Spin-isospin nuclear response using the existing microscopic Skyrme functionals

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Our paper aims at providing an answer to the question of whether one can reliably describe the properties of the most important spin-isospin nuclear excitations by using the available nonrelativistic Skyrme energy functionals. Our method, which has been introduced in a previous publication devoted to the isobaric analog states, is the self-consistent quasiparticle random-phase approximation (QRPA). The inclusion of pairing is instrumental for describing a number of experimentally measured spherical systems which are characterized by open shells. We discuss the effect of isoscalar and isovector pairing correlations. Based on the results for the Gamow-Teller resonance in ^{90}Zr, ^{208}Pb, and a few Sn isotopes, we draw definite conclusions on the performance of different Skyrme parametrizations, and we suggest improvements for future fits. We also use the spin-dipole resonance as a benchmark of our statements.

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I. INTRODUCTION

The special role played by the spin-isospin modes for the detailed understanding of the structure of nuclei has been pointed out over the past several decades. The subject has been treated in review papers [1] and textbooks [2]. Spin-isospin transitions can occur spontaneously in the case of β decay. The simplest case is that of the Gamow-Teller (GT) transitions, whose corresponding operator is

$$\vec{O}_{\mathrm{GT}\pm} = \sum_{i=1}^{A} \vec{\sigma}(i) t_{\pm}(i). \quad (1)$$

This operator is associated with a model-independent sum rule [3], namely, $m_0 \equiv m_0(t_-) - m_0(t_+) = 3(N - Z)$, where $m_0(t_\pm)$ is the total strength in the given channel. This sum rule is commonly called the Ikeda sum rule. Since the early work of K. Ikeda *et al.* [4], it has been indeed clear that in the limited energy window accessible to the β^- decay, only a limited fraction of $m_0(t_-)$ can be found. A collective state should be expected at higher energy, and this Gamow-Teller resonance (GTR) has been indeed detected in (p, n) experiments starting from the mid-1970s [5]. Later, systematic (^{3}He, t) experiments characterized by much better energy resolution were also performed. We should recall that in nuclei having neutron excess, the Ikeda sum rule is exhausted almost entirely by states in the t_- channel, as the Pauli principle hinders the t_+ excitations.

In medium-heavy nuclei, ranging from ^{90}Zr to ^{208}Pb, the GTR is located somewhat above the isobaric analog resonance (IAR) which fact is also well known from (p, n) and (^{3}He, t) experiments. This corresponds to the typical energy region of the giant resonances, that is, 10–20 MeV (we refer here to energies with respect to the ground state of the mother, or target, nucleus). At the same time, the main GT peak(s) turns out to exhaust only about 50% of the Ikeda sum rule in these medium-heavy nuclei; this percentage becomes about 70% if the whole strength in the neighboring energy region (i.e., below ≈20 MeV in the daughter, or final, nucleus) is collected [6].

The extraction of the strength from the measured cross sections is far from being straightforward. However, due to their $\Delta L = 0$ character, the GTR and IAR angular distributions are strongly peaked at 0°, and an approximate proportionality between the zero-degree cross section and the strength has been found under the hypothesis of high incident energy, zero momentum transfer, and neglect of the noncentral components of the projectile-target interaction [7].

The problem of the so-called missing GT strengh has attracted the considerable attention of nuclear physicists. Some theorists have speculated that part of the missing GT strength should be found at very high excitation energy (≈300 MeV) because of the coupling with the internal 1^+ excitation of the nucleon, i.e., the Δ isobar (1232 MeV): the reader can consult the references quoted in Ref. [1]. Other calculations [8] have shown that the usual coupling of the one particle-one hole (1p-1h) configurations involved in the GTR with two particle-two hole (2p-2h) configurations is able to shift strength outside the range accessible to experiments and explains in a more conventional fashion the missing strength. Experimentally, from the multipole-decomposition analysis (MDA) of the cross sections measured in the ^{90}Zr(p, n) experiment at $E_p =$ 295 MeV [9], it has been argued that 90% of the GT strength can be recovered below 50 MeV excitation energy, leaving little room for the coupling with the Δ isobar. However, part of the analysis (for instance, the estimate of the isovector monopole contribution) has been somehow questioned.

The coupling of simple 1p-1h configurations with more complex ones and the high-lying GT strength are not the issue of the present paper. Using the Skyrme Hamiltonian, the GTR in ^{208}Pb has been calculated beyond the simple random-phase approximation (RPA), taking into account the coupling with the continuum as well as with configurations made up with a p-h pair coupled with a collective vibration [10]. This calculation has been able to reproduce the values of the branching ratios associated with the proton decay of the GTR; at the same time, it has been shown that the position of the main GT peak does not change too much with respect to the simple RPA. In Figs. 4 and 5 of Ref. [10] one can see that the peak is indeed shifted downward by a few hundred keV. The calculations reported in Ref. [11] (also based on the coupling with phonons) are much more phenomenological, but the result is similar. The mentioned redistribution of strength is instead quite sizable, and this point should be kept in mind for the following discussion. No such complete and fully microscopic calculation, at the level of four-quasiparticle coupling, is available for the charge-exchange modes in open shell systems. We still need, and this is our first aim here, to assess in a clear way the properties of the Skyrme functionals, complemented by an effective pairing force, in the spin-isospin channel by studying the corresponding excitations within the self-consistent mean-field framework. As self-consistent calculations, we mean here quasiparticle random-phase approximation (QRPA) calculations based on a Hartree-Fock plus Bardeen-Cooper-Schrieffer (HF-BCS) description of the ground state. Our model has been introduced and applied to the IAR in Ref. [12]. Some rather preliminary results using the same model have been presented in conference proceedings [13].

There are not many self-consistent QRPA calculations available. The proton-neutron QRPA, based on Skyrme forces in the particle-hole (p-h) channel (with a simplified form, i.e., with a separable approximation) and on the use of a constant pairing gap in HF-BCS plus a free residual particle-particle (p-p) interaction, has been intensively applied to the study of both spherical and deformed nuclei [14]. The issue is to know to what extent intrinsic deformations affect the measured β-decay spectra, and the authors of Ref. [14] have explored many isotopic chains, including heavy ones [15]. Later, the first attempt to implement a self-consistent QRPA scheme based on HF-BCS has been made in Ref. [16], which is another work devoted to β decay (in this case, limited to spherical isotopes lying on the r-process nucleosynthesis path). The same group has studied the high-lying GTR and the behavior of different Skyrme parameter sets [17]; we will discuss in detail, in what follows, the comparison of that work with the present one. The charge-exchange modes have also been attacked by using relativistic charge-exchange RPA and QRPA [18–21].

But, aside from those mentioned, most of the QRPA calculations are not self-consistent. To study the GTR in ^{208}Pb, the quasiparticle-phonon model has been employed in Ref. [22] and the so-called Pyatov method in Ref. [23]. Most of the systematic calculations (done also for open-shell nuclei and for β decay) are rather based on some empirical mean field (e.g., Woods-Saxon) and residual interaction depending, in the spin-isospin channel, on a parameter g_0'. A schematic model of this type can certainly be useful in many respects. As we discuss below, predictions of the schematic RPA model based on these simple phenomenological ingredients can be regarded as a guideline while understanding our results. However, we stick to the idea of a full microscopic approach. This is of special interest nowadays. If new radioactive beam facilities aim at studying spin-isospin properties of the exotic systems, constraining this channel in the microscopic Hamiltonian must be envisaged, while sticking to phenomenological inputs may be not appropriate.

The study of spin-isospin excitations is of interest with regard to not only nuclear structure but also particle physics and astrophysics. In fact, the detailed knowledge of spin-isospin nuclear matrix elements (with ≈20% accuracy) is required to extract from the $\beta\beta$-decay experimental findings the hierarchy of neutrino masses. And, in the astrophysical sector, the details of the r-process nucleosynthesis can be understood, once more, only if nuclear masses, photonuclear cross sections, and β-decay probabilities are precisely known. Last but not least, we mention the importance of knowing the neutrino-nucleus interactions in different contexts (from the stellar environment, to the case of materials which are used for crucial experiments on the neutrinos). All these motivations lie at the basis of the recent works concerning the spin-isospin nuclear modes.

In the present work, as compared with Ref. [17], we reexamine in particular the role of the so-called spin-gradient (or J^2) terms of the Skyrme energy functionals, and we find somewhat different results for the GT strength distributions. Moreover, we perform a more general analysis, since we also study the role of the pairing residual interaction (which was neglected in Ref. [17]), and we devote some attention to the case of another kind of spin-isospin excitation, namely, the isovector spin-dipole (IVSD) resonance.

The IVSD resonance is excited by the operator

$$O_{\mathrm{IVSD}\pm,J^\pi} = \sum_{i=1}^{A} r_i [\vec{Y}_1(\hat{r}_i) \otimes \vec{\sigma}(i)]_{J^\pi M} t_\pm(i), \qquad (2)$$

where $J^\pi = 0^-, 1^-, 2^-$. The charge-exchange experimental measurements, whether (p, n) or $(^3\mathrm{He}, t)$, show indeed evidence of $L \neq 0$ strength. Most of this strength is very fragmented, and an unambiguous signature for the different multipoles (monopole, dipole, etc.) is still missing. In theoretical calculations, the spin-dipole distributions look quite broad, also because of the presence of three J^π components. Some calculations for magic nuclei have been available for a long time; the reader can refer to the phenomenological calculations of Ref. [24] or to the HF plus continuum-RPA of Ref. [25]. Recently, there has been new interest in the study of this channel: some low-lying transitions important in the $\beta\beta$ decay have in fact a first-forbidden character, and the reliability of theoretical models in predicting properties of $L \neq 0$ charge-exchange transitions is under discussion. Moreover, it has been suggested that the precise determination of the IVSD sum rule (analogous to the Ikeda sum rule) can be a unique probe of the neutron skins, as it is proportional to $N\langle r^2\rangle_n - Z\langle r^2\rangle_p$ [26]. Since it would be highly desirable to extract the key parameters governing the asymmetry part of the nuclear equation of state from the difference of the neutron and proton radii, and the experimental determination of neutron radii by means of scattering data is not very accurate, this alternative way of extracting the same quantity is potentially of great interest [27] (see also Ref. [28]). In the spirit of the present investigation, it is important of course to establish whether the conclusions about the robustness of the Skyrme-QRPA with given parameter sets remain valid when another multipolarity is studied.

