Thermal effects on nuclear symmetry energy with a momentum-dependent effective interaction

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The knowledge of the nuclear symmetry energy of hot neutron-rich matter is important for understanding the dynamical evolution of massive stars and the supernova explosion mechanisms. In particular, the electron capture rate on nuclei and/or free protons in presupernova explosions is especially sensitive to the symmetry energy at finite temperature. In view of the above, in the present work we calculate the symmetry energy as a function of the temperature for various values of the baryon density by applying a momentum-dependent effective interaction. In addition to a previous work, the thermal effects are studied separately both in the kinetic part and the interaction part of the symmetry energy. We focus also on the calculations of the mean-field potential, employed extensively in heavy-ion reaction research, both for nuclear and pure neutron matter. The proton fraction and the electron chemical potential, which are crucial quantities for representing the thermal evolution of supernova and neutron stars, are calculated for various values of the temperature. Finally, we construct a temperature dependent equation of state of β-stable nuclear matter, the basic ingredient for the evaluation of the neutron star properties.

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I. INTRODUCTION

The determination of the nuclear symmetry energy (SE) based on microscopic and/or phenomenological approaches is of great interest in nuclear physics as well as in nuclear astrophysics. For instance, it is important for the study of the structure and reactions of neutron-rich nuclei, the Type II supernova explosions, neutron-star mergers and the stability of neutron stars. In addition, the SE is the basic ingredient for the determination of the proton fraction and electron chemical potential. The above quantities determine the cooling rate and neutrino emission flux of protoneutron stars and the possibility of kaon condensation in dense matter [1,2].

Heavy-ion reactions are a unique means to produce in terrestrial laboratories hot neutron-rich matter similar to those existing in many astrophysical situations [3]. Although the behavior of the SE for densities below the saturation point still remains unknown, significant progress has been made only most recently in constraining the SE at subnormal densities and around the normal density from the isospin diffusion data in heavy-ion collisions [4,5]. This has led to a significantly more refined constraint on neutron-skin thickness of heavy nuclei [6,7] and the mass-radius correlation of neutron stars [8]. For densities above the saturation point the trend of the SE is model dependent and exhibits completely different behavior.

Up to now the main part of the calculations concerning the density dependence of the SE is related with the cold nuclear matter ($T = 0$). However, recently, there is an increasing interest for the study of the SE and the properties of neutron stars at finite temperature [3,9–15]. The motivation of the present work is to clarify the effects of finite temperature on SE and to find also the appropriate relations describing that effect. Especially we focus on the interaction part of the SE, where so far it has received little theoretical attention concerning its dependence on the temperature.

To investigate the thermal properties of the SE, we apply a momentum-dependent effective interaction model. In that way, we are able to study simultaneously thermal effects not only on the kinetic part of the symmetry energy but also on the interaction part. The present model was introduced by Gale *et al.* [16–19] to examine the influence of momentum-dependent interactions on the momentum flow of heavy-ion collisions. Over the years the model has been extensively applied in the study not only of heavy-ion collisions but also the properties of nuclear matter by a proper modification [20–23]. A review analysis of the present model is presented in Refs. [2,18].

In the present work we study the thermal properties of the nuclear symmetry energy by applying the above phenomenological model focusing mainly on the temperature dependence of the kinetic and interaction part of the SE as well as the total SE. Though it is well known how the temperature affects the kinetic part of the symmetry energy [3,24,25] the temperature dependence of the interaction part of the SE has so far received little theoretical attention. In addition, we determine the temperature dependence of the proton fraction as well as of the electron chemical potential. Both of the above quantities are related with the thermal evaluation of the supernova and the proton-neutron stars. The single-particle potential for the pure neutron matter and the symmetric nuclear matter, extensively applied in heavy-ion collision research, is also estimated for various values of the temperature. Finally, we construct the equation of state (EOS) of β-stable matter that is the basic ingredient for calculations of the neutron star properties.

The article is organized as follows. In Sec. II the model and the relative formulas are discussed and analyzed. Results are reported and discussed in Sec. III, whereas the summary of the work is given in Sec. IV.

