Single proton energies in the Si isotopes and the $Z = 14$ subshell closure

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The evidence on single proton energies in the Si isotopes is traced from $N = 12$ to $N = 28$ using available experimental data on binding energies, single proton transfer reactions, radioactive decay, and direct reaction studies with neutron-rich exotic beams. In addition to demonstrating the value of having information on single particle energies over a long sequence of isotopes, the data point out a significant discrepancy between theoretical and experimental results on the size of the $Z = 14$ subshell closure in ^{34}Si, which is located on the boundary of the island of inversion.

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Measuring and understanding the evolution of single particle energies in nuclei far from stability is one of the highest priorities for the nuclear structure community. Continuing advances in the technology for producing beams of radioactive isotopes are making it possible to determine single particle energies for long sequences of isotopes of a particular element. The evolution of single particle energies in neutron-rich nuclei has been highlighted in a number of mass regions [1–4]. A particular focus of this work has been the spin-orbit force, which in nuclei determines major shell closures [5,6].

The energies of the proton orbits in the *sd* shell in neutron-rich nuclei have been highlighted recently in studies of calcium and silicon isotopes [2,7,8]. In particular, the evolution of the $Z = 14$ and 16 shell closures plays a role in the development of collectivity in the neutron-rich silicon isotopes [2]. In the present note, we identify all available experimental information on single proton energies in the silicon isotopes. Mass information is critical for determining single particle energies. However, information on excited states in the adjacent odd-proton nuclei is important as well. The nature of the information on excited states changes from the stable isotopes ^{28,30}Si, where single proton pickup and stripping data are available, to neutron-rich nuclei such as the $N = 20$ isotope ^{34}Si, which appears doubly-magic in structure and for which some γ-ray spectroscopic information is available in the odd-proton neighbor ^{35}P.

The available experimental information is summarized in Table I and Fig. 1. The nuclear mass information that forms the basis of this study is taken from Refs. [9,10]. However, it is important from the outset to recognize that the sources of the information represented in Fig. 1 are quite different, and that the precision of the data points plotted there varies considerably.

The single proton energies in the stable isotopes ^{28,30}Si take into account the masses of the silicon ($Z = 14$) isotopes themselves and the masses of the aluminum ($Z = 13$) and phosphorus ($Z = 15$) isotones, as well as data from the single proton transfer reactions ^{28,30}Si$(t,\alpha)^{27,29}$Al and ^{28,30}Si$(^3$He$,d)^{29,31}$P [11–13]. The transfer reactions allow the location of the single proton (or proton hole) strength, even when it is fragmented among a number of states. When the strength is fragmented, the spectroscopic factors determined for the states allow the determination of the centroid.

The $d_{5/2}$ single proton energies in ^{28,30}Si are taken to be

$$D(N) = ME(Z = 14, N) - ME(Z = 13, N) - ME_p - C, \quad (1)$$

where $ME(Z, N)$ is the mass excess for the isotope with proton number Z and neutron number N, ME_p is the mass excess of the proton (7.289 MeV), and C is the centroid of the $d_{5/2}$ strength from the ^{28,30}Si$(t,\alpha)^{27,29}$Al measurements [11]. In ^{27,29}Al, the greatest concentrations of $d_{5/2}$ strength are in the ground states; however, the centroids are 0.20 and 0.61 MeV, respectively, due to additional concentrations of $d_{5/2}$ strength in states at 2.74 and 3.06 MeV. The uncertainties listed for the ^{28,30}Si $d_{5/2}$ single particle energies in Table I reflect an assumed 20% uncertainty in the spectroscopic factors determined for each of the $5/2^+$ states. The masses are known very precisely and so have an insignificant effect on uncertainties.

