Width of ^{11}B($\frac{1}{2}^+$, $T = \frac{3}{2}$)

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The latest compilation gives the width of the $1/2^+$, $T = 3/2$ level of ^{11}B as 210(20) keV. This is based in part on the value 230(65) keV obtained from fitting ^{10}Be$(p, \gamma_0)^{11}$B data. It has recently been claimed that these data can be adequately fitted with a width of 640 keV. We discuss this claim and refit the data. With noninterfering levels, a width of order 400 keV is found. A poorer fit in which a second $1/2^+$ level is included gives a width of order 600 keV.

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Fortune [1] (hereafter referred to as F) argues that the width of the ^{11}B($1/2^+$, $T = 3/2$) level should be much greater than the 210(20) keV given in the latest compilation [2]. In part, this value is based on a width of 230(65) keV obtained from fits to ^{10}Be$(p, \gamma_0)^{11}$B data [3]. F has refitted these data, with a width of 640 keV for the $1/2^+$ level. He stresses that this is not a best fit, but is it a good fit?

The data [3], at both 0° and 90°, show three peaks in the range $E_p = 0.6 - 2.1$ MeV. F fitted only the 90° data, using three levels plus a small linear background, with no interference between any of them (an alternative four-level fit by F is discussed later). The middle peak is assumed to be due to the $1/2^+$ level and the highest-energy peak to a $1/2^-$, $T = 3/2$ level, while the lowest peak is of uncertain origin.

F does not show his fit to the full data in a single figure; his Fig. 1 gives his fit to the upper two peaks, and Fig. 2 to the lowest peak (the experimental point shown at $E_{\rm lab} = 1.95$ MeV in each of F's five figures does not belong to the 90° data, but to the 0°). The combined fit would have a negative background for $E_p \approx 2$ MeV. F uses three different shapes for the three levels; the middle level is given by F's Eq. (1) with the energy dependence of the width given by a potential model, the highest level uses the same equation with an energy-independent width, while the lowest level has a Breit-Wigner (BW) shape with constant width (and no factor $1/k^2$).

Here we make a least-squares fit to the data [3], for both 0° and 90°. F used only the 90° data because then there is no interference between the $1/2^+$ and $1/2^-$ contributions; the third level could, however, interfere with one or other of these, depending on its parity (which is unknown). As in F, we ignore any such interference. The $1/2^+ - 1/2^-$ interference term can be calculated, so that the 0° data can also be fitted. A non-negative linear background is included. As in F, uncertainties on the data points are taken as 4%.

For each of the three levels, we use the one-level approximation of R-matrix theory [4]. For $E_p \leqslant 2.1$ MeV, the only isospin-allowed decay channels for $T = 3/2$ states of ^{11}B are ^{10}Be(g.s.) + p and γ-decay. As usual, contributions from γ-decay channels to the total width and level shift are neglected. Then the total width is due only to the proton channel; however, level-shift contributions can come from closed channels, such as ^{10}B$(0^+, T = 1) + n$. This ^{10}B state is the analog of ^{10}Be(g.s.), and the ratio of the reduced widths for this neutron channel to that for the proton channel should be approximately the ratio of the Clebsch-Gordan factors (C^2), which is 2. For the $1/2^+$ level, the s-wave shift factor for the loosely-bound neutron channel changes rapidly with energy, so that its contribution should not be omitted. A similar argument applies for the $1/2^-$ level, while for the lowest level the procedure is not critical. Then the cross section at angle θ can be written

$$\frac{d\sigma}{d\Omega} = \frac{1}{4k^2}[|U_1|^2 + |U_2|^2 + |U_3|^2 + Re(U_2^* U_3)\cos\theta] + A + BE, \quad (1)$$

where E is the c.m. energy in the ^{10}Be + p channel, and

$$U_j = -2ie^{i(\omega_{l_j} - \phi_{l_j})} P_{l_j}^{1/2} k_\gamma^{3/2} \frac{\gamma_j \gamma_{\gamma j}}{E_j - E - \gamma_j^2\left[S_{l_j}(p) - B_j(p) + f_j^2\{S_{l_j}(n) - B_j(n)\} + iP_{l_j}\right]}, \quad (2)$$

with $f_j^2 \approx 2$ for the $T = 3/2$ levels. We actually take $f_1^2 = 0$, $f_2^2 = 1.656$, and $f_3^2 = 1.549$, because the single-particle reduced widths are different in the neutron and proton channels. Level 2 is the $1/2^+$ level, with $l_2 = 0$, level 3 is the $1/2^-$ level with $l_3 = 1$, and we somewhat arbitrarily use $l_1 = 1$ for level 1. Then $\gamma_{\gamma 1}$ could depend on θ. We use the conventional value of the channel radius $a = 4.574$ fm. For given B_j, the variable parameters are E_j, γ_j, and $\gamma_{\gamma j}$ ($j = 1 - 3$), together with A and B, with the restriction that the background is non-negative. Initially the 0° data (25 points) and the 90° data (31 points) are fitted separately.