II. THE MODEL

The schematic potential model, used in the present work, is designed to reproduce the results of the more microscopic calculations of both nuclear and neutron-rich matter at zero temperature and can be extended to finite temperature [2]. The energy density of the asymmetric nuclear matter (ANM) is given by the relation

$$\epsilon(n_n, n_p, T) = \epsilon^n_{\rm kin}(n_n, T) + \epsilon^p_{\rm kin}(n_p, T) + V_{\rm int}(n_n, n_p, T), \quad (1)$$

where n_n (n_p) is the neutron (proton) density and the total baryon density is $n = n_n + n_p$. The contribution of the kinetic parts are

$$\epsilon^n_{\rm kin}(n_n, T) + \epsilon^p_{\rm kin}(n_p, T) = 2\int \frac{d^3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m}[f_n(n_n, k, T) + f_p(n_p, k, T)], \quad (2)$$

where f_τ, (for $\tau = n, p$) is the Fermi-Dirac distribution function with the form

$$f_\tau(n_\tau, k, T) = \left\{1 + \exp\left[\frac{e_\tau(n, k, T) - \mu_\tau(n, T)}{T}\right]\right\}^{-1}. \quad (3)$$

The nucleon density n_τ is evaluated from the following integral

$$n_\tau = 2\int \frac{d^3k}{(2\pi)^3} f_\tau(n_\tau, k, T) = 2\int \frac{d^3k}{(2\pi)^3}\left\{1 + \exp\left[\frac{e_\tau(n, k, T) - \mu_\tau(n, T)}{T}\right]\right\}^{-1}. \quad (4)$$

In Eq. (3), $e_\tau(n, k, T)$ is the single-particle energy (SPE) and $\mu_\tau(n, T)$ stands for the chemical potential of each species. The SPE has the form

$$e_\tau(n, k, T) = \frac{\hbar^2k^2}{2m} + U_\tau(n, k, T), \quad (5)$$

where the single-particle potential $U_\tau(n, k, T)$ is obtained by differentiating $V_{\rm int}$, i.e., $U_\tau = \partial V_{\rm int}(n_n, n_p, T)/\partial n_\tau$. Including the effect of finite-range forces between nucleons, to avoid acausal behavior at high densities, the potential contribution is parameterized as follows [2]

$$\begin{aligned} V_{\rm int}&(n_n, n_p, T) \\ &= \frac{1}{3}An_0\left[\frac{3}{2} - \left(\frac{1}{2} + x_0\right)(1-2x)^2\right]u^2 \\ &+ \frac{\frac{2}{3}Bn_0\left[\frac{3}{2} - \left(\frac{1}{2} + x_3\right)(1-2x)^2\right]u^{\sigma+1}}{1 + \frac{2}{3}B'\left[\frac{3}{2} - \left(\frac{1}{2} + x_3\right)(1-2x)^2\right]u^{\sigma-1}} \\ &+ \frac{2}{5}u\sum_{i=1,2}\left[(2C_i + 4Z_i)2\int\frac{d^3k}{(2\pi)^3}g(k, \Lambda_i)(f_n + f_p)\right. \\ &+ \left.(C_i - 8Z_i)2\int\frac{d^3k}{(2\pi)^3}g(k, \Lambda_i)(f_n(1-x) + f_px)\right], \end{aligned} \quad (6)$$

where $x = n_p/n$ is the proton fraction and $u = n/n_0$, with n_0 denoting the equilibrium symmetric nuclear matter density $n_0 = 0.16\ {\rm fm}^{-3}$. The constants A, B, σ, C_1, C_2, and B', which enter in the description of symmetric nuclear matter and the additional parameters x_0, x_3, Z_1, and Z_2, used to determine the properties of asymmetric nuclear matter, are treated as parameters constrained by empirical knowledge [2]. The function $g(k, \Lambda_i)$ suitably chosen to simulate finite range effects is of the following form

$$g(k, \Lambda_i) = \left[1 + \left(\frac{k}{\Lambda_i}\right)^2\right]^{-1}, \quad (7)$$

where the finite-range parameters are $\Lambda_1 = 1.5k_F^0$ and $\Lambda_2 = 3k_F^0$ and k_F^0 is the Fermi momentum at the saturation point n_0.

