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Why is the equation of state for tin so soft?

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The distribution of isoscalar monopole strength in the neutron-even $^{112-124}$Sn isotopes has been computed using a relativistic random-phase-approximation approach. The accurately-calibrated model used here ("FSUGold") has been successful in reproducing both ground-state observables as well as collective excitations—including the giant monopole resonance (GMR) in ^{90}Zr, ^{144}Sm, and ^{208}Pb. Yet this same model significantly overestimates the GMR energies in the Sn isotopes. It is argued that the question of "Why is tin so soft?" becomes an important challenge to the field and one that should be answered without sacrificing the success already achieved by several theoretical models.

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The compression modulus of nuclear matter (also known as the nuclear incompressibility) is a fundamental parameter of the equation of state that controls small density fluctuations around the saturation point. While existing ground-state observables have accurately constrained the binding energy per nucleon ($B/A \simeq -16$ MeV) and the baryon density ($\rho \simeq 0.15\ \text{fm}^{-3}$) of symmetric nuclear matter at saturation, the extraction of the compression modulus (K) requires to probe the response of the nuclear system to small density fluctuations. It is generally agreed that the *nuclear compressional modes*—particularly the isoscalar giant monopole resonance (GMR)—provide the optimal route to the determination of the nuclear incompressibility [1]. Moreover, the field has attained a level of maturity and sophistication that demands strict standards in doing so. It is now demanded that the same microscopic model that predicts a particular value for the compression modulus of infinite nuclear matter (an experimentally *inaccessible* quantity) be able to accurately reproduce the experimental distribution of monopole strength.

Earlier attempts at extracting the compression modulus of symmetric nuclear matter relied primarily on the distribution of isoscalar monopole strength in ^{208}Pb—a heavy nucleus with a well developed giant resonance peak [2,3]. However, as was pointed out recently in Refs. [4,5]—and confirmed since then by several other groups [6–8]—the GMR in ^{208}Pb does not provide a clean determination of the compression modulus of *symmetric* nuclear matter. Rather, it constraints the nuclear incompressibility of *neutron-rich* matter at the particular value of the neutron excess found in ^{208}Pb, namely, $b \equiv (N-Z)/A = 0.21$. As such, the GMR in ^{208}Pb is sensitive to the *density dependence of the symmetry energy*. The symmetry energy represents a penalty levied on the system as it departs from the symmetric limit of equal number of neutrons and protons. As the infinite nuclear system becomes neutron rich, the saturation density moves to lower densities, the binding energy weakens, and the nuclear incompressibility *softens* [9]. Thus, the compression modulus of a neutron rich system having the same neutron excess as ^{208}Pb is *lower* than the compression modulus of symmetric nuclear matter. We note in passing that the symmetry energy is to an excellent approximation equal to the difference between the energy of pure neutron matter (with $b \equiv 1$) and that of symmetric nuclear matter (with $b \equiv 0$).

The alluded sensitivity of the distribution of isoscalar monopole strength to the density dependence of the symmetry energy proved instrumental in resolving a puzzle involving K: how can accurately calibrated models that reproduce ground state data as well as the distribution of monopole strength in ^{208}Pb, predict values for K that differ by as much as 25%? (Note that accurately-calibrated relativistic models used to predict a compression modulus as high as $K \approx 270$ MeV while their nonrelativistic counterpart suggested values as low as $K \approx 215$ MeV.) This discrepancy is now attributed to the poorly determined density dependence of the symmetry energy [4]. Indeed, models that predict a stiffer symmetry energy (one that increases faster with density) consistently predict higher compression moduli than those with a softer symmetry energy. Thus, the success of some models in reproducing the GMR in ^{208}Pb was accidental, as it resulted from a combination of both a stiff equation of state for symmetric nuclear matter and a stiff symmetry energy [5]. Since then, the large differences in the predicted value of K have been reconciled and a "consensus" has been reached that places the value of the incompressibility coefficient of *symmetric* nuclear matter at $K = 230 \pm 10$ MeV [7,8,10,11]. Note that while some Skyrme and relativistic mean-field models do not display a clear correlation between K and the density dependence of the symmetry energy [12], we trust that once those models are further constrained to reproduce the experimental distribution of isoscalar monopole strength in ^{208}Pb, the alluded correlation will reemerge [4,5].