The $s_{1/2}$ and $d_{3/2}$ single proton energies in ^{28,30}Si are calculated using

$$D(N) = ME(Z = 15, N) - ME(Z = 14, N) - ME_p + C, \quad (2)$$

where C is the centroid of either the $s_{1/2}$ or $d_{3/2}$ strength observed in ^{28,30}Si$(^3$He$,d)^{29,31}$P measurements [12,13]. The ^{28}Si$(^3$He$,d)^{29}$P data of Ref. [12] give only one $s_{1/2}$ state—the ground state, so the $s_{1/2}$ energy adopted here for ^{28}Si is simply that given by Eq. (2) with $C = 0$. Only one $d_{3/2}$ state is reported in Ref. [12], located at 1.38 MeV, so $C = 1.38$ MeV for the calculation of the $d_{3/2}$ energy in ^{28}Si. In both these cases, there is no experimental uncertainty regarding the location of the centroids, so the uncertainties listed in Table I for ^{28}Si are zero (the uncertainties in the masses are less than 2 keV).

For ^{30}Si, we adopt the ^{30}Si$(^3$He$,d)^{31}$P results of [13]. The authors of Ref. [13] list five states having $T = 1/2$ $s_{1/2}$ strength, with 86% of the strength located in the ground state, but fragments of strength located as high as 8.5 MeV. The centroid of the $T = 1/2$ $s_{1/2}$ strength is 0.70 MeV, and the experimental uncertainty once again reflects an assumption of a 20% uncertainty in the measurement of the spectroscopic factor for each $1/2^+$ state. The centroid of the five

TABLE I. Single proton energies in the Si isotopes.

Nucleus	$E(d_{5/2})$ (MeV)	$E(s_{1/2})$ (MeV)	$E(d_{3/2})$ (MeV)
^{26}Si	−5.87(35)[a]	−0.51(35)[b]	
^{28}Si	−11.79(5)[c]	−2.75(0)[d]	−1.36(0)[d]
^{30}Si	−14.11(14)[c]	−6.60(14)[d]	−5.74(6)[d]
^{32}Si	−16.77(35)[a]	−9.20(35)[b]	−7.76(35)[e]
^{34}Si	−19.07(35)[a]	−11.84(35)[b]	−9.45(35)[e]
^{36}Si	−19.85(37)[a]	−13.51(36)[b]	
^{38}Si	−21.62(37)[a]	−15.66(37)[b]	
^{40}Si	−22.56(71)[a]	−17.40(41)[b]	
^{42}Si		−17.28(76)[b]	−17.10(76)[e]

[a] $d_{5/2}$ single proton energy based on mass excesses of Si, Al isotones, and 0.35 MeV adjustment (and 0.35 MeV uncertainty) for absence of data on fragmentation from proton pickup reaction into Al isotone, as explained in text. Experimental uncertainty also reflects uncertainties in masses.

[b] $s_{1/2}$ single proton energy based on mass excesses of Si, P isotones, and 0.35 MeV adjustment (and 0.35 MeV uncertainty) for absence of data on fragmentation from proton pickup reaction into P isotone, as explained in text. Experimental uncertainty also reflects uncertainties in masses.

[c] $d_{5/2}$ single proton energy based on mass excesses of Si, Al isotones, and centroid of $d_{5/2}$ strength from Si(t,α) reaction. Uncertainty from (t,α) data. Uncertainty from mass measurements is insignificant.

[d] $s_{1/2}$ and $d_{3/2}$ single proton energies based on mass excesses of Si, P isotones, and centroids of $s_{1/2}$ and $d_{3/2}$ strengths from Si(^{3}He, d) reactions. Uncertainties from (^{3}He, d) data. Uncertainty from mass measurements is insignificant.

[e] $d_{3/2}$ single proton energy based on mass excesses of Si, P isotones, energy of excited $3/2^+$ state, and 0.35 MeV adjustment (and 0.35 MeV uncertainty) for absence of data on fragmentation from proton pickup reaction in to P isotone, as explained in text. Experimental uncertainty also reflects uncertainties in masses.