The entropy density $s_\tau(n, T)$ required for the calculations of the total pressure and for the EOS, has the same functional form as that of a noninteracting gas system, that is

$$s_\tau(n, T) = -2\int\frac{d^3k}{(2\pi)^3}\left[f_\tau \ln f_\tau + (1 - f_\tau)\ln(1 - f_\tau)\right]. \quad (8)$$

The ratio entropy/baryon is given by $S_\tau(n, T) = s_\tau(n, T)/n$. The baryon pressure $P_b(n, T)$, needed to construct the EOS, is given by

$$P_b(n, T) = T\sum_{\tau=p,n} s_\tau(n, T) + \sum_{\tau=p,n} n_\tau\mu_\tau(n, T) - \epsilon_{\rm anm}(n, T). \quad (9)$$

Finally, the total energy density and pressure of charge neutral and chemically equilibrium nuclear matter are

$$\epsilon_{\rm tot}(n, T) = \epsilon_b(n, T) + \sum_{l=e^-,\mu^-}\epsilon_l(n, T), \quad (10)$$

$$P_{\rm tot}(n, T) = P_b(n, T) + \sum_{l=e^-,\mu^-}P_l(n, T). \quad (11)$$

The leptons (electrons and muons) originating from the condition of the β-stable matter are considered as noninteracting Fermi gases.

The above analysis holds in general for the asymmetric nuclear matter. Below, to calculate the thermal effect on the SE, we will focus our study on two cases, i.e., the symmetric nuclear matter (SNM) and the pure neutron matter (PNM).

A. Symmetric nuclear matter

The energy density of SNM is given by Eqs. (1) and (6) by setting $x = 1/2$, that is [2]

$$\begin{aligned}\epsilon_{\rm snm}(n, T) &= 2\int\frac{d^3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m}f_n + 2\int\frac{d^3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m}f_p \\ &+ \frac{1}{2}An_0u^2 + \frac{Bn_0u^{\sigma+1}}{1 + B'u^{\sigma-1}} \\ &+ u\sum_{i=1,2}C_i2\int\frac{d^3k}{(2\pi)^3}g(k, \Lambda_i)f_n \\ &+ u\sum_{i=1,2}C_i2\int\frac{d^3k}{(2\pi)^3}g(k, \Lambda_i)f_p. \end{aligned} \quad (12)$$

In addition, the single-particle potential $U^\tau_{\rm snm}(n, k, T)$ in the case of SNM, defined from the relation $U^\tau_{\rm snm} = \partial V_{\rm snm}/\partial n_\tau$, is easily calculated and given by

$$U^{\tau}_{\rm snm}(n,k,T)=\tilde{U}^{\tau}_{\rm snm}(n,T)+u\sum_{i=1,2}C_i\left[1+\left(\frac{k}{\Lambda_i}\right)^2\right]^{-1}. \tag{13}$$

It is obvious from Eq. (13) that $U^{\tau}_{\rm snm}(n,k,T)$ is separated in two terms. The first one corresponds to the momentum-independent part, whereas the second one corresponds to the momentum-dependent one. The term $\tilde{U}^{\tau}_{\rm snm}(n,T)$ has the following form

$$\tilde{U}^{\tau}_{\rm snm}(n,T)={\rm A}u+\frac{Bu^{\sigma}(\sigma+1+2B'u^{\sigma-1})}{(1+B'u^{\sigma-1})^2}+\frac{2}{n_0}\sum_{i=1,2}C_i2\int\frac{d^3k}{(2\pi)^3}\left[1+\left(\frac{k}{\Lambda_i}\right)^2\right]^{-1}f_{\tau},\quad \tau=p,n. \tag{14}$$

At zero temperature ($T=0$), where $f_\tau=\theta(k_{F_\tau}-k)$, the integrals in Eqs. (12) and (14) are calculated analytically (see Appendix A for more details).