For each level, one can calculate the resonance energy

$$E_{jr} = E_j - \gamma_j^2\left[S_{l_j}(p) - B_j(p) + f_j^2\{S_{l_j}(n) - B_j(n)\}\right]_{E_{jr}} \quad (3)$$

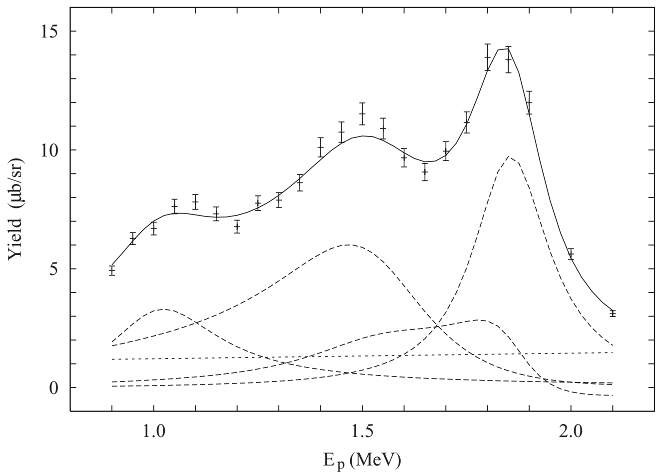

FIG. 1. ^{10}Be$(p,\gamma_0)^{11}$B cross section at 0° as a function of proton energy. The experimental points are from Goosman *et al.* [3]. The solid curve is the best simultaneous R-matrix fit (e) to the 0° and 90° data. The dashed curves show the contributions of the individual levels and of the interference between the upper two. The dotted line is the background.

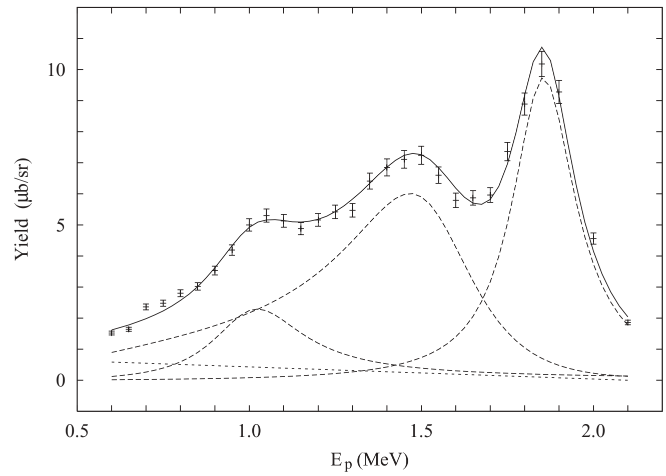

FIG. 2. ^{10}Be$(p,\gamma_0)^{11}$B cross section at 90° as a function of proton energy. The experimental points are from Goosman *et al.* [3]. The solid curve is the best simultaneous R-matrix fit (e) to the 0° and 90° data. The dashed curves show the contributions of the individual levels. The dotted line is the background.

and the observed width

$$\Gamma_j^0 = \frac{2\gamma_j^2 P_{l_j}(E_{jr})}{1+\gamma_j^2\left[\mathrm{d}S_{l_j}(p)/\mathrm{d}E + f_j^2\mathrm{d}S_{l_j}(n)/\mathrm{d}E\right]_{E_{jr}}}. \quad (4)$$

Resultant values are given in Table I, which also includes values of the peak energy E_{jm} and full width at half maximum (FWHM) $\Gamma_{j1/2}$ of the contributions of the individual levels to the cross section. For 0°, there are two "best fits", almost equally good, with constructive or destructive interference between the $1/2^+$ and $1/2^-$ levels. The constructive interference case (a) gives parameter values reasonably close to those obtained in the 90° fit (c). We therefore fit simultaneously the 0° and 90° data. For convenience, we include only data at energies for which both 0° and 90° values are available (thus

TABLE I. Parameter values from R-matrix fits to ^{10}Be$(p,\gamma_0)^{11}$B data [3]. In pairs of numbers, the first number refers to 0°, the second to 90°.