The outline of our paper is the following. We first provide the basic information about our formalism in Sec. II by limiting ourselves to what is essential for understanding the rest of the discussion. One part of the Skyrme functionals that we employ here, namely, that associated with the so-called J^2 terms, has been discussed recently, also in Ref. [17]; for this reason, we discuss at length the point of view emerging from our calculations and results in Sec. III. We can then analyze the results for the GT and IVSD strength distributions, respectively, in Secs. IV and V, and draw relevant conclusions on the performances of the existing Skyrme sets as well as make suggestions for the future fits. Considerations on the pairing correlations are made in Sec. VI, before coming to the overall conclusions of Sec. VII.

II. FORMALISM

Our model has been introduced in Ref. [12], and we will focus here only on those aspects which are important to the understanding of our results. We start by dealing with the HF-BCS coupled problem; that is, at each iteration we solve in real space the HF equations and the BCS gap and number equations. For ^{90}Zr and ^{208}Pb, pairing is neglected. For the Sn isotopes, the pairing window is the 50–82 neutron shell, and the pairing force is the same as that fitted in Ref. [12], namely, a zero-range, density-dependent interaction of the type

$$V = V_0\left[1-\left(\frac{\rho\left(\frac{\vec r_1+\vec r_2}{2}\right)}{\rho_C}\right)^{\gamma}\right]\cdot\delta(\vec r_1-\vec r_2), \tag{3}$$

with $V_0 = 680$ MeV fm^3, $\rho_C = 0.16$ fm^{-3}, and $\gamma = 1$. It has been checked that when this pairing force is used in connection with different Skyrme forces (we consider in this work the parameter sets SIII [29], SGII [30], SLy5 [31], and SkO′ [32]), the resulting pairing gaps do not vary too much along the Sn isotope chain.

All the states at positive energy (either those in the BCS pairing window or those outside this window, which have occupation factors v^2 equal to zero) are calculated using box boundary conditions; that is, our continuum is discretized. Two quasiparticle configurations (or particle-hole, in the cases in which pairing is absent) with proper J^π are built, and the QRPA matrix equations

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X^{(n)} \\ Y^{(n)} \end{pmatrix} = E_n \begin{pmatrix} X^{(n)} \\ Y^{(n)} \end{pmatrix} \tag{4}$$

are solved in this model space. The upper limit for the configurations is chosen so that the results are stable against variations, and the proper sum rules, which are expected to hold in full self-consistent calculations, are indeed exhausted with high accuracy. In the charge-exchange case, it is known that these sum rules are the difference of the non-energy-weighted sum rules in the two isospin channels $m_0 \equiv m_0(t_-) - m_0(t_+)$, and the sum of the energy-weighted sum rules $m_1 \equiv m_1(t_-) + m_1(t_+)$. The analytic values of these sum rules in the case of the Skyrme forces can be found, e.g., in Ref. [33].

In the p-h channel, for the charge-exchange modes, the residual interaction reads

$$v^{ph}(\mathbf{\vec r}_1,\mathbf{\vec r}_2) = \delta(\mathbf{\vec r}_1-\mathbf{\vec r}_2)\big[v_{01}(r)+v_{11}(r)+v'_{01} + v'_{11} + v_1^{(\text{s.o.})}\big]. \tag{5}$$

In this formula, the two indices for each of the first four terms in square brackets refer to the projection in a given $\sigma\tau$ channel. The terms with (without) a prime are those which are (are not) velocity dependent. The last term is the isovector part of the spin-orbit residual interaction.

In the following work, our considerations will focus on the spin-isospin terms of the p-h residual interaction, the spin-independent terms being far from dominant or even not active. For the sake of completeness, we provide anyway the detailed expressions of all terms:

$$\begin{aligned}
v_{01}(r) &= 2C_1^{\rho}[\rho(r)]\vec\tau_1\vec\tau_2,\\
v_{11}(r) &= 2C_1^{S}[\rho(r)]\vec\sigma_1\vec\sigma_2\vec\tau_1\vec\tau_2,\\
v'_{01} &= [(\vec k'^2+\vec k^2)\tfrac{1}{2}\big(C_1^{\tau}-4C_1^{\Delta\rho}\big)\\
&\quad + \vec k'\vec k\big(3C_1^{\tau}+4C_1^{\Delta\rho}\big)]\vec\tau_1\vec\tau_2,\\
v'_{11} &= \big[(\vec k'^2+\vec k^2)\tfrac{1}{2}\big(C_1^{T}-4C_1^{\Delta S}\big)\\
&\quad + \vec k'\vec k\big(3C_1^{T}+4C_1^{\Delta S}\big)\big]\vec\sigma_1\vec\sigma_2\vec\tau_1\vec\tau_2,\\
v_1^{(\text{s.o.})} &= -2iC_1^{\nabla J}(\vec\sigma_1+\vec\sigma_2)\vec k'\times\vec k\,\vec\tau_1\vec\tau_2.
\end{aligned} \tag{6}$$

We recall that

$$\begin{aligned}
\vec k' &= -\frac{1}{2i}(\vec\nabla'_1-\vec\nabla'_2),\\
\vec k &= \frac{1}{2i}(\vec\nabla_1-\vec\nabla_2),
\end{aligned} \tag{7}$$

with the operators acting at right (left) in the case of k (k'). The parameters entering the above expressions can be written in terms of those of the Skyrme force. For the convenience of the reader, this correspondence is explicitly provided in the Appendix.

In the p-p channel, we fix self-consistently the residual isovector pairing force by exploiting the isospin symmetry; that is, we take the isovector proton-neutron pairing interaction to be the same as the neutron-neutron one used in the BCS description of the ground state. The proton-neutron isoscalar pairing cannot be constrained: presently, we miss a clear indication from empirical data about the parameter of the isoscalar pairing force. Using a quite conservative approach, we present in the following results which, unless otherwise stated, correspond to an isoscalar pairing force equal to the isovector one. We have tried to give some indication about the sensitivity of our results when in the isoscalar channel a strength $V_0^{(T=0)}$ different from $V_0^{(T=1)}$ is adopted. This kind of study has been done in connection with the Relativistic Mean Field (RMF) analysis of the charge-exchange modes. No such analysis has been available so far in the case of the Skyrme calculations; we have found results which are to some extent consistent with those associated with the RMF study. In that case, a finite-range Gogny pairing force is employed, but this does not seem to produce macroscopic differences with respect to the use of zero-range effective pairing forces.

III. TREATMENT OF THE J^2 TERMS OF THE ENERGY FUNCTIONAL

As mentioned above, the energy functional includes the so-called spin-gradient, or J^2, terms which are built on the spin-orbit densities. They arise from the exchange part of the central Skyrme interaction [34]. The spin-orbit densities vanish in the ground state of spin-saturated nuclei, but they provide a contribution to the spin and spin-isospin parts of the residual p-h interaction. In the past (with some exceptions), the J^2 terms were neglected when fitting the Skyrme parameters; some more recent parametrizations include them, and in particular we will consider in the following the sets SLy5 and SkO′. In the discussion below, for the sake of simplicity, we will call type I forces the Skyrme sets which do not include the J^2 terms in the fit, and type II forces those which do include them. To our knowledge, there is not a clear indication emerging from the nuclear phenomenology of whether these J^2 terms must be included in a physically sound energy functional. Of course, if the functional is derived from a two-body force of the Skyrme type, which has a momentum dependence, it looks questionable to drop the J^2 terms. We should also notice that the J^2 terms are neither hard to evaluate nor time-consuming if the HF calculation is performed in coordinate space as in the present case.

In the past, many RPA calculations have been performed with type I forces. The authors of Ref. [17] pointed out that such calculations (e.g., those of Ref. [30]) do not respect the full self-consistency, since the contributions from the J^2 terms are included in the residual interaction but not in the mean field (the authors of Ref. [30] also neglected the spin-orbit residual interaction, but this has practically no effect). To respect the Galilean invariance, the authors of Ref. [17], when employing type I forces in their work, adopted the prescription of removing from the residual interaction not only the contribution from the J^2 term but also that from the so-called $S \cdot T$ term in the functional. This amounts to setting C_1^T equal to zero [cf. Eq. (6)] and leads to a substantial quenching of the velocity-dependent part in the spin-isospin channel, since $C_1^{\triangle S}$ is not as large. Therefore, we deem that the issue should be further discussed here.

We start from the fact that fitting the Skyrme parameters is usually done by using, in addition to nuclear (or neutron) matter quantities, binding energies and charge radii of a few selected isotopes (with the spin-orbit strength W_0 separately adjusted). In ^{208}Pb, the binding energy (charge radius) changes by 0.22% (0.15%), using the force SLy4, when the J^2 terms are omitted or inserted. These variations are too small to allow a clear statement about the manifestation of the J^2 terms in the benchmarks used for the fit, because it must be noted that in the protocol for the parameter fitting presented in Ref. [31], larger errors on binding energies and charge radii are imposed in the χ^2 formula to let the fit converge (we mean here, larger than the experimental error bars and larger than the ≈0.1–0.2% variations we just mentioned). Even in ^{120}Sn, which is not used for the parameter fitting but is studied in the present paper, we find a similar pattern.

On the other hand, the effect of the J^2 terms on the single-particle spectrum becomes appreciable. In Fig. 1 we display the highest occupied and lowest unoccupied proton and neutron levels in ^{208}Pb. With few exceptions the spectra of SLy4 and SLy5 are similar, the proton (neutron) levels being in general slightly lower (higher) in energy in the case of SLy4. The spectrum associated with SLy4 plus the J^2 terms is instead somewhat different; in fact, one notices that the $j_>$ ($j_<$) spin-orbit partners are raised (lowered) in energy, both for protons and neutrons, up to 400 keV. Accordingly, the unperturbed energies of the $j_> \rightarrow j_<$ configurations are reduced. The net overall effect is that the main GTR peak varies only by 60 keV