B. Pure neutron matter

The energy density of PNM is given by Eqs. (1) and (6) by setting $x=0$ and $f_p=0$, that is [2]

$$\epsilon_{\rm pnm}(n,T)=2\int\frac{d^3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m}f_n+\frac{1}{3}An_0(1-x_0)u^2+\frac{\frac{2}{3}Bn_0(1-x_3)u^{\sigma+1}}{1+\frac{2}{3}B'(1-x_3)u^{\sigma-1}}+\frac{2}{5}u\sum_{i=1,2}(3C_i-4Z_i)2\times\int\frac{d^3k}{(2\pi)^3}g(k,\Lambda_i)f_n. \tag{15}$$

The single-particle potential $U^n_{\rm pnm}(n,k,T)$ in the case of PNM is defined from the relation $U^n_{\rm pnm}=\partial V_{\rm pnm}/\partial n_n$ is written as

$$U^n_{\rm pnm}(n,k,T)=\tilde{U}^n_{\rm pnm}(n,T)+\frac{2}{5}u\sum_{i=1,2}(3C_i-4Z_i)\times\left[1+\left(\frac{k}{\Lambda_i}\right)^2\right]^{-1}. \tag{16}$$

The momentum-independent part is

$$\tilde{U}^n_{\rm pnm}(n,T)=\frac{2}{3}A(1-x_0)u+\frac{\frac{2}{3}B(1-x_3)u^{\sigma}}{\left[1+\frac{2}{3}B'(1-x_3)u^{\sigma-1}\right]^2}\times\left[(\sigma+1)+\frac{4}{3}B'(1-x_3)u^{\sigma-1}\right]+\frac{2}{5n_0}\sum_{i=1,2}(3C_i-4Z_i)2\times\int\frac{d^3k}{(2\pi)^3}\left[1+\left(\frac{k}{\Lambda_i}\right)^2\right]^{-1}f_n. \tag{17}$$

The integrals in Eqs. (15) and (17), similarly to the case of SNM, at $T=0$ are calculated analytically (see Appendix A for more details).

C. Asymmetric nuclear matter-nuclear symmetry energy

The energy density of ANM at density n and temperature T, in a good approximation, is expressed as

$$\epsilon_{\rm anm}(n,T,x)=\epsilon_{\rm snm}(n,T,x=1/2)+\epsilon_{\rm sym}(n,T,x), \tag{18}$$

where

$$\epsilon_{\rm sym}(n,T,x)=n(1-2x)^2E^{\rm tot}_{\rm sym}(n,T)=n(1-2x)^2\times\left[E^{\rm kin}_{\rm sym}(n,T)+E^{\rm int}_{\rm sym}(n,T)\right]. \tag{19}$$

In Eq. (19) the nuclear symmetry energy $E^{\rm tot}_{\rm sym}(n,T)$ is separated in two parts corresponding to the kinetic contribution $E^{\rm kin}_{\rm sym}(n,T)$ and the interaction contribution $E^{\rm int}_{\rm sym}(n,T)$. In the present work we will concentrate on the systematic study of the thermal properties of the above two quantities.

From Eqs. (18) and (19) and setting $x=0$ we obtain that the nuclear symmetry energy $E^{\rm tot}_{\rm sym}(n,T)$ is given by

$$E^{\rm tot}_{\rm sym}(n,T)=\frac{1}{n}[\epsilon_{\rm pnm}(n,T)-\epsilon_{\rm snm}(n,T)]. \tag{20}$$