$T = 1/2$ $d_{3/2}$ states is located at 1.56 MeV, with 92% of the strength concentrated in a state at 1.27 MeV.

The proton transfer data with ^{28,30}Si targets provide the ideal situation for determining single proton energies. However, such data are not yet available for the exotic Si isotopes, so we must make the best use of the data that are available in these nuclei. For example, for ^{32}Si we have the precisely-known masses of the isotones ^{31}Al, ^{32}Si, and ^{33}P. The ground state of ^{31}Al has $J^\pi = 5/2^+$, as would be expected for the $d_{5/2}$ hole configuration. The ground state of ^{33}P has $J^\pi = 1/2^+$, reflecting its $s_{1/2}$ origin. The lowest $J^\pi = 3/2^+$ state in ^{33}P is located at 1.43 MeV. Given the concentration of $d_{3/2}$ strength in the 1.27 MeV state in ^{31}P, it seems reasonable to conclude that a similar amount of $d_{3/2}$ strength is located in the 1.43 MeV $3/2^+_1$ state in ^{33}P.

The ^{28,30}Si proton transfer data clearly demonstrate that uncertainties are introduced into the determination of single proton energies by the absence of the transfer results. However, the same results for ^{28,30}Si can be used to devise a procedure for estimating single particle energies and specifying uncertainties in the absence of proton transfer data.

If we had set the single proton energies in ^{28,30}Si simply by using the lowest energy states of the correct spin and parity in the Al and P isotones, we would have made errors of between 0 and 0.70 MeV, according to the proton transfer results. The ^{28,30}Si(t,α) results demonstrate that the $d_{5/2}$ orbit is 0.20 and 0.61 MeV more bound in ^{28,30}Si, respectively, than we would have determined had we assumed that the ground states of ^{27,29}Al held all the $d_{5/2}$ strength. The ^{28}Si(^{3}He, d) data show that the $s_{1/2}$ orbit is concentrated in the ground state of ^{29}P, and therefore that the error resulting from assigning the lowest $1/2^+$ state (ground state) as the $s_{1/2}$ state would have been zero. The corresponding result for ^{30}Si gives the centroid of $s_{1/2}$ strength in ^{31}P at 0.70 MeV—so the error from assuming that the $s_{1/2}$ strength is concentrated in the ground state would have been 0.70 MeV. The errors in assuming the $d_{3/2}$ proton strength is concentrated in the $3/2^+_1$ states would have been zero in ^{28}Si (with the strength in a single state at 1.38 MeV) and 0.29 MeV in ^{30}Si (with the centroid at 1.56 MeV and the $3/2^+_1$ state at 1.27 MeV).

In short, the errors from assuming that the single particle strength is located in the single lowest lying state in the Al or P isotone range from 0 to 0.70 MeV. For $d_{5/2}$, the orbit is always more bound than the single-state estimate would give; for $s_{1/2}$ and $d_{3/2}$, less bound. To take this into account, we specify this procedure: we shift the single proton energy by 0.35 MeV (half of 0.70 MeV) in the appropriate direction (more bound for $d_{5/2}$, less bound for $s_{1/2}$ and $d_{3/2}$) and assign an uncertainty of 0.35 MeV to the result. This uncertainty is added in quadrature with the uncertainty from the mass measurements.

To return to the example of ^{32}Si: Assuming that the $d_{5/2}$ strength is concentrated in the ground state of ^{31}Al would give a single proton energy of −16.42 MeV (the uncertainty resulting from the mass measurements is only