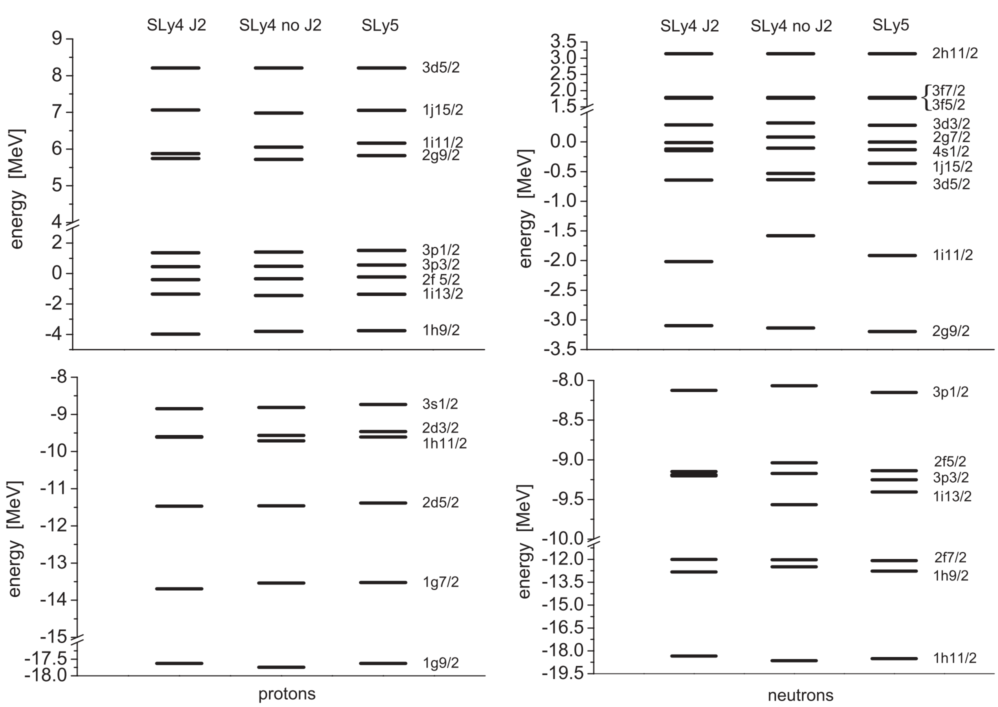

FIG. 1. Single-particle levels of ^{208}Pb, calculated using Skyrme-HF and employing, respectively, the parametrization SLy4 with and without the J^2 terms and the parametrization SLy5. The left (right) part refers to protons (neutrons). The lower (upper) panels include levels below (above) the Fermi energy.

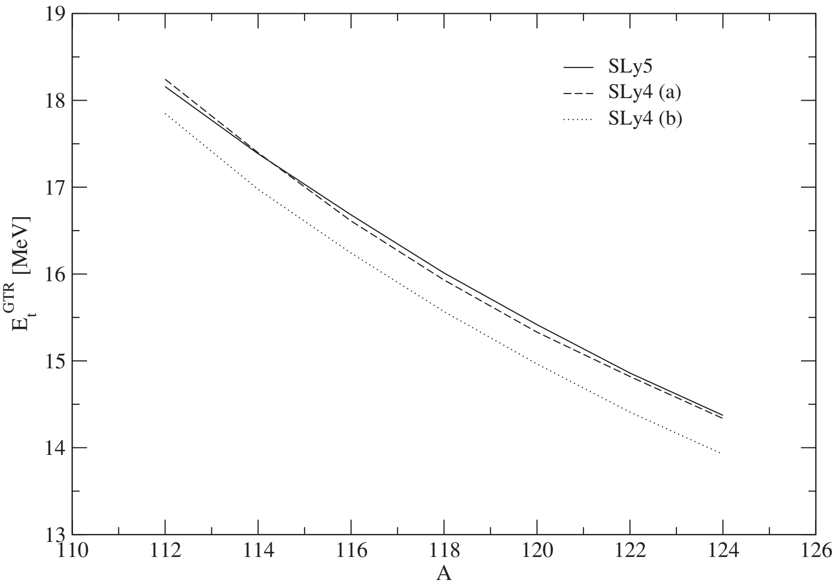

FIG. 2. GTR peak energies, along the Sn isotope chain, calculated by using either the force SLy5 (full line) or the force SLy4 with (dotted line) and without (dashed line) the contribution associated with the J^2 terms in the mean field. See the text for the discussion.

between SLy4 and SLy5, but it varies by 0.5 MeV when the J^2 terms are added to the SLy4 mean field. This is because only $j_> \rightarrow j_<$ configurations are present in the GTR wave function calculated with the Lyon parameter sets. We show the variation of the GTR peak energy along the Sn isotopes in Fig. 2. A stronger effect of the J^2 terms is that associated with the spin-isospin residual p-h force. In fact, if we remove the part corresponding to the J^2 term in the energy functional from the p-h interaction, the GTR peak energy changes by about 2 MeV. Qualitatively similar conclusions can be deduced from the study of the Sn isotopes.

This detailed study led us to the following conclusions. The J^2 terms do not manifest themselves so much in the ground state observables used for the fit of the Skyrme parameters, but they do affect some other properties of the nuclear ground state such as the spin-orbit splitting. Moreover, they play a major role when GT calculations are performed, mainly because of their contribution to the p-h interaction. Looking at our results, we believe that the most natural and physical choice is to omit the contribution of the J^2 terms when calculating the ground state with type I forces, but retain the corresponding contribution in the residual p-h force. For nuclei which are not spin saturated, we agree with Ref. [17] that this choice breaks self-consistency. If one insists on self-consistency, the choice of inserting the J^2 contribution into the ground state alters the GTR energy by about 0.5 MeV, whereas the alternative choice of neglecting the J^2 contribution systematically appears to be quite unnatural. After all, we definitely suggest that fits of new Skyrme parameters are systematically done by inserting the J^2 terms.

We conclude this section by mentioning that in the recent literature there have been claims about the necessity of complementing the usual Skyrme forces with tensor terms. Together with other collaborators, we have shown that the contribution of the tensor force can reduce qualitative discrepancies between the single-particle levels predicted within the Skyrme framework and those which are experimentally observed [35]. Similar discussions can be found in Ref. [36]. The reason for mentioning this here is that the two-body zero-range tensor force gives the same kind of contribution to the mean field of even-even nuclei as the J^2 terms. The tensor force may affect the GT centroid energy. A simple estimate based on sum rule arguments can be found in Ref. [35]. Since the aim of this work is the discussion of the performance of the existing functionals, we do not return to this point in the following. If a new general fit of Skyrme functionals plus tensor contribution is made, and the corresponding (Q)RPA becomes available, new steps can be undertaken.

IV. RESULTS FOR THE GAMOW-TELLER RESPONSE

As stated in the Introduction, the strength distributions associated with the Gamow-Teller operator $\sum_{i=1}^{A} \vec{\sigma}(i) t_-(i)$ are expected to display a main resonance located at energy $E_{\rm GTR}$. In Fig. 3 we show the behavior of $E_{\rm GTR} - E_{\rm IAR}$, where $E_{\rm IAR}$ is the isobaric analog energy, as function of $(N - Z)/A$. Experimental data are from Refs. [37–39]. The theoretical (Q)RPA calculations have been performed with some of the most recent and/or widely used Skyrme interactions, that is, SIII, SGII, SLy5, and SkO′. For SIII and SGII, based on the discussion in the previous section, the J^2 terms are included in the residual interaction and not in the mean field. When our calculations produce a resonance which is fragmented in more than one peak, the exact definition of the values of $E_{\rm GTR}$ used in the figure is the centroid $m_1(t_-)/m_0(t_-)$ where the two sum rules are evaluated in the interval of the resonance. This interval is 15–24 MeV for Pb and 12–22 MeV for the Sn isotopes (in Zr, there is a single GT main state). In some cases, we face the well-known problem of (Q)RPA instabilities, and (quasiparticle) Tamm-Dancoff approximation ((Q)TDA) values are reported (in particular, this happens for ^{90}Zr and ^{118,120}Sn when the force SkO′ is employed, and for ^{114}Sn when using SGII). In Ref. [12] we showed that our model provides

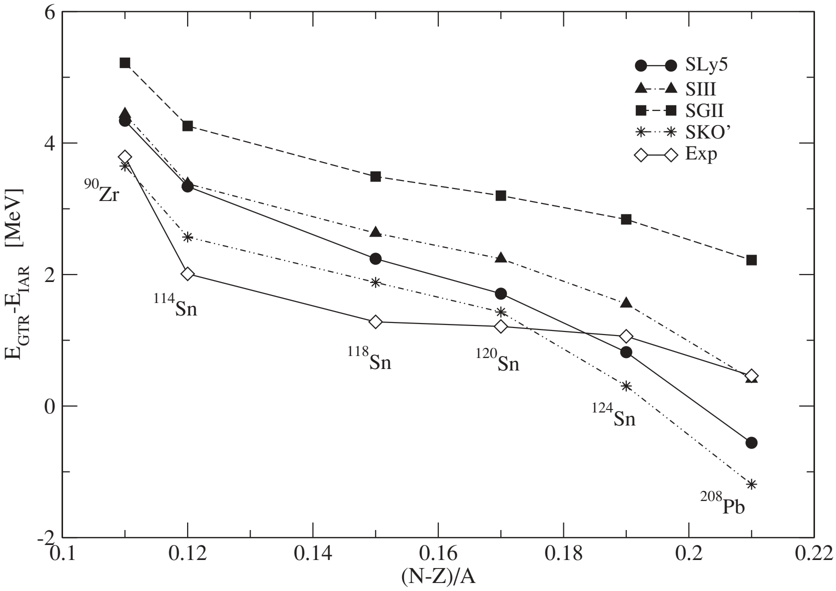

FIG. 3. Differences between the GTR and the IAR energies in some selected spherical nuclei as a function of $(N-Z)/A$. Theoretical results associated with different Skyrme parametrizations are compared with experimental values from Refs. [37–39]. The related discussion, including details on how the energies have been defined, can be found in the text.

quite accurate values of E_{IAR}, but in the figure, for simplicity, we used the experimental values for this quantity.

From Fig. 3, we can draw two initial conclusions. First, one should notice that the linear behavior of $E_{\mathrm{GTR}} - E_{\mathrm{IAR}}$ vs. $(N-Z)/A$ was already checked, on the experimental data, in Ref. [40] as it was expected on the ground of simple schematic models [1,41]. In fact, if one performs a simple RPA calculation using a separable interaction in a restricted space (made up with the excess neutrons and the proton spin-orbit partners), one finds that

$$E_{\mathrm{GTR}} - E_{\mathrm{IAR}} = \Delta E_{ls} + 2\frac{\kappa_{\sigma\tau} - \kappa_{\tau}}{A}(N-Z), \qquad (8)$$

where ΔE_{ls} is (an average value of) the spin-orbit splitting and κ_{τ}/A ($\kappa_{\sigma\tau}/A$) is the coupling constant of the separable schematic isospin (spin-isospin) residual force. The result of Fig. 3 suggests that our calculations, which are microscopically based and much more sophisticated, obey in first approximation this simple pattern.

Second, one would also infer from the figure that some forces account well for the experimental findings while others perform less well. SkO′ and SLy5 lie close to experiment, although their predictions drop below the experimental trend in ^{208}Pb. SGII and SIII tend to overestimate the experimental energies, but the trend of SGII looks qualitatively similar to the experimental one on the neutron-rich side. The result obtained with the force SIII corresponds, within ≈400 keV, to the one found in Ref. [42]. The trend associated with the energy location of the GTR is not the only significant experimental observable: we should also analyze the fraction of $m_0(t_-)$ or collectivity of the GTR.

