 energies obtained within nonrelativistic and relativistic approaches. In Sec. III we concentrate on the nonrelativistic case and perform a detailed analysis of the results. The different contributions to $\Delta \epsilon$ are isolated by analyzing them with the equivalent potential in the Schroedinger-like HF equations. In Sec. IV the effect of the tensor force is estimated in the framework of the SLy5-HF [12] model. Finally, conclusions are drawn in Sec. V.

II. EVOLUTION OF 2s1/2 AND 1d3/2 PROTON STATES WITHIN SKYRME-HF AND RMF

We first perform a preliminary study with HF calculations of ⁴⁸Ca using different Skyrme interactions. We then choose three representative forces: SkI5 [13], which gives a 2s1/2-1d3/2 inversion with an energy difference $\Delta\epsilon$ of ~800 keV; SGII [14], which also reproduces the inversion ($\Delta\epsilon \sim 200$ keV); and SLy4 [12] for which there is no inversion. With the three selected parametrizations we have systematically analyzed the Ca isotopes from ⁴⁰Ca up to the HF two-neutron drip line. We recall that, in the three considered Skyrme parametrizations there is no explicit tensor force.

We show in Fig. 1 the difference $\Delta \epsilon$ between the energies of the proton states 2s1/2 and 1d3/2 for the three Skyrme forces. The inversion takes place where $\Delta \epsilon$ is positive. Corrections to the individual energies due to the coupling of single-particle motion with collective vibrations, which are neglected in our



FIG. 1. Difference between the energies of the 2s1/2 and 1d3/2 proton states calculated with the Skyrme interactions SkI5, SGII, and SLy4 for Ca isotopes.

treatment, should be expected (see, for instance, Ref. [15]). However, by considering the energy difference instead of the individual single-particle energies the effects of these corrections should be reduced, the coupling to vibrations having the tendency of shifting upward the energies of both occupied states.

We observe that the SLy4-HF calculations never lead to the 2s1/2-1d3/2 inversion. However, both SkI5-HF and SGII-HF models present this inversion around ⁴⁸Ca as well as for more neutron-rich isotopes starting from ⁵⁸Ca up to the drip line. The HF two-neutron drip line is located at ⁸²Ca, ⁷⁸Ca, and ⁶⁰Ca with SkI5, SGII, and SLy4, respectively. The two experimental points for ⁴⁰Ca and ⁴⁸Ca are also included in the figure. These points represent the energy splitting between the $1/2^+$ and $3/2^+$ states in ³⁹K and ⁴⁷K, respectively. The three sets of results globally present the same behavior. Indeed, in all three cases the quantity $\Delta \epsilon$ starts from a negative value and increases from A = 40 to A = 48. This generates a 2s1/2-1d3/2 inversion with SkI5 (in ⁴⁶Ca, ⁴⁸Ca, and ⁵⁰Ca) and with SGII (in ⁴⁸Ca). Going from ⁴⁸Ca up to ⁵²Ca the states cross again with SkI5 and SGII, whereas the distance between them increases with SLy4. These results agree with the experimental indications related to first-forbidden β -decay measurements [16]: the ground state of ⁵⁰K has been assigned as $J^{\pi} = 0^{-}$ and low-energy levels in ⁵⁰K are dominated by the $(\pi d3/2)^{-1} (\nu p3/2)^{1}$ configuration. This is an indication that the inversion is not present at N = 31 in the K chain and, thus, at N = 32 in the Ca chain. Beyond ⁵²Ca $\Delta \epsilon$ increases again with the three parametrizations. This generates another inversion with SkI5 and SGII starting from ⁵⁸Ca. We notice also that the nuclei for which $\Delta \epsilon$ presents maxima or minima are the same for the three Skyrme forces.