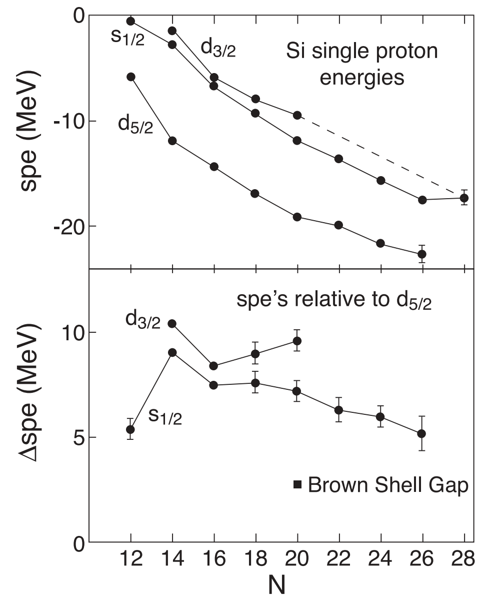

FIG. 1. Experimental single proton energies in the silicon isotopes (top panel), and these energies relative to the $d_{5/2}$ orbit (bottom panel). Discussion of the extraction of the data points from experimental data is given in the text. The dashed line in the top panel connects the $d_{3/2}$ energy at $N = 20$ to the corresponding energy at $N = 28$. The $d_{3/2}$ energies for $N = 22$–26 are not known.

0.020 MeV). Applying the procedure specified above yields a single proton energy of −16.77 MeV with an uncertainty of 0.35 MeV. The assumption of the concentration of $s_{1/2}$ strength in the ground state of ^{33}P would yield a single proton energy of −9.55 MeV; the adjustment procedure yields −9.20(35) MeV. Assuming the $d_{3/2}$ strength is concentrated in the $3/2^+_1$ state at 1.43 MeV would give a single proton energy of −8.11 MeV, but the adjustment procedure gives −7.76(35) MeV.

This procedure is applied for the states listed in Table I for ^{26,32,34,36,38,40,42}Si. In the heaviest isotopes, the uncertainties in the mass measurements become significant and the resulting uncertainties in the single proton energies become larger than 0.35 MeV, reaching 0.76 MeV in ^{42}Si.

In the $N = 20$ isotope ^{34}Si, which appears from its spectroscopy to be doubly-magic, we extract the $d_{3/2}$ single proton energy by noting the $3/2^+$ state at 2.4 MeV in the isotone ^{35}P observed in ^{35}Si β-decay and proton pickup reactions on ^{36}S. No other $3/2^+$ state is presently known in ^{35}P. Figure 1 also depicts a data point for the $d_{3/2}$ single proton energy in the near-dripline nucleus ^{42}Si. A 184 keV γ-ray was observed in a study of the single-proton knockout reaction on ^{44}S and was interpreted as connecting the $d_{3/2}$ and $s_{1/2}$ single proton states [2]. The comparison of the ratio of the observed cross sections for the two states to a theoretical calculation led to the assignment of the ground state as the $s_{1/2}$ state and the state at 184 keV as the $d_{3/2}$ state.

No $d_{3/2}$ single-proton energies are shown in Fig. 1 for $N = 22, 24$ despite the fact that $J^\pi = 3/2^+$ states have been identified in ^{37,39}P [14] (these states have also been cataloged in the study of Gade *et al.* [15]). The fragmentation reactions used by Sorlin *et al.* to identify the $3/2^+$ states do not preferentially populate single proton states as the proton transfer reactions cited here for ^{29,31}P do. Even the proton knockout reaction cited here for ^{43}P preferentially populates single proton states. Therefore, there is no reason to interpret the $3/2^+$ states seen in ^{37,39}P by Sorlin *et al.* [14] as $d_{3/2}$ single proton states. Indeed, the shell model calculations presented in Ref. [14] demonstrate that the $3/2^+$ states observed in these midshell nuclei can be complex mixtures of single proton states with rotational excitations built on the $s_{1/2}$ proton.