In Figs. 4 and 5 we show the GT strength distributions, for ^{208}Pb and ^{120}Sn, respectively, associated with different forces. The strength functions in Sn display more fragmentation,

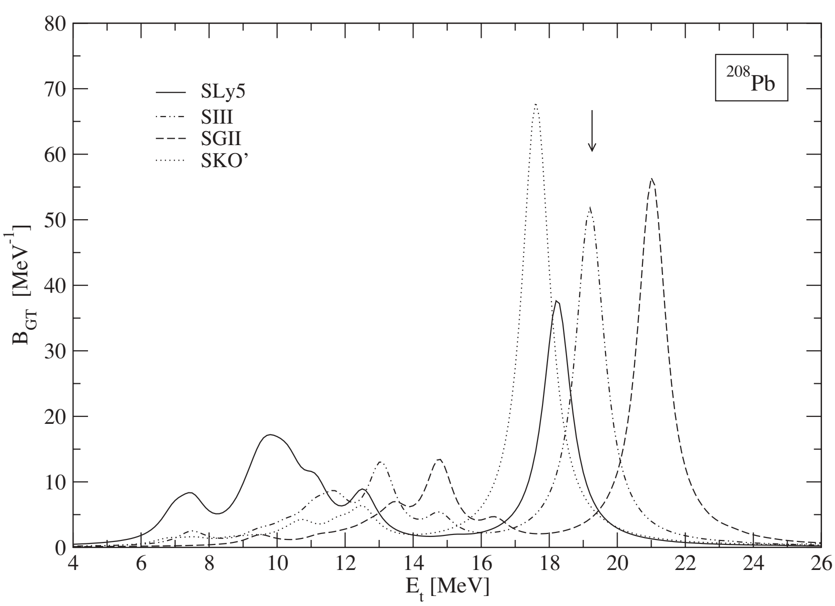

FIG. 4. Gamow-Teller strength distributions in ^{208}Pb, calculated using different Skyrme forces within HF plus RPA. The results are displayed as function of the energy in the target nucleus (E_t). The discrete RPA peaks have been smeared out using Lorentzian functions having 1 MeV width. The arrow corresponds to the experimental energy.

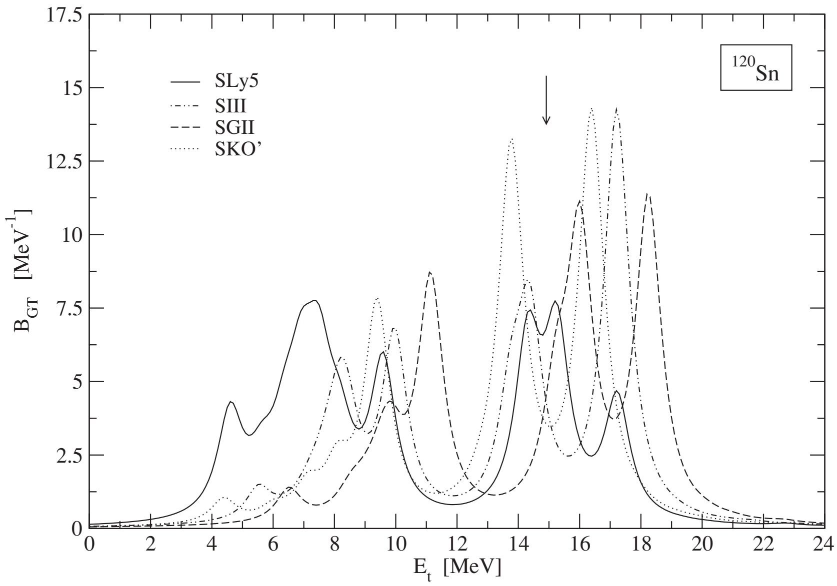

FIG. 5. Same as Fig. 4, but for the open-shell nucleus ^{120}Sn.

as expected in keeping with its open-shell character. As mentioned in the Introduction, the experimental result is that about 60% of the total strength is exhausted by the GTR. In Table I we show the fraction of strength in the resonance region, for the different forces, both for ^{208}Pb (where the result obtained with force SIII is very close to the 63.6% of Ref. [42]) and a few selected Sn isotopes. The results present a clear systematics: all forces concentrate ≈60–70% of the strength in the resonance region, apart from SLy5 (we recall again that our model does not include the coupling with 2p-2h).

Looking at the results for the GTR associated with the different forces, we ask ourselves if their performances depend more on the features of the associated mean field, on the effective interaction in the spin-isospin channel, or on a delicate balance between the two ingredients. As far as the GTR energies are concerned, we did not find clear correlations between them and any simple parameter. On the other hand, interesting correlations are found if one analyzes the GT collectivity. This will allow us to draw quite strong conclusions about the Skyrme parameter sets under study.

In the cases of the three forces SIII, SGII, and SkO′, the wave functions are qualitatively similar, i.e., they display a large number of p-h components: the wave function associated to the main GT state, in the case of ^{208}Pb and of the force SkO′, is reported in Table II. The wave function resulting from SLy5, shown for the same nucleus in Table III, instead displays fewer components. It has been checked that the reduced collectivity of the GTR calculated using SLy5 (and characteristic of not only Pb but also the Sn isotopes) cannot be explained simply in terms of the differences between the unperturbed energies associated with this parameter set and those of the other sets. Indeed, we have observed that the p-h matrix elements of the SLy5 force are, on the average, smaller than those of the other forces.

In our analysis, we have also singled out the role of the velocity-dependent terms. In particular, we observed that the $(\vec{k}'^2 + \vec{k}^2)$ and the $\vec{k}'\vec{k}$ contributions [cf. Eq. (6)] are comparable. If we drop these terms from the SLy5 p-h interaction, the GTR wave function becomes closer to that of the other forces, leading to an increase in the strength of ≈20% (the GT energy is of course also affected). In the case of the force SKO′, the increase in collectivity when the velocity-dependent terms are dropped is extremely small. In fact, for SkO′, the coefficient C_1^T, characterized by a positive

TABLE I. Percentages of the sum rule $m_0(t_-)$ exhausted in the giant resonance region. This region is 12–22 MeV in the Sn isotopes and 15–24 MeV in ^{208}Pb.

	^{114}Sn	^{118}Sn	^{120}Sn	^{124}Sn	^{208}Pb
SIII	60.44	60.98	61.44	62.76	60.68
SGII	61.75	61.30	61.49	63.36	67.24
SLy5	46.38	42.06	41.16	41.41	44.76
SKO′	66.06	67.08	67.19	72.76	79.80

TABLE II. Wave function of the main GT state in ^{208}Pb obtained with the SkO′ force. Under the label "weight" we report the absolute value of the quantity $X_{ph} + (-)^{S(J+L)} Y_{ph}$, which enter the calculation of the B(GT) value.

Configuration	Weight
$\nu i_{13/2} \rightarrow \pi i_{11/2}$	0.69
$\nu h_{11/2} \rightarrow \pi h_{9/2}$	0.49
$\nu f_{7/2} \rightarrow \pi f_{5/2}$	0.28
$\nu i_{13/2} \rightarrow \pi i_{13/2}$	0.20
$\nu f_{7/2} \rightarrow \pi f_{7/2}$	0.16
$\nu h_{9/2} \rightarrow \pi h_{9/2}$	0.16
$\nu p_{3/2} \rightarrow \pi p_{1/2}$	0.13
$\nu p_{3/2} \rightarrow \pi p_{3/2}$	0.13
$\nu f_{5/2} \rightarrow \pi f_{5/2}$	0.11
$\nu f_{5/2} \rightarrow \pi f_{7/2}$	0.15
$\nu p_{1/2} \rightarrow \pi p_{3/2}$	0.11

TABLE III. Same as Table II, but for the SLy5 force.

Configuration	Weight
$\nu i_{13/2} \rightarrow \pi i_{11/2}$	0.79
$\nu h_{11/2} \rightarrow \pi h_{11/2}$	0.59
$\nu f_{7/2} \rightarrow \pi f_{5/2}$	0.11
$\nu i_{13/2} \rightarrow \pi i_{13/2}$	0.04

value of t_2, is smaller than that of the other forces. We conclude that both the velocity-independent and velocity-dependent terms in the residual interaction are important.

This discussion already points out that, although it is not our purpose here to discuss in too much detail the strategy for improving the fits of effective Skyrme forces, we would like to strongly encourage the use of realistic constraints coming from the GT properties. We show in Fig. 6 direct correlations between the percentage of $m_0(t_-)$ associated with the GTR and combinations of Skyrme parameters (actually, we find correlation also with the t_1, t_2 parameters separately and with the quantity Θ_S defined in Ref. [31]). We are well aware that the (t_0, t_3) part of the interaction is mainly connected to the saturation properties of symmetric nuclear matter and the related value of the incompressibility, and the t_1, t_2 part must be fitted together with finite nuclei ground state properties. The best choice should probably be to check *a posteriori* that the value of t_0 and t_3 are compatible with the upper panel of Fig. 6, and impose *a priori* the constraint associated with the lower panels on the t_1, t_2 part, together with the other ones which are usually imposed. An alternative strategy is represented by the possibility of fixing the odd parameter C_1^T in an independent way with respect to the even part of the functional.

In some works, values of the Landau parameters have been fitted. Therefore, in Fig. 7 we show the correlation between the percentage of $m_0(t_-)$ exhausted by the GTR and either g_0' or g_1'. Future fits of Skyrme parameter sets can certainly also benefit from the use of one of these two constraints, which set either g_0' or g_1' around 0.45 or 0.5. We believe that this estimate is more appropriate than the one based on the empirical g_0' since this latter is, as a rule, extracted from calculations based on a Woods-Saxon mean field instead of a Hartree-Fock one.

In summary, our results show clearly how the differences in the residual spin-isospin interaction (in particular in the velocity-dependent part), between various Skyrme parameter sets, manifest themselves if one studies the collectivity of the GTR. In particular, we point to the necessity of new fits which include the GT data as an additional constraint, mainly to cure such forces as SLy5 which display a kind of anomaly in this respect.