A natural question to ask is whether the above general trends are specific of the Skyrme-HF approach. It is well known that the RMF approach gives a spin-orbit potential whose (N-Z) dependence is somewhat different from that of Skyrme-HF models [17]. We have performed RMF calculations with different parametrizations for the same set of Ca isotopes using the parametrizations DDME1 [18], NL3 [8], and NLB2 [19]. The latter one is chosen as an example of RMF model that does not lead to a 2s1/2-1d3/2 level inversion. The calculated values of $\Delta \epsilon$ are shown in Fig. 2 up to ⁶⁰Ca which is the two-neutron drip line isotope predicted by DDME1 and NL3. Globally, we observe for $\Delta \epsilon$ the same trend as that obtained within the nonrelativistic HF, with maxima and minima corresponding to the same nuclei, ⁴⁸Ca and ⁵²Ca. Because comparable trends are obtained in both nonrelativistic and relativistic approaches we conclude that the calculated evolution of 2s1/2 and 1d3/2 states is a generic behavior. We can thus explore more in detail the results by considering only the nonrelativistic case.

III. ANALYSIS OF THE CONTRIBUTIONS TO $\Delta \epsilon$

We now concentrate on the maxima and minima of $\Delta \epsilon$. They correspond to nuclei with neutron closed shells or subshells: the maximum at ⁴⁸Ca corresponds to the closure of the neutron 1 f7/2 orbital, whereas the minimum at ⁵²Ca



FIG. 2. Difference between the energies of the 2s1/2 and 1d3/2 proton states in Ca isotopes calculated in RMF with the parametrizations DDME1, NL3, and NLB2.

corresponds to the filling of the neutron 2p3/2 state. In the nonrelativistic Skyrme-HF model the radial HF equations can be expressed in terms of an energy-dependent equivalent potential V_{eq}^{lj} :

$$\frac{\hbar^2}{2m} \left[-\frac{d^2}{dr^2} \psi(r) + \frac{l(l+1)}{r^2} \psi(r) \right] + V_{\text{eq}}^{lj}(r,\epsilon) \psi(r) = \epsilon \psi(r),$$
(1)

where

$$V_{\rm eq}^{lj}(r,\epsilon) = V_{\rm eq}^{\rm centr.} + \frac{m^*(r)}{m} U_{\rm so}^{lj}(r) + \left[1 - \frac{m^*(r)}{m}\right]\epsilon, \quad (2)$$

with $U_{so}^{lj}(r) = U_{so}(r) \times [j(j+1) - l(l+1) - 3/4]$. $U_{so}(r)$ is the spin-orbit HF potential and $V_{eq}^{centr.}$ is

$$V_{\rm eq}^{\rm centr.} = \frac{m^*(r)}{m} U_0(r) - \frac{m^{*2}(r)}{2m\hbar^2} \left[\frac{\hbar^2}{2m^*(r)}\right]^{\prime 2} + \frac{m^*(r)}{2m} \left[\frac{\hbar^2}{2m^*(r)}\right]^{\prime \prime},$$
(3)

where $U_0(r)$ is the central HF potentials and $m^*(r)$ is the effective mass [12]. For protons U_0 includes the Coulomb potential. Up to a normalization factor the HF radial wave function ϕ of energy ϵ is related to the solution ψ of Eq. (1) by the relation $\psi = (m^*/m)^{1/2}\phi$. From Eqs. (1)–(3) we can write $\Delta\epsilon$ as

$$\Delta \epsilon = \left[\frac{\langle T \rangle_s}{\langle m^*/m \rangle_s} - \frac{\langle T \rangle_d}{\langle m^*/m \rangle_d}\right] + \left[\frac{\langle V_{eq}^{\text{centr.}} \rangle_s}{\langle m^*/m \rangle_s} - \frac{\langle V_{eq}^{\text{centr.}} \rangle_d}{\langle m^*/m \rangle_d}\right] - \frac{\langle (m^*/m) U_{so}^{d3/2} \rangle}{\langle m^*/m \rangle_d},\tag{4}$$

where *T* is the kinetic contribution. The three terms of the right-hand side of Eq. (4)—kinetic, central, and spin-orbit—are plotted in Fig. 3 for the force SkI5 and the nuclei ⁴⁰Ca, ⁴⁸Ca, ⁵²Ca, and ⁷⁰Ca. Similar results are obtained with SLy4 and SGII. We mention that the mean value of the effective mass in the denominators of Eq. (4) has very little *A* dependence from ⁴⁰Ca to⁷⁰Ca. From Fig. 3, one notices that the spin-orbit