An example of how this consensus was reached is depicted in Fig. 1 where the distribution of isoscalar monopole strength in ^{90}Zr, ^{116}Sn, ^{144}Sm, and ^{208}Pb at the small momentum transfer of $q = 45.5$ MeV (or $q = 0.23\ \text{fm}^{-1}$) is displayed for the relativistic FSUGold model of Ref. [10]—a model that predicts an incompressibility coefficient for symmetric nuclear matter of $K = 230$ MeV. Note that the distribution of strength was obtained from a relativistic random-phase-approximation (RPA) approach as described in detail in Ref. [13]. Further, the inset on Fig. 1 shows a comparison of the theoretical predictions against the experimental centroid energies reported

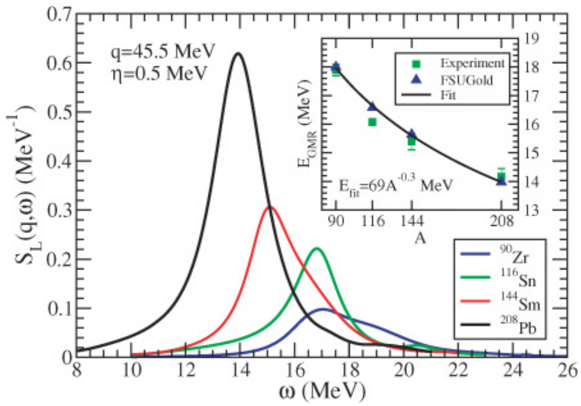

FIG. 1. (Color online) Distribution of isoscalar monopole strength predicted by the FSUGold model of Ref. [10]. The inset includes a comparison against the experimental centroid energies reported in Ref. [14], with the solid line providing the best fit to the theoretical predictions.

in Ref. [14]. Finally, the solid line in the inset provides a fit to the mass dependence of the theoretical predictions that yields $E_{\rm GMR}(A) \approx [69/A^{0.3}]$ MeV.

The isoscalar monopole strength displayed in Fig. 1 is extracted from the low momentum transfer behavior of the longitudinal response defined as follows:

$$S_{\rm L}(\mathbf{q},\omega) = \sum_n |\langle\Psi_n|\hat{\rho}(\mathbf{q})|\Psi_0\rangle|^2\delta(\omega-\omega_n)\ . \qquad (1)$$

Here Ψ_0 is the exact nuclear ground state, Ψ_n is an excited state with excitation energy ω_n, and $\hat{\rho}(\mathbf{q})$ is the Fourier transform of the isoscalar baryon density. That is,

$$\hat{\rho}(\mathbf{q}) = \int d^3r\, e^{-i\mathbf{q}\cdot\mathbf{r}}\bar{\psi}(\mathbf{r})\gamma^0\tau_0\psi(\mathbf{r}), \qquad (2)$$

where $\psi(\mathbf{r})$ is an isodoublet nucleon field, γ^0 is the timelike (or *zeroth*) component of the Dirac gamma matrices, and $\tau_0 \equiv \mathbf{1}$ is the identity matrix in isospin space.

The important realization that the distribution of monopole strength in heavy nuclei is sensitive to the density dependence of the symmetry energy has motivated a recent experimental study of the GMR along the isotopic chain in tin. Indeed, the distribution of isoscalar monopole strength in the neutron-even $^{112\text{–}124}$Sn isotopes has been measured at the Research Center for Nuclear Physics (RCNP) in Osaka, Japan [11,15]. This important experiment probes the incompressibility of *asymmetric* nuclear matter by measuring the distribution of isoscalar strength in a chain of isotopes with a neutron excess ranging from $b = 0.11$ (in ^{112}Sn) to $b = 0.19$ (in ^{124}Sn). The experiment represents a hadronic complement to the *purely electroweak* parity radius experiment (PREX) at the Jefferson Laboratory that aims to measure the neutron radius of ^{208}Pb accurately and model independently via parity-violating electron scattering [16,17]. Such an accurate determination will have far-reaching implications in areas as diverse as nuclear structure [18], heavy-ion collisions [19–24], atomic parity violation [18,25,26], and nuclear astrophysics [27–30]. While this important experiment gets off the ground, a significant effort has been devoted to constrain the neutron radius of a heavy nucleus by alternative (hadronic) means. One such effort uses nuclear giant and pygmy resonances in neutron-rich nuclei to constrain the density dependence of the symmetry energy [4–8]. Another promising approach is the use of the spin dipole sum rule—a quantity that is highly sensitive to the difference between neutron and proton mean square radii [31]. Indeed, Yako, Sagawa, and Sakai recently used the spin dipole sum rule to extract the neutron skin of ^{90}Zr and obtained a result that is consistent with that obtained through significantly different means [32].