Before concluding, we would like to show another kind of correlation with a physical parameter (cf. Fig. 8). In fact, the GT collectivity is also related to the quantity which we denote by $a_{\sigma\tau}$. This quantity is analogous, in the spin-isospin case, to the well-known asymmetry parameter a_τ (we recall that sometimes notations like a_4 or J are used for this latter quantity). It is

$$a_{\sigma\tau} = \frac{1}{2}\frac{\partial^2}{\partial\rho_{11}^2}\frac{E}{A}, \tag{9}$$

where we consider infinite matter with a generic spin and isospin asymmetry, and variations with respect to the spin-isospin density ρ_{11} defined as

$$\rho_{11} = \frac{\rho_{n\uparrow} - \rho_{n\downarrow} - \rho_{p\uparrow} + \rho_{p\downarrow}}{\rho}. \tag{10}$$

Although the spirit of our discussion is connected with the points raised in Ref. [17], our conclusions are different. In fact, we find different results than those published in Ref. [17]. We have tried to analyze in detail the sources of this difference, and in particular we have checked the numerical effects in the case of ^{90}Zr [43]. First, the energies in charge-exchange QRPA are naturally defined with respect to the target nucleus ground state. Since the experimental values of the charge-exchange resonances are provided in the final, or daughter, systems, we find it quite straightforward (as we did in the past and as other authors do) to transform the experimental value into a corresponding value with respect to the target nucleus ground

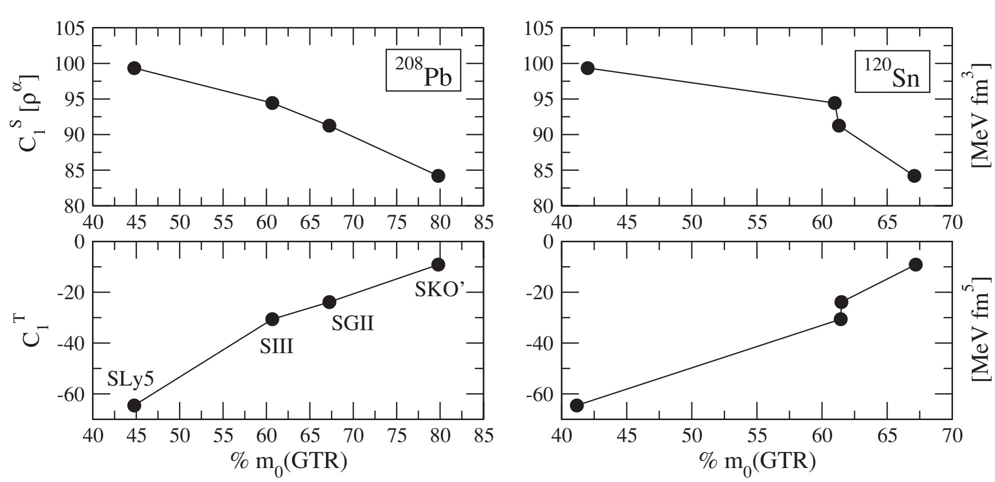

FIG. 6. Correlations between relevant parameters of the residual p-h interaction [cf. Eq. (6)] and the percentage of $m_0(t_-)$ exhausted by the GTR. The left (right) panel refers to ^{208}Pb (^{120}Sn). In the upper part of the figure, the coefficient C_1^S has been evaluated at $\rho = 0.16$ fm^{-3}.

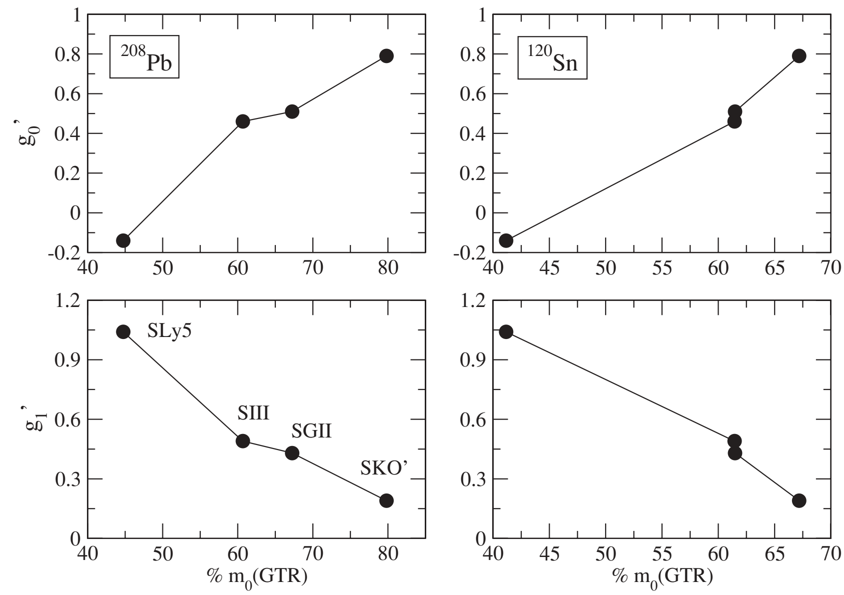

FIG. 7. Correlations between the Landau parameters g_0' or g_1' and the percentage of $m_0(t_-)$ exhausted by the Gamow-Teller resonance. The left (right) panel refers to ^{208}Pb (^{120}Sn).

state by using experimental binding energies. However, this is not done in Ref. [17] where a theoretical estimate of the binding energy difference is carried out. In ^{90}Zr, the two alternative choices produce a discrepancy of 1.2 MeV. A second source of difference, already discussed, is the treatment of the J^2 terms; in the case of ^{90}Zr, this produces another $\approx$1 MeV of difference. After considering these two facts, part of the discrepancy (in ^{90}Zr, another 1.3 MeV for SLy4 and 0.6 MeV for SkO′) has remained unexplained, and it is quite hard to attribute it simply to the different numerical implementations.

V. RESULTS FOR THE SPIN-DIPOLE RESPONSE

The spin-dipole strength is not extracted experimentally in a straightforward manner. In the absence of a well-established proportionality between the cross section at a given angle and the dipole strength, either spectra subtraction or multipole decomposition analysis has to be attempted. Furthermore, the three different J^π components are mixed; the similarity of the associated angular distributions would require sophisticated techniques to disentangle these components, such as the use of polarized beams or the study of the γ decay of the IVSD to the GTR and to low-lying states, performed with high energy resolution and high γ-ray detection efficiency [44].

Theoretically, a systematic clear picture of the IVSD is still missing. The two references [24,25] mentioned in the Introduction predict, respectively, the IVSD in ^{208}Pb to lie at 21.3 and 24.0 MeV. Only recently self-consistent calculations have been carried out in the same nucleus [42], but we learned from the previous discussion on the GTR that we need to consider several isotopes and to extract a global trend if we wish to understand which interactions provide reliable results. Therefore, our present discussion is quite timely.

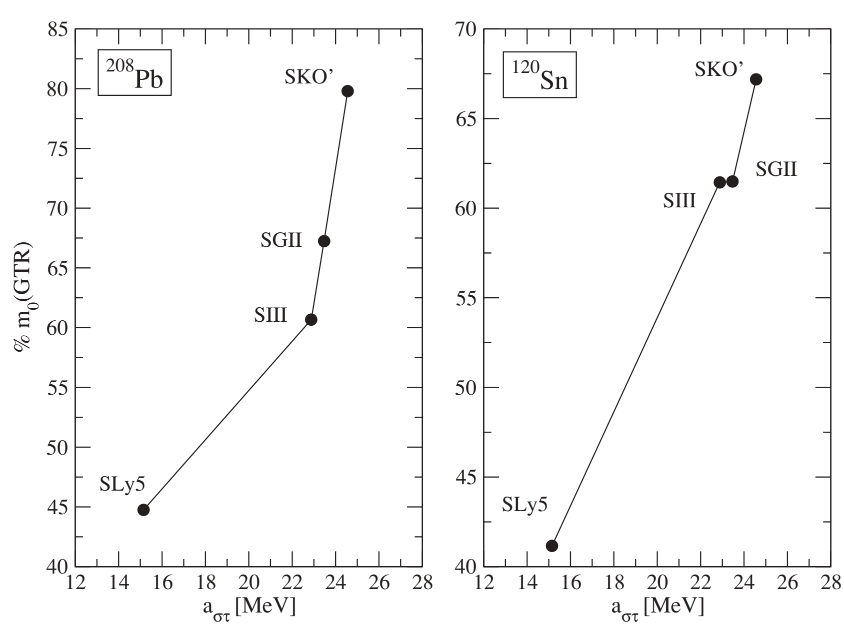

FIG. 8. Correlation between the parameter $a_{\sigma\tau}$ defined in the text and the percentage of $m_0(t_-)$ exhausted by the Gamow-Teller resonance. The left (right) panel refers to ^{208}Pb (^{120}Sn).

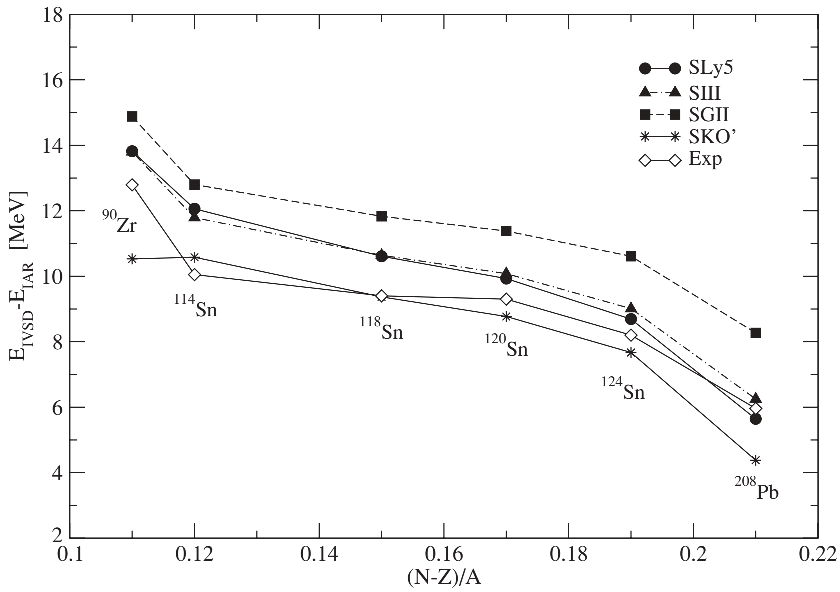

FIG. 9. Same as Fig. 3, but for the spin-dipole case. Experimental data are from Refs. [9,26,41,45].