θ	Case	j	E_{jr} (MeV)	γ_j (MeV$^{1/2}$)	$\gamma_{\gamma j}$	A	B	χ^2	Γ_j^0 (MeV)	E_{jm} (MeV)	$\Gamma_{j1/2}$ (MeV)	$\mathcal{S}_j$
0°	a	1	1.014	2.09	0.260				0.498	0.951	0.386	
		2	1.372	0.626	0.204	1.14	−0.60	14.4	0.364	1.352	0.360	0.49
		3	1.682	0.582	0.176				0.205	1.677	0.208	0.73
	b	1	1.008	1.68	0.191				0.413	0.960	0.343	
		2	1.375	0.608	0.260	3.08	−1.61	13.9	0.350	1.357	0.347	0.47
		3	1.678	0.591	−0.260				0.210	1.672	0.212	0.76
90°	c	1	1.027	2.82	0.264				0.605	0.941	0.435	
		2	1.363	0.685	0.194	1.58	−0.83	76.3	0.407	1.337	0.401	0.59
		3	1.691	0.530	0.161				0.180	1.685	0.190	0.61
0° + 90°	d	1	1.036	2.63	0.258,0.255				0.596	0.952	0.443	
		2	1.372	0.650	0.188	4.31,1.59	−1.88,−0.83	156.2	0.383	1.346	0.378	0.53
		3	1.683	0.544	0.164				0.186	1.679	0.195	0.64
	e	1	1.208	1.546	0.141,0.118				0.579	0.931	0.292	
		2	1.420	0.678	0.394	0.98,0.82	0.26,−0.43	85.1	0.415	1.332	0.469	0.58
		3	1.693	0.575	−0.188				0.204	1.686	0.175	0.72
	f	2	1.432	0.8[a]	0.452			94.5	0.510	1.322	0.558	0.81
	g	2	1.453	0.9[a]	0.611			106.0	0.587	1.315	0.641	1.02
	h	2	1.464	1.0[a]	0.658			117.0	0.642	1.310	0.691	1.26
	i	1	0.944	0.757	0.016				0.404			0.72
		2	1.550	0.824	0.636	4.73,1.18	−1.98,−0.62	112.9	0.591			0.86
		3	1.687	0.578	0.185				0.205	1.681	0.180	0.72

[a]Fixed value.

excluding 0° points at $E_p = 1.95$ and 2.05 MeV, and 90° points at $E_p = 1.825$ and 1.875 MeV). The best-fit parameter values are given in Table I, case (d). The fit to the 0° data is much poorer than in case (a), with $\chi^2(0°) = 62.1$. In each of these fits, the background decreases as E increases, and most often vanishes at the highest energy data point at $E_p = 2.1$ MeV.

So far the fits have all used standard R-matrix formulas [4], which are not really justified for reactions involving γ-rays. Modified R-matrix formulas suitable for electric-multipole radiative-capture reactions have been given [5], and are now used for the $1/2^+$ contribution to the ^{10}Be$(p, \gamma_0)^{11}$B cross section. These formulas contain channel contributions, which include a nonresonant amplitude that could reduce the background necessary to fit the data. One additional parameter is involved, the dimensionless reduced-width amplitude θ_f of the ^{11}B ground state for the ^{10}Be + p channel. A much better fit to the combined 0° and 90° data is obtained, with smaller background, and with the parameter values given in Table I, case (e), together with $\theta_f = -0.487$. This fit is shown in Figs. 1 and 2. The contribution to χ^2 from the 0° data is $\chi^2(0°) = 25.9$. The observed width of the $1/2^+$ level, $\Gamma_2^0 = 415$ keV, is about midway between the original value of 230 keV [3] and F's value of 640 keV. In Γ_2^0, the neutron contribution to the denominator of Eq. (4) is 0.50, which is about seven times the proton contribution; F's potential model neglects any effect from the neutron channel. To see if acceptable fits can be obtained with larger values of the $1/2^+$ width, fits are made with some larger, fixed values of γ_2; the resultant parameter values for level 2 are given in Table I, cases (f–h). For a $1/2^+$ width Γ_2^0 of about 640 keV, χ^2 is about 37% above the best-fit value. On most criteria, this would not be considered an acceptable fit.

Our best fit (e) gives $\theta_f = -0.487$. A calculated value of θ_f may be obtained using $\theta_f^2 = (C^2\mathcal{S})_f\theta_{f,\mathrm{sp}}^2$, with $C^2 = 2/3$, $\mathcal{S}_f = 0.6449$ [6], and $\theta_{f,\mathrm{sp}}^2 = 0.0960$, giving $|\theta_f| = 0.203$. If θ_f is fixed at -0.203, the best fit to the combined 0° and 90° data gives $\chi^2 = 102.7$.