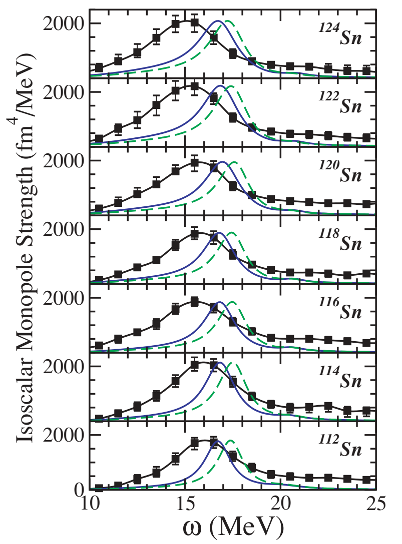

FIG. 2. (Color online) Comparison between the distribution of isoscalar monopole strength in all neutron-even ^{112}Sn-^{124}Sn isotopes extracted from experiment (black solid squares) and the theoretical predictions of the FSUGold (blue solid line) and NL3 (green dashed line) models.

In Fig. 2 the experimental distribution of isoscalar monopole strength measured at the RCNP [11,15] is compared against the predictions of the highly successful NL3 [33,34] and FSUGold [10] models. Note that for completeness, we have listed in Table I the most important bulk parameters of neutron-rich matter (for a precise definitions of each term see, for example, Ref. [12]). As one is only interested in comparing the shape of the distribution and a particular ratio of its moments, the maximum of the theoretical curves—computed

TABLE I. Binding energy per nucleon and compression modulus of symmetric nuclear matter for the two mean-field models employed in this work. Also shown are values for the symmetry energy, its slope, and its curvature at saturation density. All quantities are in MeV. For the precise definition of these terms see Ref. [12].

Model	ϵ_0	K	J	L	$K_{\rm sym}$
NL3	−16.2	271	37.3	118.2	337.3
FSUGold	−16.3	230	32.6	60.5	69.7

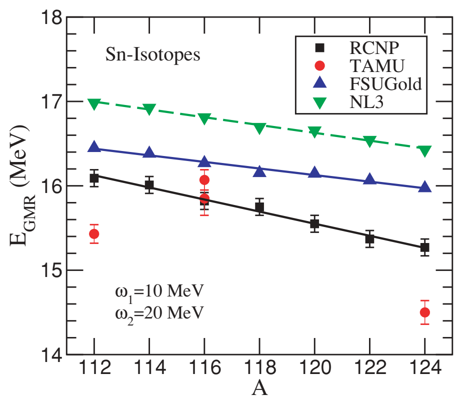

FIG. 3. (Color online) Comparison between the GMR centroid energies (m_1/m_0) in all neutron-even ^{112}Sn-^{124}Sn isotopes extracted from experiment (black solid squares) and the theoretical predictions of the FSUGold (blue up-triangles) and NL3 (green down-triangles) models. Also shown (filled red circles) are results from the Texas A&M group [14,35,36] for the cases of ^{112}Sn, ^{116}Sn, and ^{124}Sn.

from the longitudinal response as described in the text—has been normalized to the experimental data. The A-dependence of the corresponding centroid energies is also displayed in Fig. 3 and compiled in Table II. Note that the centroid energy is computed from the ratio of the m_1 moment to that of the m_0 moment. That is,

$$E_{\rm GMR} \equiv \frac{m_1}{m_0} = \frac{\int_{\omega_1}^{\omega_2} \omega S_{\rm L}(q_0,\omega)d\omega}{\int_{\omega_1}^{\omega_2} S_{\rm L}(q_0,\omega)d\omega}, \tag{3}$$

where, consistent with the experimental analysis [11,15], the limits of integration have been chosen to be $\omega_1 = 10$ MeV and $\omega_2 = 20$ MeV. Further, to mimic the forward-angle experiment, the longitudinal response was evaluated at the "small" momentum transfer of $q_0 = 0.23$ fm^{-1}.