We of course can separate the three J^π components; however, to compare with experiment, we have to make a global average of the different J^π centroids. In particular, we estimate

$$\overline{E}_{\mathrm{IVSD}_-} = \frac{\sum_{J^\pi=0^-,1^-,2^-} m_1(J^\pi) m_1(J^\pi)(t_-)}{\sum_{J^\pi=0^-,1^-,2^-} m_0(J^\pi) m_0(J^\pi)(t_-)}, \qquad (11)$$

for different nuclei. We evaluate the sum rules in the whole energy region where the transition strength is not negligible. We report the difference between these energies and the IAR energies in Fig. 9, and we compare these values with experimental data from Refs. [9,26,41,45].

It is rather satisfactory to have found that the different Skyrme forces behave quite similarly, as far as the IVSD is concerned, to how they behave for the simpler GTR. We have also looked in more detail to the strength distributions obtained by using the forces SkO′ and SLy5. These distributions, for the nuclei ^{90}Zr, ^{208}Pb, and ^{120}Sn, are displayed in Figs. 10–12 (SkO′) and Figs. 13–15 (SLy5). The QRPA strength functions are shown in the upper panels and compared with the unperturbed strength functions which appear in the lower panels. The integral features of the distributions are resumed in Tables IV and V, for the two forces, respectively. It is evident that the unperturbed centroids, whose values are reported in parentheses, follow the known energy hierarchy [24], the 2^- being the lowest and the 0^- the highest centroid. This is because the 0^- wave functions are entirely composed of particle-hole (or two-quasiparticle) excitations between proton-neutron states with opposite parity and the same total angular momentum, which are in general widely separated in energy. This trend is retained when the residual interaction is turned on, pushing up the centroids. The comparison between the unperturbed and the QRPA distributions highlights the large values of the repulsive matrix elements of the residual interaction.

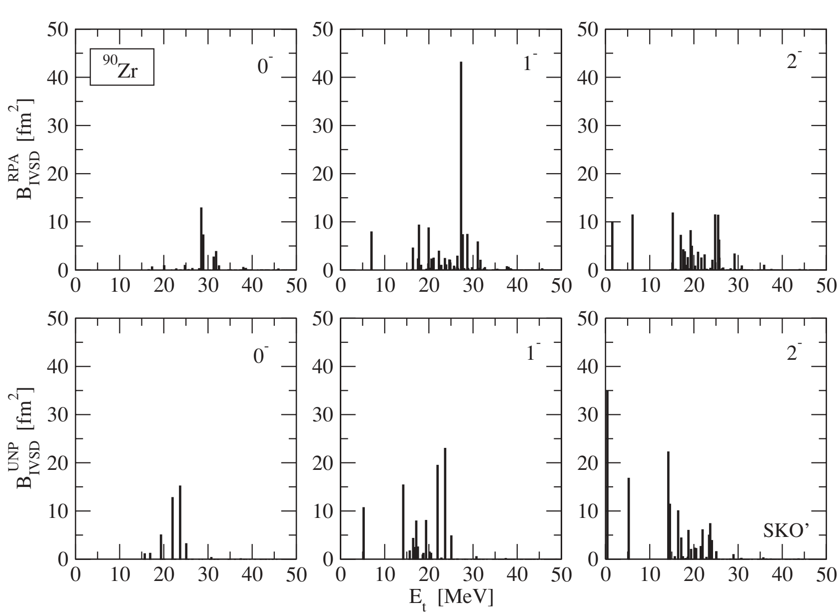

FIG. 10. RPA (upper panels) and unperturbed (lower panels) strength distributions for the three spin components of the IVSD. They are calculated using the force SkO′ in the nucleus ^{90}Zr.

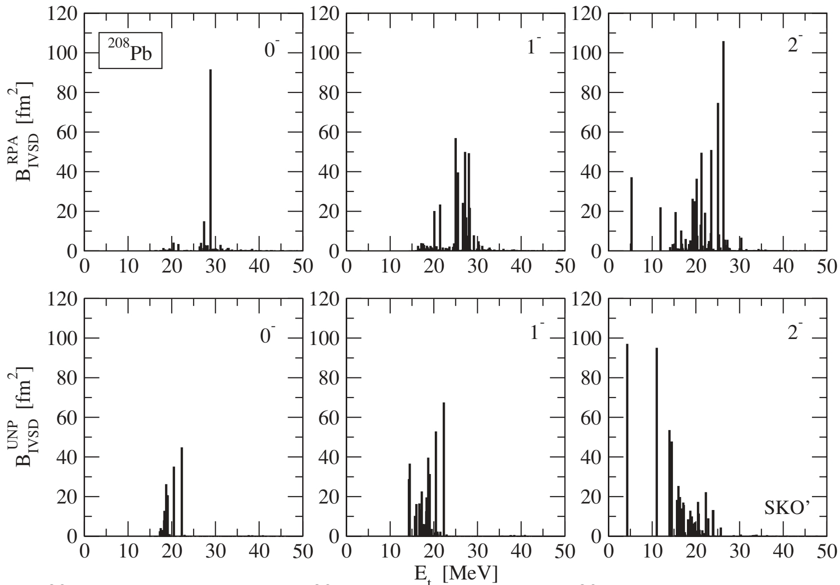

FIG. 11. Same as Fig. 10, but for the nucleus ^{208}Pb.

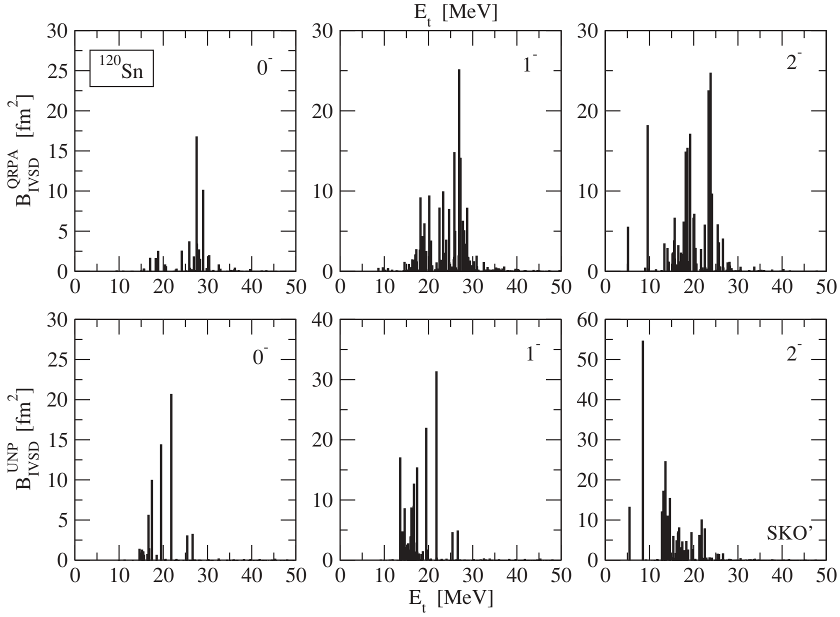

FIG. 12. Same as Fig. 10, but for the nucleus ^{120}Sn.

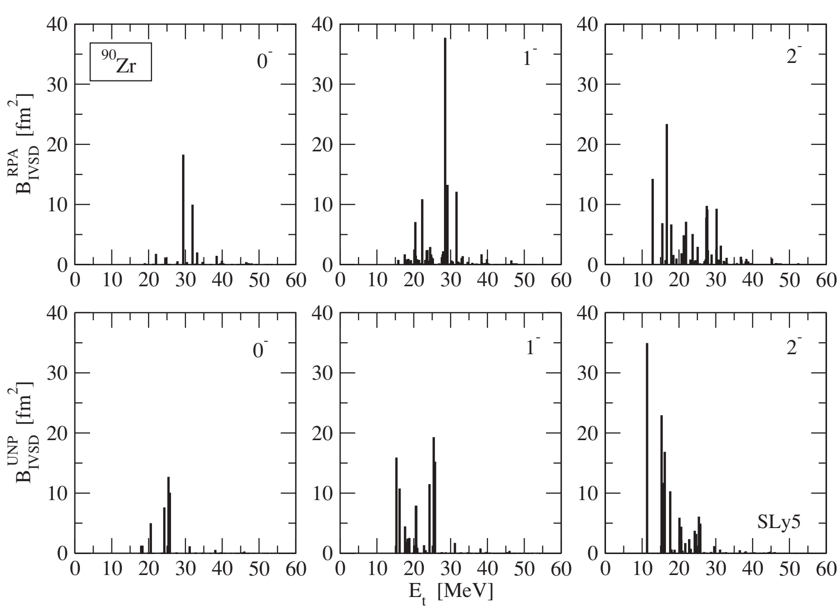

FIG. 13. RPA (upper panels) and unperturbed (lower panels) strength distributions for the three spin components of the IVSD. They are calculated using the force SLy5 in the nucleus ^{90}Zr.

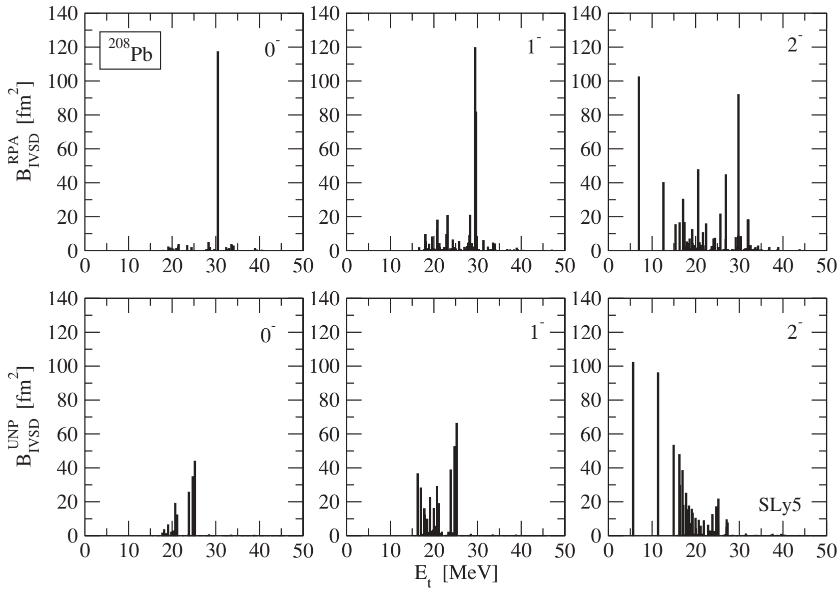

FIG. 14. Same as Fig. 13, but for the nucleus ^{208}Pb.

The IVSD spectra are rather fragmented. This fragmentation increases with the value of L, the 2^- distribution being broader than the 1^- and 0^-. Because of the degeneracy factor, when the energy is averaged over the three spin components, the contribution from 0^- is less weighted than the 1^- and 2^-. It has been checked that the sum rules of 2^-, 1^-, and 0^- respect the ratio 5:3:1.