FIG. 3. Kinetic, central, and spin-orbit contributions of Eq. (4) for SkI5 in 40 Ca, 48 Ca, 52 Ca, and 70 Ca.

and kinetic terms present a regular behavior as a function of A. Both of them are weakened with increasing isospin asymmetry favoring the inversion in very neutron-rich isotopes. The spinorbit term is weakened because the neutron surface becomes more diffuse with increasing A. In general, the kinetic energy of an orbital depends on the mean distance between its singleparticle energy and the bottom of the potential in the region where the wave function is localized. For the two states 2s1/2and 1d3/2 we can look at the difference $\epsilon_{lj} - V_{eq}^{lj}(r_0)$, where r_0 is the rms radius of the corresponding wave function. The evolution of V_{eq}^{lj} with increasing A is governed by two effects: (i) the lowering of the proton potential due to the symmetry term and (ii) the formation of a neutron skin that modifies the proton distribution by pulling it toward larger radii. The intensity of these two effects depends on the quantum numbers of the neutron orbitals that are filled and of the proton wave function under study. As an illustration, we consider ⁵²Ca and ⁷⁰Ca. The rms radii r_0 and the values $\epsilon_{lj} - V_{eq}^{lj}(r_0)$ are shown in Table I for the 2s1/2 and 1d3/2 states. From ⁵²Ca to ⁷⁰Ca the difference $\epsilon_{lj} - V_{eq}^{lj}(r_0)$ is reduced more for 1d3/2 (4.9%) than for 2s1/2 (0.4%). This analysis is confirmed by the evolution of the two rms radii. It is evident that, going from ⁵²Ca to 70 Ca, the 1d3/2 wave function is more affected than 2s1/2 by the enlarging of the potential due to the formation of a thick neutron skin. This explains the increase of the kinetic contribution to $\Delta \epsilon$ with the neutron number. We consider now the central term of Eq. (4) that is responsible for the maxima and minima of $\Delta \epsilon$, and concentrate on the maximum at ⁴⁸Ca. We mention that the major role played by the central term in

TABLE I. The values of r_0 and $\epsilon - V_{eq}^{lj}(r_0)$, for the states 2s1/2 and 1d3/2 in ⁵²Ca and ⁷⁰Ca. The interaction is SkI5.

A	<i>r</i> ₀ (fm)	<i>r</i> ₀ (fm)	$\frac{\epsilon - V_{\rm eq}^{lj}(r_0)}{({\rm MeV})}$	$\frac{\epsilon - V_{\rm eq}^{lj}(r_0)}{({\rm MeV})}$
	2s1/2	1d3/2	2 <i>s</i> 1/2	1d3/2
52	3.71	3.70	23.11	19.73
70	3.81	3.89	23.01	18.76



FIG. 4. V_0 , V_1 , and V_2 calculated with SkI5 in ⁴⁰Ca, ⁴⁸Ca, and ⁵²Ca.

modifying the single-particle energies has been also underlined by Gaudefroy *et al.* [4]. We introduce the quantities V_0 , V_1 , and V_2 that correspond to the contributions of the three terms of Eq. (3). They are plotted in Fig. 4 for ⁴⁰Ca, ⁴⁸Ca, and ⁵²Ca. It turns out that the term mainly affected by the neutron shell structure is V_0 , which contains the Hartree-Fock potential.