Thus, from Eqs. (12) and (15) and by a suitable choice of the parameters x_0, x_3, Z_1, and Z_2, we can obtain different forms for the density dependence of the symmetry energy $E^{\rm tot}_{\rm sym}(n,T)$. It is well known that the need to explore different forms for $E^{\rm tot}_{\rm sym}(n,T)$ stems from the uncertain behavior at high density [2]. In the present work, because we are interested mainly in the study of thermal effects on the SE, we choose a specific form of the SE enabling us to reproduce accurately the results of many other theoretical studies [26]. According to this choice the SE, at $T=0$, is expressed as

$$E^{\rm tot}_{\rm sym}(n,T=0)=\underbrace{13u^{2/3}}_{\rm Kinetic}+\underbrace{17F(u)}_{\rm Interaction}=\underbrace{13u^{2/3}}_{\rm Kinetic}+\underbrace{17u}_{\rm Interaction}, \tag{21}$$

where the contributions of the kinetic and the interaction term are separated clearly. The parameters x_0, x_3, Z_1, and Z_2 are chosen in order that Eq. (20), for $T=0$, to reproduce the results of Eq. (21). In addition, the parameters A, B, σ, C_1, C_2, and B' are determined in order that $E(n=n_0)-mc^2=-16$ (MeV), $n_0=0.16$ fm^{-3}, and the incompressibility to be $K_0=240$ MeV.

The single-particle potential $U^{\tau}_{\rm anm}(n,k,T)$, in the case of ANM defined from the relation $U^{\tau}_{\rm anm}=\partial V_{\rm anm}/\partial n_\tau$, is written as

$$U^{\tau}_{\rm anm}(n,k,T)=U^{\tau}_{\rm snm}(n,k,T)+\frac{\partial V_{\rm sym}}{\partial n_\tau}=U^{\tau}_{\rm snm}(n,k,T)+U^{\tau}_{\rm sym}(n,T,x), \tag{22}$$

where

$$V_{\rm sym}(n,T,x)=(1-2x)^2nE^{\rm int}_{\rm sym}(n,T). \tag{23}$$

It is easy to find that the term $U^{\tau}_{\rm sym}(n,T)$, in the case of $T=0$ and by applying expression (21), is given by (see also Ref. [27])

$$U^{\tau}_{\rm sym}(n,T,x) = \pm 34u(1-2x), \tag{24}$$

where $+$ and $-$ stand for neutrons and protons, respectively. In the general case where thermal effects are included in our calculations, the $E^{\rm int}_{\rm sym}(n,T)$ takes the form

$$E^{\rm int}_{\rm sym}(n,T) = au^{b}, \tag{25}$$

where a and b are temperature-dependent constants [see Eq. (41) on Sec. III]. Thus, after some algebra, we get in a good approximation, the relation

$$U^{\tau}_{\rm sym}(n,T,x) \simeq \pm 2au^{b}(1-2x). \tag{26}$$

The above relation is needed for the calculation of the single-particle energy $e_\tau(n,k,T)$ in the β-stable matter and afterwards for the calculation of the Fermi-Dirac function $f_\tau(n,T)$ which is the basic ingredient for the determination of the entropy density $s_\tau(n,T)$.

D. Proton fraction-electron chemical potential

The key quantity for the determination of the equation of state in β-stable matter is the proton fraction x, which is a basic ingredient of Eq. (19). In β-stable matter the processes [28]

$$n \longrightarrow p + e^- + \bar{\nu}_e, \quad p + e^- \longrightarrow n + \nu_e, \tag{27}$$

take place simultaneously. We assume that neutrinos generated in these reactions have left the system. This implies that

$$\hat{\mu} = \mu_n - \mu_p = \mu_e, \tag{28}$$

where μ_n, μ_p, and μ_e are the chemical potentials of the neutron, proton, and electron respectively. Given the total energy density $\epsilon \equiv \epsilon(n_n,n_p)$, the neutron and proton chemical potentials can be defined as

$$\mu_n = \left.\frac{\partial\epsilon}{\partial n_n}\right|_{n_p}, \quad \mu_p = \left.\frac{\partial\epsilon}{\partial n_p}\right|_{n_n}. \tag{29}$$

Hence we can show that

$$\hat{\mu} = \mu_n - \mu_p = -\left.\frac{\partial\epsilon/n}{\partial x}\right|_{n} = -\left.\frac{\partial E}{\partial x}\right|_{n}. \tag{30}$$