In the spectrum of ^{90}Zr, most of the IVSD strength predicted by the two forces we employed lies in the energy range between 12 and 40 MeV with respect to the target ground state. This is in agreement with the experimental trend [27]. In addition, some low-lying IVSD strength shows up clearly. In particular, with the force SkO′, the strength distribution displays a 1^- peak around 1 MeV (with respect to the daughter nucleus ground state), which absorbs 6% of the total strength in the t_- channel. Experimentally, it has been already pointed out in Ref. [46] that in the (p, n) reaction, a state at 1 MeV is visible, which does not have a completely 1^+ character. The presence of $L \geqslant 1$ components in the transitions between 1 and 3 MeV is somehow confirmed in Ref. [9] as well. In Ref. [47], the presence of $L = 1$ peaks has been reported, in the (n, p) reaction, at about 5 and 30 MeV. If we look at our t_+ strength distributions, we find strength up to around 13.5 MeV. The ratio $m_0(t_-)/m_0(t_+)$ is equal to 2.25 (2.03) in the case of the force SLy5 (SkO′), in good agreement with the experimental prediction of 2.52 [27].

In the 2^- spectrum of ^{208}Pb, it is possible to recognize a low-lying state, due to the $\nu i_{13/2} \rightarrow \pi h_{9/2}$ particle-hole transition. Our findings are in reasonable agreement with the experimental peak observed, for the first time, at 2.8 MeV (6.5 MeV referred to the target ground state) in Ref. [46].

It can be concluded that the behavior of the considered Skyrme forces seems to be quite robust in reproducing properties of the isovector resonances which involve the spin-isospin

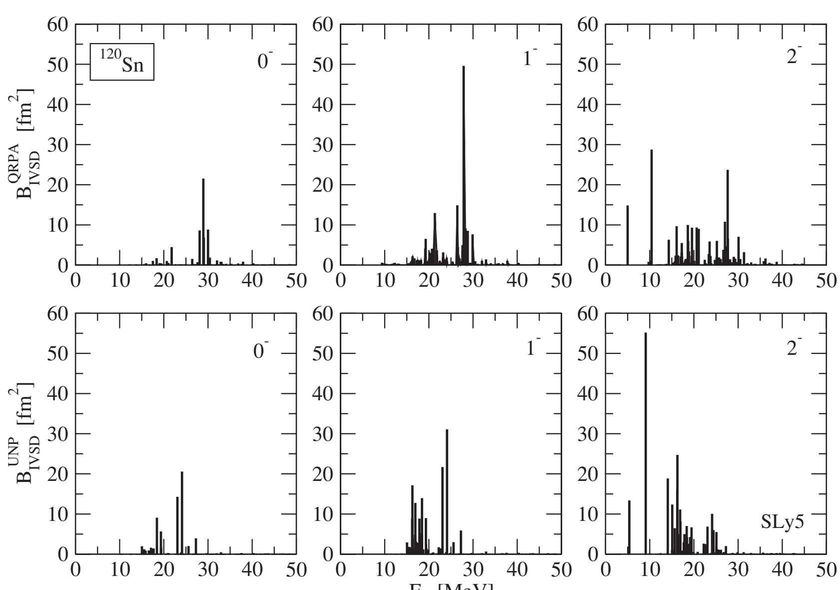

FIG. 15. Same as Fig. 13, but for the nucleus ^{120}Sn.

TABLE IV. QRPA (HF-BCS) summed transition strengths and centroid energies for the three spin-dipole components. The total centroid defined by Eq. (11) is also reported. All results correspond to the SkO′ force, as in Figs. 11 and 12.

	J^π	$m_{J^\pi}(0)\,(t_-)$ (fm^2)	$m_{J^\pi}(1)/m_{J^\pi}(0)\,(t_-)$ (MeV)
	0^-	33.0 (39.0)	29.14 (22.40)
^{90}Zr	1^-	132.3 (106.3)	24.20 (18.58)
	2^-	119.0 (142.7)	18.63 (12.51)
Tot.		284.3 (288.0)	22.44 (16.09)
	0^-	147.9 (162.5)	28.21 (20.16)
^{208}Pb	1^-	467.9 (436.0)	25.84 (18.49)
	2^-	650.0 (667.0)	21.32 (14.62)
Tot.		1265.8 (1265.5)	23.18 (16.67)
	0^-	57.6 (65.9)	26.82 (20.11)
^{120}Sn	1^-	207.3 (179.5)	24.58 (18.19)
	2^-	235.2 (256.5)	19.54 (14.49)
Tot.		500.2 (501.9)	22.47 (16.55)

degrees of freedom. Our results, reported in the figures and tables for different forces, can be compared with detailed forthcoming experimental findings (cf., e.g., Ref. [48]).

VI. THE EFFECT OF ISOVECTOR AND ISOSCALAR PAIRING

Our calculations are in principle sensitive to the effect of both isovector and isoscalar pairing. We recall that the empirical evidence of isovector pairing, in the ground state of open-shell nuclei as well as in their low-lying excitations, has been clear for a long time; but in connection with microscopic calculations based on energy functionals, there is still debate about the proper pairing force (for instance, whether it should have volume, or surface, or mixed character). About isoscalar pairing, the situation is much less clear. The existence of a $T = 0$ condensate has been questioned; if any exists, it is expected to show up only in the ground state of nuclei having an equal number of protons and neutrons, or others lying very near. In our HF-BCS calculations, as stated in Sec. II, we fix the $T = 1$ pairing force in order to have reasonable values for the empirical pairing gaps. The corresponding residual p-p force has been fixed by using the isospin invariance. If we change its strength, even by producing a drastic change on the pairing gap, the energy of the GTR is only slightly affected (≈200 keV). As already said, in keeping with the lack of possible constraints, we vary the strength of the $T = 0$ residual p-p interaction.

TABLE V. Same as Table IV, but for the SLy5 force.

	J^π	$m_{J^\pi}(0)\,(t_-)$ (fm^2)	$m_{J^\pi}(1)/m_{J^\pi}(0)\,(t_-)$ (MeV)
	0^-	37.9 (39.6)	30.81 (24.80)
^{90}Zr	1^-	115.2 (107.8)	27.45 (21.66)
	2^-	137.9 (142.8)	22.91 (17.54)
Tot.		290.9 (290.2)	25.73 (20.06)
	0^-	158.8 (159.8)	29.84 (23.30)
^{208}Pb	1^-	432.7 (428.0)	27.21 (21.16)
	2^-	645.8 (653.2)	21.25 (16.14)
Tot.		1237.3 (1241.1)	24.44 (18.79)
	0^-	64.8 (66.5)	28.31 (22.17)
^{120}Sn	1^-	187.7 (181.4)	25.72 (20.08)
	2^-	249.7 (257.9)	20.83 (15.93)
Tot.		502.1 (505.9)	23.63 (18.24)

In the case of the GTR, that is, in the 1^+ channel, only the isoscalar residual pairing is active when a zero-range force is assumed. We studied the effects of the pairing correlations on the GT strength distributions and found qualitatively similar outcomes in connection with different Skyrme forces. In the following, we will mention some specific results emerging from the calculations carried out using SLy5, just for illustrative purposes. Since SLy5 does not produce highly collective GT states, the analysis of the effects produced by pairing is simpler, but our general conclusions will remain valid for other Skyrme sets.

The effect of the residual p-p isoscalar (IS) pairing is shown for the isotope ^{118}Sn in Fig. 16. This effect is clearly visible, but it is small for the main peak, which varies only by 300 keV when the pairing strength is changed from zero to a value equal to that of the $T = 1$ pairing (i.e., 680 MeV fm^3). The IS pairing does not affect the total collectivity of the GTR, leaving the considerations made in Sec. IV basically unchanged.

In the absence of residual pairing, two peaks appear above 15 MeV: the first one at 15.30 MeV is mainly due to the $|\nu g_{9/2}, \pi g_{7/2}\rangle$ configuration, while the second one at 18.47 MeV is dominated by the $|\nu h_{11/2}, \pi h_{9/2}\rangle$ configuration. This so-called configuration splitting has been predicted [19,49], but it is smaller than the spreading width of the GTR. The $|\nu h_{9/2}, \pi h_{11/2}\rangle$ configuration gives a small QRPA solution at 18.68 MeV, which is not visible in the figure because of its negligible strength.

When the IS pairing is turned on, three new QRPA states show up, in which the mentioned configurations are mixed (cf. Table VI). The reduction of the configuration splitting (already remarked in Ref. [19], and which we have observed as a linear function of the pairing strength), and the mixing of spin-flip and back spin-flip configurations associated with the h orbitals, can be understood by analyzing the matrix elements

$$
\begin{aligned}
V^{J,ph}_{a\tilde{b}c\tilde{d}} &= \langle (ad^{-1})J|V_{ph}|(cb^{-1})J\rangle (u_a v_d u_c v_b \\
&\quad + v_a u_d v_c u_b) \\
V^{J,pp}_{abcd} &= \langle (ab)J|V_{pp}|(cd)J\rangle (u_a u_{p_2} u_{p'_1} u_c \\
&\quad + v_a v_{p_2} v_{p'_1} v_c).
\end{aligned} \tag{12}
$$

In the case at hand, with normal proton and superfluid neutron components, the previous equations reduce to

$$
\begin{aligned}
V^{J,ph}_{p\tilde{n}p'\tilde{n}'} &= \langle (pn'^{-1})J|V_{ph}|(p'n^{-1})J\rangle v_n v_{n'} \\
V^{J,pp}_{pnp'n'} &= \langle (pn)J|V_{pp}|(p'n')J\rangle u_n u_{n'}.
\end{aligned} \tag{13}
$$

The $|\nu g_{9/2}, \pi g_{7/2}\rangle$ configuration is not very sensitive to the isoscalar pairing, because the associated p-p matrix elements are weighted by factors which include a very small u_n. On the other hand, the u_n factors associated with the $h_{11/2}$ and $h_{9/2}$ are not so small, and the $|\nu h_{11/2}, \pi h_{9/2}\rangle$ and $|\nu h_{9/2}, \pi h_{11/2}\rangle$ configurations have p-p matrix elements larger than the

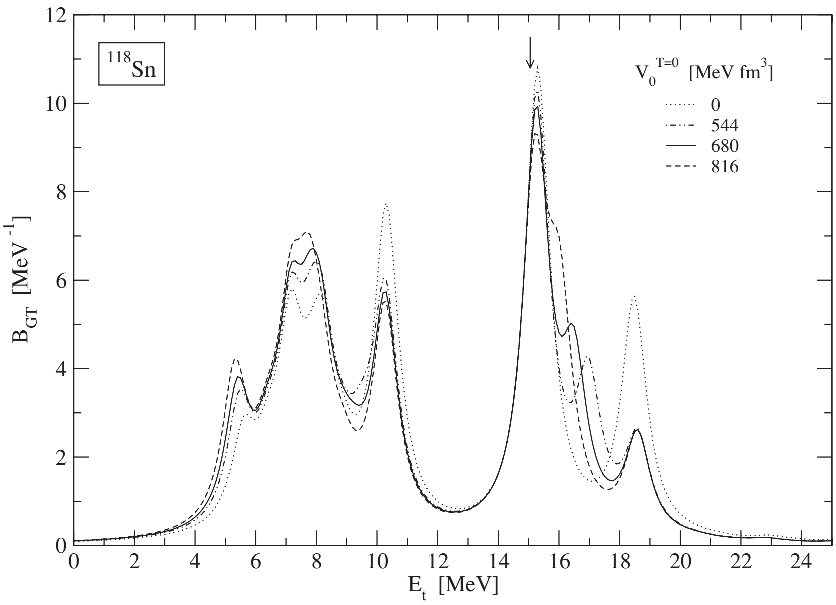

FIG. 16. GT strength distributions obtained in the nucleus [118]Sn by using the force SLy5 and varying the strength of the residual p-p isoscalar force.

corresponding p-h ones, which are about one-half or negligible. Therefore, the $|\nu h_{9/2}, \pi h_{11/2}\rangle$ is exclusively admixed in the GT wave function by the residual (isoscalar) pairing.