We can separate the energy contributions of the N = Z =20 core from those of the excess neutrons. For instance, the total nucleon density ρ is a sum of ρ_{core} and ρ_{excess} , and similarly for the other types of densities. Then, for any HF quantity the core contribution is obtained by replacing in its expression the total densities by core densities, whereas the neutron excess contribution corresponds to the rest. We show in Fig. 5 the core and neutron excess contributions to V_0 . It is clear that the change of slope at ⁴⁸Ca is mainly due to the neutron excess contribution. We have further verified that the term mainly responsible is the t_0 term of the Skyrme force. The density-dependent term $(t_3 \text{ term})$ is also sensitive to the neutron shell structure but with an opposite behavior reducing the effect due to the t_0 term. Hence, the main parameters that influence the behavior of $\Delta \epsilon$ are x_0 and t_0 as well as x_3 , t_3 , and α . We have checked that the role played by the other terms of the HF potential is negligible.



FIG. 5. Core and neutron excess contributions to V_0 for SkI5 in 40 Ca, 48 Ca, and 52 Ca.



FIG. 6. (Color online) Potential $P_{\rm HF}$ (see text) (top), square of 1d3/2 and 2s1/2 radial wave functions times r^2 (middle) and their product (bottom) calculated with SkI5 in ⁴⁴Ca, ⁴⁸Ca, and ⁵²Ca.

To complete our analysis we consider separately the two single-particle energies 2s1/2 and 1d3/2. We have verified that the maximum of $\Delta \epsilon$ at ⁴⁸Ca is mostly due to the energy of the 1d3/2 state that decreases less rapidly from ⁴⁸Ca to ⁵²Ca than from ⁴⁰Ca to ⁴⁸Ca. This behavior is explained in Fig. 6. In the top panels the neutron excess contribution $P_{\rm HF}$ to the HF potential due to the t_0 and t_3 terms is plotted for ⁴⁴Ca, ⁴⁸Ca, and ⁵²Ca (see, e.g., Ref. [12] for the expressions in terms of the Skyrme force parameters). In the middle panels we show the squares of the 1d3/2 and 2s1/2 radial wave functions multiplied by r^2 . In the bottom panels the products $P_{\rm HF} |\phi|^2 r^2$ are shown. Because $P_{\rm HF}$ is negative (the t_0 contribution is negative and the largest in absolute value) the proton single-particle energies are lowered with increasing N-Z, as expected. We observe that in ⁴⁴Ca and ⁴⁸Ca the potential related to the neutron excess (1 f 7/2 neutron orbital)is concentrated in the region where the 1d3/2 wave function is localized. The overlap with this wave function is thus the largest and this explains why the filling of the neutron 1 f 7/2orbital has an important effect on the proton 1d3/2 energy that is strongly lowered. However, when the neutron 2p3/2 orbital is filled (from ⁴⁸Ca to ⁵²Ca) the potential changes very little in the region where the 1d3/2 wave function is appreciable. This explains why the 1d3/2 energy decreases more from A = 44 to 48 than from A = 48 to 52. The energy of the 2s1/2proton state is much less sensitive to the neutron shell structure in these isotopes and it decreases rather monotonically from A = 44 to 52.

IV. TENSOR FORCE EFFECT

The tensor force plays certainly a role in the evolution of single-particle states. This is discussed, e.g., in the framework of the shell model in Ref. [3]. In a mean-field approach, the tensor effect originates from the π -nucleon and ρ -nucleon contributions to the Fock terms [20,21], and it can be introduced phenomenologically in the parametrizations of effective interactions built for HF models [22–24]. Recent progress have been made in determining the tensor terms of



FIG. 7. Difference between the energies of 2s1/2 and 1d3/2 proton states in ⁴⁰Ca, ⁴⁸Ca, ⁵²Ca, and ⁷⁰Ca calculated with SLy5 with and without the tensor contribution.

Skyrme interactions [25,26] by adjusting the single-particle spectra measured in N = 82 isotones and Z = 50 isotopes [27].