A subtle telltale problem with tin barely discernible in the inset on Fig. 1, becomes magnified in Fig. 2 as one compares the experimentally extracted distribution of monopole strength against the theoretical predictions. While mean-field plus RPA calculations are typically unable to describe the experimental width—which is in general composed of both an escape (particle-hole) and a spreading (multiparticle-multihole) width—such is not the case for the description of the centroid energies. Indeed, accurately calibrated models, both nonrelativistic [8] and relativistic (see Fig. 1), provide an adequate description of the GMR centroid energies in both ^{90}Zr (with $b = 0.11$) and ^{208}Pb (with $b = 0.21$)—nuclei with a neutron excess similar to those at the two extremes of the isotopic chain considered here. Why is then that both nonrelativistic [11,15,37] and relativistic models consistently overestimate the centroid energies in the Sn isotopes? Or more colloquially, *why is tin so soft?* And why is that the discrepancy between theory and experiment continues to grow as the neutron excess increases? A stiff symmetry energy leads to a rapid softening of the nuclear incompressibility [9]. This is the main reason behind the slightly larger (negative) slope displayed by NL3 relative to FSUGold in Fig. 3. The even larger (by more than 50%) slope displayed by the experimental data is unlikely to be solely related to the stiffness of the symmetry energy, as NL3 already predicts a neutron skin thickness in ^{208}Pb that appears overly large [10]. Note that in a recent paper, Sagawa and collaborators seem to reach similar conclusions to ours, namely, a theoretical distribution of isoscalar monopole strength in the tin isotopes that is significantly stiffer than experiment [38].

So why is tin so soft and why does it become even softer with an increase in the neutron excess? Could there be a systematic error in the experimental extraction? While possible, this is unlikely as an earlier independent measurement on ^{116}Sn [14] appears to confirm the present (RCNP) result (see Fig. 3). Although note that recent data by the Texas A&M group on ^{112}Sn and ^{124}Sn [36] deviates significantly from the RCNP data. Could the GMR in tin probe physics that has not been already constrained by nuclear observables? This also appears unlikely as existing density functionals are successful at describing a host of ground-state observables as well as collective excitations—including the GMR in ^{90}Zr, ^{144}Sm, and ^{208}Pb (see Fig. 1 and Ref. [10]). Could tin be sensitive to pairing correlations and more complicated multiparticle-multihole excitations? The answer at present is not clear, but if it turns out to be positive, why should tin be sensitive to these effects but not Zr, Sm, and Pb? Clearly, the distribution of isoscalar monopole strength in the Sn isotopes poses a serious theoretical challenge, perhaps suitable for the new Universal Nuclear Energy Density Functional (UNEDF) initiative. Whatever the theoretical approach, however, one must remember that the challenge is not solely to describe the distribution of monopole strength along the isotopic chain in tin, but rather, to do so without sacrificing the enormous success already achieved in reproducing a host of ground-state observables and collective modes.

TABLE II. Giant monopole resonance centroid energies (in MeV) computed from the ratio of moments (m_1/m_0) as described in the text. All moments were obtained from integrating the distribution of strength over the $10 \leqslant \omega \leqslant 20$ MeV interval.

Nucleus	NL3	FSUGold	Experiment
^{112}Sn	16.98	16.45	16.2 ± 0.1
^{114}Sn	16.92	16.38	16.1 ± 0.1
^{116}Sn	16.81	16.27	15.8 ± 0.1
^{118}Sn	16.70	16.15	15.8 ± 0.1
^{120}Sn	16.66	16.14	15.7 ± 0.1
^{122}Sn	16.54	16.07	15.4 ± 0.1
^{124}Sn	16.43	15.97	15.3 ± 0.1

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