In β equilibrium one has

$$\frac{\partial E}{\partial x} = \frac{\partial}{\partial x}[E_b(n,x) + E_e(x)] = 0, \tag{31}$$

where $E_b(n,x)$ the energy per baryon and $E_e(x)$ the electron energy. The charge condition implies that $n_e = n_p = nx$ or $k_{F_e} = k_{F_p}$. Combining relations (18), (19), and (30) we get

$$\mu_e(n,T) = \hat{\mu}(n,T) = 4(1-2x)E^{\rm tot}_{\rm sym}(n,T). \tag{32}$$

From Eq. (32) it is obvious that the proton fraction x is not only a function of the baryon density n but, in addition, depends on the temperature T, i.e., $x = x(n,T)$.

For relativistic nondegenerate free electrons we have

$$n_e = xn = \frac{2}{(2\pi^3)}\int \frac{d^3k}{1+\exp\left[\frac{\sqrt{\hbar^2k^2c^2+m_e^2c^4}-\mu_e(n,T)}{T}\right]}. \tag{33}$$

Or, using Eq. (32) and performing the angular integration we get

$$n_e = xn = \frac{1}{\pi^2}\int_0^{\infty} \frac{k^2dk}{1+\exp\left[\frac{\sqrt{\hbar^2k^2c^2+m_e^2c^4}-4(1-2x)E^{\rm tot}_{\rm sym}(n,T)}{T}\right]}. \tag{34}$$

Equation (34) determines the equilibrium electron (proton) fraction $x(n,T)$ because the density and momentum-dependent symmetry energy $E^{\rm tot}_{\rm sym}(n,T)$ is known.

E. Calculations recipe

We focus our attention on the calculation of the $E^{\rm tot}_{\rm sym}(n,T)$ with the help of Eq. (20). Thus, one has to calculate first the energy densities in pure and in symmetric nuclear matter as a function of the density n and for fixed values of temperature T. As an example of the calculations procedure at finite temperature (the results for $T=0$ are included in the Appendix A), we consider the case of pure neutron matter. The procedure is similar in the case of symmetric nuclear matter (see Ref. [2]).

The outline of our approach is the following: For a fixed neutron density n_n and temperature T, Eq. (4) may be solved iteratively to calculate the variable

$$\eta(n;T) = \frac{\mu_\tau(n;T) - \tilde{U}(n;T)}{T}. \tag{35}$$

The knowledge of $\eta(n,T)$ allows the last term in Eq. (17) to be evaluated, yielding $\tilde{U}(n;T)$, which may then be used to infer the chemical potential from

$$\mu_\tau(n;T) = T\eta(n;T) + \tilde{U}(n;T), \tag{36}$$

required as an input to the calculation of the single-particle spectrum $e_\tau(n,k,T)$ in Eq. (5). Using $e_\tau(n,k;T)$, the energy density in Eq. (15) is evaluated.

III. RESULTS AND DISCUSSION

According to our calculation recipe, given in the previous subsection, we calculate the energy densities of PNM and SNM as functions of the density, for various values of the temperature T. As a second step, we calculate the $E^{\rm tot}_{\rm sym}(n,T)$ from Eq. (20). The knowledge of $E^{\rm tot}_{\rm sym}(n,T)$ is required for the evaluation of the proton fraction x from Eq. (34) as well as for the electron chemical potential $\mu_e = \hat{\mu}$ from Eq. (32). Finally from Eqs. (9), (10), and (11) we construct the EOS of β-stable matter for various values of the temperature T. It is worth pointing out that in the present work we do not include the muon case, since we restrict ourselves mainly on the temperature dependent behavior of the SE. According to our plan, in future work we will extend the treatment to include also the muon case in order to study the detailed composition and the thermal properties of neutron-rich matter with applications in neutron star structure and thermal evaluation.