When this force is included the spin-orbit potential presents an additional term depending on the spin density J, namely,

$$U_{\rm so}^q = \frac{W_0}{2} \left(\nabla \rho + \nabla \rho_q \right) + \alpha J_q + \beta J_{q'},\tag{5}$$

where q stands for neutrons (protons) and q' for protons (neutrons), α and β consist of a sum of central and tensor contributions: $\alpha = \alpha_C + \alpha_T$, $\beta = \beta_C + \beta_T$. The central contributions depend only on the velocity-dependent part of the Skyrme force, whereas the tensor contributions are generated by the tensor component of the Skyrme force [23,26]. To estimate the effect of the tensor force in our case we use the Skyrme force SLy5 that already contains in the fitting protocol the terms α_C and β_C . For α_T and β_T we adopt the values determined in Ref. [25] by comparing the Skyrme-HF predictions with the data of Ref. [27]. These values are $\alpha_T = -170 \text{ MeV fm}^5$ and $\beta_T = 100 \text{ MeV fm}^5$. We expect that the tensor force favors the inversion in ⁴⁸Ca (see, for instance, Fig. 4 of Ref. [3]). Actually, from ⁴⁰Ca to ⁴⁸Ca the 1 f7/2 neutron orbital is filled. The interaction between the proton orbital 1d3/2 and the neutron orbital 1f7/2 is attractive and its effect is to lower the energy of 1d3/2, thus favoring the crossing with 2s1/2. As an illustration we performed SLy5-HF calculations for ${}^{40}Ca$, ${}^{48}Ca$, ${}^{52}Ca$, and ${}^{70}Ca$ (${}^{70}Ca$ is still bound within SLy5-HF). We show in Fig. 7 the values of $\Delta \epsilon$ obtained with and without the tensor contribution. As expected, the tensor force increases the slope going from ⁴⁰Ca to ⁴⁸Ca, bringing the two states close together and improving the agreement with the experimental data. The improvement is quite strong because the value of $\Delta \epsilon$ in ⁴⁸Ca is equal to -0.26 MeV and -1.41 MeV with and without the tensor contribution, respectively. We can also expect that, by adding the tensor contribution within, for instance, the SkI5 model, the

value of $\Delta \epsilon$ at ⁴⁸Ca (Fig. 1) would be further increased and this result would be more similar to the relativistic case DDME1 or NL3 (Fig. 2). However, a precise conclusion cannot be drawn because, to perform a proper calculation, including the tensor effect in the SkI5 model, all the other parameters of the force should be also readjusted.

V. SUMMARY

In this article we have analyzed the modification of the proton single-particle states 2s1/2 and 1d3/2 in Ca isotopes within the nonrelativistic Skyrme-HF and the relativistic RMF models. Pairing effects have been neglected because Ca isotopes are proton closed-shell. We are interested in the evolution of proton states and the inclusion of neutron pairing does not affect significantly the global trend of our results. Both models, HF and RMF, lead to the same evolution with increasing A for the difference $\Delta \epsilon$ of the energies of the two states. This evolution depends on the neutron orbitals that are filled, $\Delta \epsilon$ presenting maxima and minima corresponding to neutron shell or subshell closures. In particular, going from ⁴⁰Ca to ⁴⁸Ca the two proton states come closer to each other and they can sometimes cross in some models. By performing an analysis based on the equivalent potential in the nonrelativistic Skyrme-HF approach, we have shown that the kinetic and spin-orbit contributions present quite a regular behavior with increasing A. They both strongly favor the inversion of the two states in very neutron-rich nuclei. We have also verified that the contribution that is mostly responsible for the maximum of $\Delta \epsilon$ at ⁴⁸Ca (and leading to an inversion for some models) is the central HF potential and, in particular the t_0 and t_3 terms. The former term favors the crossing of the two states near ⁴⁸Ca, whereas the latter acts against it. The net effect is that the two states get closer and can cross each other in some models.

We have finally analyzed the role of the tensor force within the SLy5-HF model and found that its contribution goes in the same direction as the t_0 term of the HF potential, favoring the inversion of the states near ⁴⁸Ca.

Our analysis was restricted to a purely mean-field picture. Work should be done to include effects beyond mean field. For instance, particle-phonon coupling, which has been neglected here, is expected to improve the quality of the theoretical results in the study of single-particle states evolution.

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