The fit (e) still includes nonzero background contributions. If one assumes that there is no background, the fit is much worse with $\chi^2 \approx 248$. F shows a fit to the 90° data without background, with contributions from four resonances—three narrow and one broad, the last representing the $1/2^+$ level. This has an energy-independent width of 700 keV, although the s-wave penetration factor increases by nearly 80% between 1.0 and 1.7 MeV. This fit looks reasonable at most energies, but the two lowest-energy points would contribute more than 100 to χ^2.

Values of spectroscopic factors may be obtained from

$$\gamma_j^2 = (C^2\mathcal{S})_j\theta_{l_j,\mathrm{sp}}^2\hbar^2/\mu a^2, \tag{5}$$

with $C^2 = 1/3$ and $\hbar^2/\mu a^2 = 2.18$ MeV. For the $1/2^+$ level, $\theta_{0,\mathrm{sp}}^2 = 1.090$, and for the $1/2^-$ level, $\theta_{1,\mathrm{sp}}^2 = 0.635$. Values of $\mathcal{S}_j$ are given in the last column of Table I. Our best fit (e) gives $\mathcal{S}_2 = 0.58$. This is less than the value $\mathcal{S}_2 = 0.75$ assumed by F and shell-model values of 0.82 [7], 0.74 [8], and 0.74 [9], while experimental values found for the analog state ^{11}Be$(1/2^+)$ are 0.73 ± 0.06 [10], 0.77 [11], 0.66 – 0.79 [12] (various effects are mentioned that would reduce these values by factors 0.6 – 0.7), 0.19 ± 0.02 [13], and ≈ 0.74 [9]. Wide variations have been reported for the width of the analog state in ^{11}N; the most recent experiment gives $\mathcal{S}_2 \approx 0.5$ [14].

A possible reason why $\mathcal{S}_2$ for the $1/2^+$ level of ^{11}B in the ^{10}Be + p channel should be less than the calculated values, and also less than the experimental values for ^{11}Be$(1/2^+)$, has been suggested by Millener [15]. He points out that Teeters and Kurath [7] predict two $1/2^+$, $T = 1/2$ states in ^{11}B just below the $T = 3/2$ state, and that the calculated space-spin structure of the $T = 3/2$ state is also present in the $T = 1/2$ states, so that isospin mixing is expected to occur. This would make the upper (mainly $T = 3/2$) mixed state more "neutron-like" and the lower more "proton-like" [15].

It is tempting to consider the possibility that the lowest observed peak [3] is due to the lower isospin-mixed $1/2^+$ state. One requirement of this would be isotropy of the level 1 contribution, implying equal values of $\gamma_{\gamma 1}$ at 0° and 90°; the values for cases (a,c,d) are consistent with this, and those for case (e) may not exclude it. We have fitted the data assuming two coherent $1/2^+$ levels (1 and 2) and one $1/2^-$ level (3). The best fit found gives the parameter values shown in Table I, case (i), together with $\theta_f = -0.326$. Some new definitions are needed for these parameters. In the fitting process, the parameters for the $1/2^+$ levels use $B(c)\,(c = p, n)$ the same for levels 1 and 2 [actually $B(c) = S_0(c, E = 1.32\ \mathrm{MeV})$], and use $f_1 = -0.643$ (as for a $T = 1/2$ level) and $f_2 = 1.287$. The values given in the table are for $B_j(c) = S_0(c, E_{jr})$. Values of E_m and $\Gamma_{1/2}$ could not be obtained as the total $1/2^+$ contribution does not show two distinct peaks. In the isospin-mixing model of these two $1/2^+$ states, one might expect $\mathcal{S}_2 < \mathcal{S}_1$, but the opposite occurs for case (i); also $\mathcal{S}_2$ is larger than the calculated values and the experimental values for ^{11}Be$(1/2^+)$. Thus there is some lack of consistency in the identification of the lowest peak with the lower isospin-mixed $1/2^+$ state; also the fit (i) is poorer than fit (e) and the backgrounds are appreciably larger.

We may note that all the fits in Table I give about 200 keV for the width of the $1/2^-$ level, in agreement with the value of 200(25) keV adopted in the latest compilation [2].

Summarizing, our best fits to the ^{10}Be$(p, \gamma_0)^{11}$B data [3] with three non-interfering levels give a smaller width and smaller spectroscopic factor for the $1/2^+$ level than those favored by Fortune [1]. A larger spectroscopic factor can be obtained by assuming two coherent $1/2^+$ levels in addition to a $1/2^-$ level, but the fit is somewhat poorer.

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