By looking also at the neighboring isotope ^{120}Sn, and comparing it with the results obtained with the force SkO′, we have reached the following conclusion. Although the presence of the nonspin-flip components in the GT wave function depends on the p-h interaction (as discussed in Sec. IV), the isoscalar pairing favors this admixture. Moreover, if we increase the strength of the isoscalar pairing force, more back-spin-flip configurations (which are energetically less favored) mix in the GT wave function.

In summary, the effect of pairing (both $T = 0$ and $T = 1$ pairing, the latter being responsible for the u and v factors) in the resonance region mainly concerns the detailed microscopic structure of the RPA states, besides their individual strength and energy, the GTR centroid energy being less affected and the associated total strength much less. In principle, particle decay experiments could shed light on the microscopic structure of the GTR; quantifying the presence of other components than the pure direct spin-flip ones in the GT wave function may highlight the features of corresponding pairing matrix elements. Accordingly, the theoretical framework based on the RPA plus the coupling with the continuum and the more complex configurations, which has explained the proton decay from the GTR of ^{208}Pb, should be extended to superfluid systems. This is left for future work. The present study of the behavior of different Skyrme sets is one of the requirements before going to more ambitious calculations.

We have also checked the effect of isoscalar pairing on the IVSD. The shifts on the $J^\pi = 0^-$, 1^-, and 2^- centroids, induced by the $T = 0$ pairing with $V_0^{(T=0)} = V_0^{(T=1)}$, amounts to a few hundred keV. In ^{118}Sn, the total IVSD centroid is affected by 500 keV. This effect is not negligible but remains smaller than the variations associated with the choice of the p-h interaction.

TABLE VI. Wave functions of the QRPA states obtained for the GTR in ^{118}Sn, with the interaction SLy5, when the isoscalar residual pairing is included and its strength is set equal to that of the isovector pairing (namely, 680 MeV fm^3).

Energy (percentage of $m_0(t_1)$)	Configuration	Weight
	$\nu g_{9/2}, \pi g_{7/2}$	0.93
15.25 MeV (26.4%)	$\nu d_{5/2}, \pi d_{3/2}$	0.07
	$\nu h_{11/2}, \pi h_{9/2}$	0.31
	$\nu g_{9/2}, \pi g_{7/2}$	0.30
16.49 MeV (9.7%)	$\nu h_{11/2}, \pi h_{9/2}$	0.62
	$\nu h_{11/2}, \pi h_{11/2}$	0.11
	$\nu h_{9/2}, \pi h_{11/2}$	0.67
	$\nu g_{9/2}, \pi g_{7/2}$	0.15
18.59 MeV (6.0%)	$\nu h_{11/2}, \pi h_{9/2}$	0.68
	$\nu h_{9/2}, \pi h_{11/2}$	0.69

VII. CONCLUSIONS

In this work, we have tried to shed light on the systematic behavior of the nuclear collective spin-isospin response, in different spherical medium-heavy nuclei, calculated by using the microscopic Skyrme functionals. Our model is a self-consistent QRPA based on HF-BCS, and we have studied both the Gamow-Teller and the spin-dipole strength distributions. We believe that the importance of our work stems from the fact that constraining the microscopic functionals in the spin-isospin channel is highly desirable if studies of exotic nuclei and applications for particle physics or astrophysics are envisaged, in which the spin-isospin transitions must be accurately obtained.

Pairing must be considered if the study has to be extended to different systems for which experimental measurements are available. The resonance properties depend of course mainly on the p-h interaction. We have not only elucidated the features of the existing functionals, but also made suggestions for future fits. In fact, the Lyon force SLy5 does not predict the correct GT collectivity. The other forces we have considered more or less reproduce this collectivity (within our mean field approximation), SGII and SIII overpredicting somewhat the GT centroid, and SkO′ lying closer to it. We have found a clear correlation between the GT collectivity and either selected combinations of Skyrme parameters or Landau parameters. These correlations may be used to improve the existing Skyrme parametrizations.

The IVSD has been systematically studied using our microscopic QRPA. No such study is available in the literature so far. The IVSD behavior does not introduce new constraints but somewhat confirms what has been deduced from the study of the GTR.

Finally, we have also singled out the effect of pairing (mainly its contribution to the residual proton-neutron interaction). Its effect is not large enough to alter the conclusions which have been drawn concerning the interaction in the p-h channel. However, some conclusions of this part are also interesting. Even if pairing does not affect so much the GT centroid and collectivity, it induces specific admixtures in the wave functions. If experimental evidence coming, e.g., from the particle decay was available, we could say that the microscopic structure of the collective spin-isospin states may help pin down the features of the effective proton-neutron force in the p-p channel, which is one of the open questions in nuclear structure.

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APPENDIX: EXPLICIT FORM OF THE RESIDUAL p-h INTERACTION

The coefficients appearing in Eq. (6) are

$$\begin{aligned}
C_1^{\rho}[\rho] &= -\frac{1}{8}(t_0 - 2t_0x_0) - \frac{1}{48}\rho^{\alpha}(t_3 + 2t_3x_3),\\
C_1^{S}[\rho] &= -\frac{t_0}{8} - \frac{t_3}{48}\rho^{\alpha},\\
C_1^{\tau} &= \frac{1}{64}(-4t_1 - 8t_1x_1 + 4t_2 + 8t_2x_2),\\
C_1^{\triangle\rho} &= \frac{1}{64}(3t_1 + 6t_1x_1 + t_2 + 2t_2x_2),\\
C_1^{T} &= \frac{1}{16}(-t_1 + t_2),\\
C_1^{\triangle S} &= \frac{1}{64}(3t_1 + t_2),\\
C_1^{\nabla J} &= -\frac{1}{4}W_0
\end{aligned} \tag{A1}$$

(When the spin-orbit part of the functional is generalized by introducing the parameters b_4 and b'_4 [50], the last expression becomes $-\frac{1}{2}b'_4$.) If we insert these expressions into Eq. (6), we find

$$\begin{aligned}
v_{01}(r) &= \left[2\left(-\frac{t_0}{8} - \frac{1}{4}t_0x_0\right) - \frac{1}{24}\rho^{\alpha}(r)(t_3 + 2t_3x_3)\right]\vec{\tau}_1\vec{\tau}_2,\\
v_{11}(r) &= \left[-\frac{t_0}{4} - \frac{t_3}{24}\rho^{\alpha}(r)\right]\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2,\\
v'_{01} &= \left[-\frac{t_1}{8}(2x_1+1)(\vec{k}'^2 + \vec{k}^2) + \frac{t_2}{4}(2x_2+1)(\vec{k}'\vec{k})\right]\vec{\tau}_1\vec{\tau}_2,\\
v'_{11} &= \left[-\frac{t_1}{8}(\vec{k}'^2 + \vec{k}^2) + \frac{t_2}{4}\vec{k}'\vec{k}\right]\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2,\\
v_1^{(\mathrm{s.o.})} &= \frac{iW_0}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)\vec{k}' \times \vec{k}\vec{\tau}_1\vec{\tau}_2,
\end{aligned} \tag{A2}$$

keeping the same notation as used in Sec. II.

The choice of neglecting the contribution to the residual interaction from the J^2 terms amounts to writing

$$\begin{aligned}
v'_{01} &= \left[\frac{1}{16}[\vec{\nabla}_1\vec{\nabla}_2 + \vec{\nabla}'_1\vec{\nabla}'_2](2x_1t_1 - t_1)\right.\\
&\quad \left. + \frac{1}{4}\vec{k}'\vec{k}(2x_2t_2 + t_2)\right]\vec{\sigma}_1\vec{\sigma}_2,\\
v'_{11} &= \left[-\frac{t_1}{16}[\vec{\nabla}_1\vec{\nabla}_2 + \vec{\nabla}'_1\vec{\nabla}'_2] + \frac{t_2}{4}\vec{k}'\vec{k}\right]\vec{\tau}_1\vec{\tau}_2\vec{\sigma}_1\vec{\sigma}_2.
\end{aligned} \tag{A3}$$

As mentioned in Sec. II, it is appropriate to give here the expressions for the Landau parameters discussed in the paper. In symmetric nuclear matter, the $\ell = 0$ and 1 spin-isospin parameters are

$$\begin{aligned}
g'_0 &= -N_0\left[\frac{1}{4}t_0 + \frac{1}{24}t_3\rho^{\alpha} + \frac{1}{8}k_F^2(t_1 - t_2)\right],\\
g'_1 &= N_0\left(\frac{t_1}{8} - \frac{t_2}{8}\right)k_F^2,
\end{aligned} \tag{A4}$$

where $N_0 = 2k_F m^*/\pi^2\hbar^2$ and k_F is the Fermi momentum. The Landau parameters are zero for $\ell > 1$. If we rewrite the Landau parameters in terms of the coefficients of Eq. (6), they read

$$\begin{aligned}
g'_0 &= N_0\left(2C_1^S + 2C_1^T k_F^2\right),\\
g'_1 &= -2N_0C_1^T k_F^2.
\end{aligned} \tag{A5}$$

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