In Fig. 1 we check the validity of approximation (18). We plot the difference $E(n,T,x) - E(n,T,x=1/2)$ as a

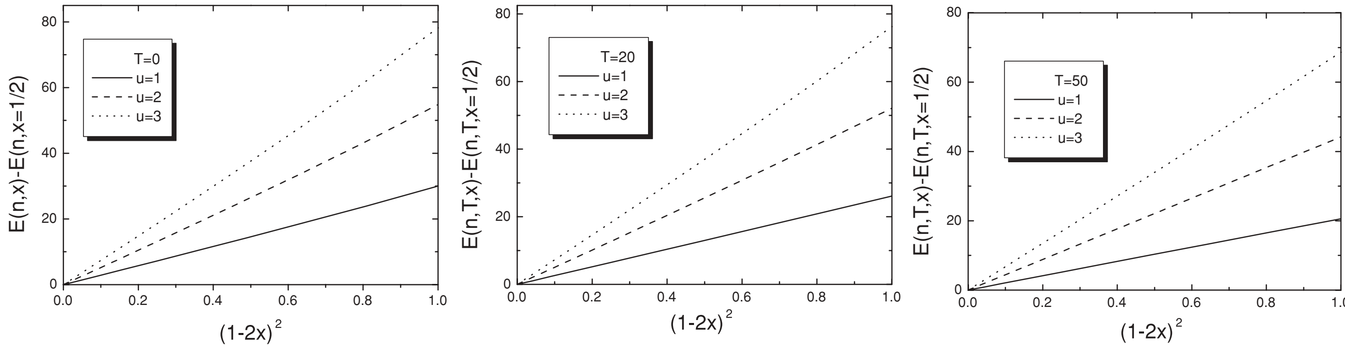

FIG. 1. The difference $E(n, T, x) - E(n, T, x = 1/2)$ as a function of $(1 - 2x)^2$ at temperature $T = 0$, $T = 20$, and $T = 50$ MeV for three baryon number fractions $u = 1$, $u = 2$, and $u = 3$.

function of $(1 - 2x)^2$ at temperature $T = 0$, $T = 20$, and $T = 50$ MeV for three baryon number fractions, i.e., $u = 1$, $u = 2$, and $u = 3$. It is seen that an almost linear relation holds between $E(n, T, x) - E(n, T, x = 1/2)$ and $(1 - 2x)^2$, even closer to the case of pure neutron matter ($x = 0$), indicating the validity of approximation (18).

In Fig. 2 we indicate the behavior of the SE as a function of the temperature T for various fixed values of the baryon