The most easily discerned issue in Fig. 1 is the size of the gaps between the $d_{5/2}$ proton orbit and the $s_{1/2}$ and $d_{3/2}$ orbits in the neutron-rich nucleus ^{34}Si, which has a neutron number (20) that is magic at the line of stability. The calculation of the gap between the $d_{5/2}$ and $s_{1/2}$ proton orbits in ^{34}Si by Brown [16] is 2.8 MeV, considerably less than the empirical gap of 7.2 MeV shown in Fig. 1 (the Brown value is shown in Fig. 1 as well). The empirical spin-orbit splitting between the $d_{5/2}$ and $d_{3/2}$ proton orbits is approximately 9.5 MeV.

Shell structure in ^{34}Si is an important physics issue because this nucleus is a neighbor of the “island of inversion” [17–19]. One of the outstanding empirical features of the island of inversion phenomenon is the abrupt shift from the apparently spherical *sd* shell spectroscopy of ^{34}Si to the radically different spectroscopy of the isotone ^{32}Mg. With a major closed neutron shell and a closed $Z = 14$ subshell, it is not at all surprising that the energy of the 2^+_1 state in ^{34}Si is quite high (3328 keV) [20]. However, it is at first surprising that the 2^+_1 state in ^{32}Mg is at 885 keV [21], an energy that suggests an interpretation as a strongly deformed rotor in which the $N = 20$ major shell closure is broken. Indeed, several measurements [22–25] of the $B(E2; 0^+_{\mathrm{g.s.}} \to 2^+_1)$ value in ^{32}Mg are consistent with a strongly deformed intrinsic shape with β_2 close to 0.5. It is important to note that the first experimental evidence for the island of inversion phenomenon came from the observation of binding energies in ^{31,32}Na considerably larger than those expected from conventional shell model calculations that took into account only the *sd* shell [26].

Given the importance of the recent results on the effect of the tensor force on single particle energies by Otsuka *et al.* [8,27], it would be gratifying to be able to discern the effects of the tensor force in the single proton energies illustrated in Fig. 1. However, $s_{1/2}$ single particle energies are not affected by the tensor force, and the effect of the occupation of the $d_{3/2}$ neutron orbit on the single particle energy of the $d_{3/2}$ proton orbit is complicated by isospin symmetry [8]. It would be expected that the influence of the occupation of the $f_{7/2}$ neutron orbit on the *d* proton orbits could be discerned as discussed in Refs. [7,8], but at present there is not sufficient data on the energies of these orbits in the neutron-rich Si isotopes to track changes in the spin-orbit splitting.

The absence of the $d_{5/2}$ single proton energy for $N = 28$ in Fig. 1 also highlights the importance of measuring the mass of ^{41}Al. The existence of ^{41}Al was established by Sakarai *et al.* [28].

Obtaining more single proton energies in the entire sequence of Si isotopes would require the systematic application of experimental probes that select single proton strength, as with the use of the (t, α) and $(^3\mathrm{He}, d)$ reactions with the stable isotopes ^{28,30}Si described above. The recent dissertation work of Roeder [29] suggests that the (d, n) reaction in inverse kinematics would provide a practical probe for single proton strengths with fast radioactive beams. Using the solid deuterium target developed by Ryuto *et al.* [30], fast (100 MeV/nucleon) beams of ^{40,42}S and ^{48}Ca and the γ-ray array and magnetic spectrograph available at the National Superconducting Cyclotron Laboratory [31,32], Roeder and collaborators measured cross sections of 0.5–0.7 mb for the inverse kinematics reactions $d(^{40}\mathrm{S}, n)^{41}\mathrm{Cl}$, $d(^{42}\mathrm{S}, n)^{43}\mathrm{Cl}$, and $d(^{48}\mathrm{Cl}, n)^{49}\mathrm{Sc}$. These cross sections make spectroscopic studies with a wide range of proton- and neutron-rich beams practical.

In summary, the present examination of single proton energies in the Si isotopes—from proton-rich to neutron-rich—points out a discrepancy between theoretical and experimental results for the size of the proton subshell closure in ^{34}Si. Fast beam reactions such as (d, n) with exotic beams may provide a means for more precisely determining single proton energies off the line of stability.

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