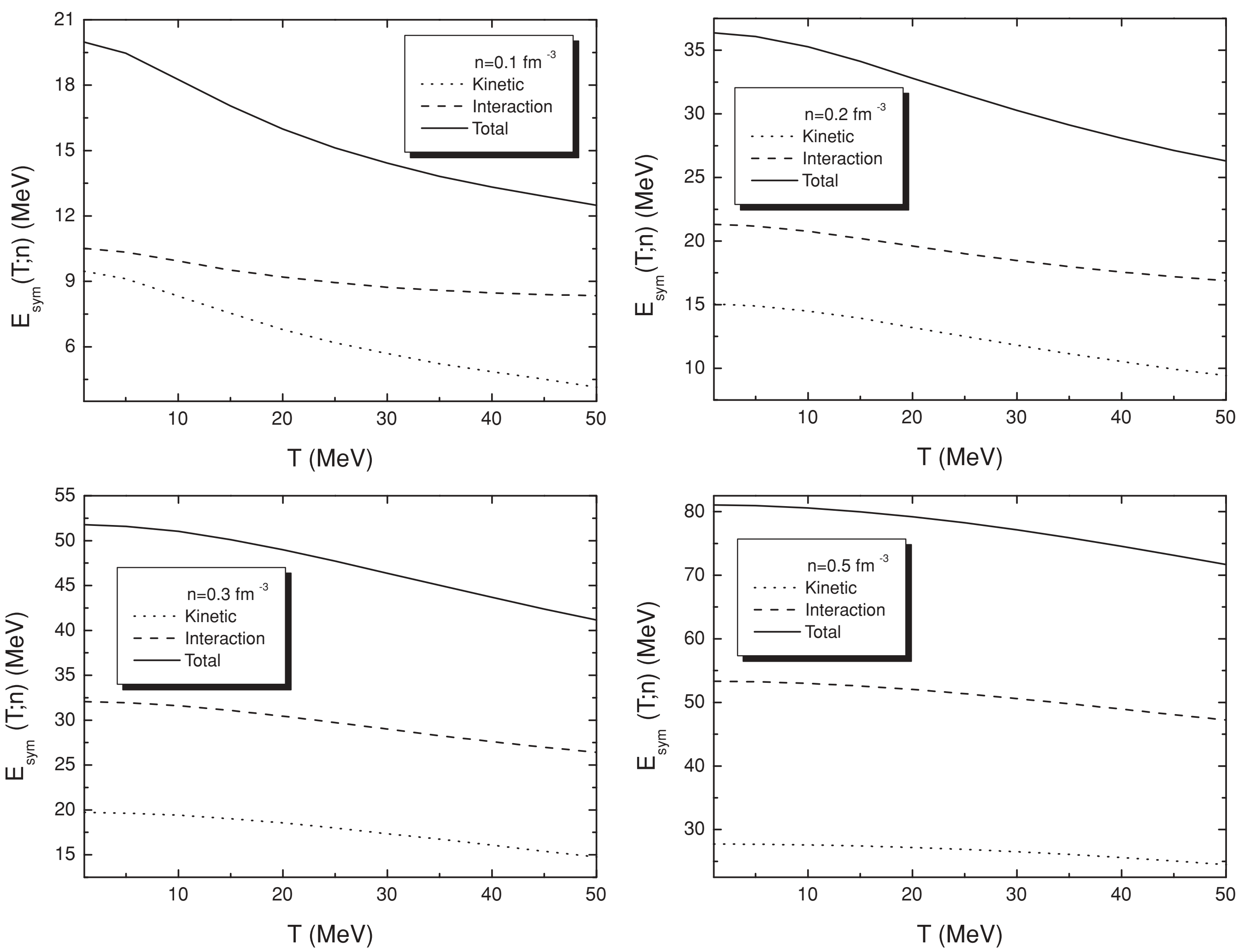

FIG. 2. Temperature dependence of the total nuclear symmetry energy and its interaction and kinetic energy part for various values of the baryon density n.

TABLE I. The values of the density dependent parameters A, B, T_0, and c, for $E_{\rm sym}^{\rm tot}(u;T)$, $E_{\rm sym}^{\rm kin}(u;T)$ and $E_{\rm sym}^{\rm int}(u;T)$ for $n = 0.1, 0.3, 0.5\ {\rm fm}^{-3}$. For more details see text.

Parameters	$n = 0.1\ {\rm fm}^{-3}$			$n = 0.3\ {\rm fm}^{-3}$			$n = 0.5\ {\rm fm}^{-3}$		
	$E_{\rm sym}^{\rm tot}$	$E_{\rm sym}^{\rm kin}$	$E_{\rm sym}^{\rm int}$	$E_{\rm sym}^{\rm tot}$	$E_{\rm sym}^{\rm kin}$	$E_{\rm sym}^{\rm int}$	$E_{\rm sym}^{\rm tot}$	$E_{\rm sym}^{\rm kin}$	$E_{\rm sym}^{\rm int}$
A	10.105	7.864	2.559	17.079	15.230	10.832	44.164	20.548	19.442
B	9.969	1.679	7.969	19.328	4.504	21.240	36.895	7.162	33.887
T_0	25.692	30.549	19.027	41.004	73.143	47.772	57.011	109.193	73.551
c	1.610	1.518	1.866	1.856	1.904	1.992	1.982	2.156	2.026

density n. More precisely, in any case, we plot $E_{\rm sym}^{\rm tot}(T;n)$, as well as $E_{\rm sym}^{\rm kin}(T;n)$ and $E_{\rm sym}^{\rm int}(T;n)$ as a function of T for $n = 0.1, 0.2, 0.3, 0.5\ {\rm fm}^{-3}$. The most striking feature of the above analysis is a decrease of the SE (total, kinetic, and interaction part) by increasing the temperature. This is consistent with the predictions of microscopic and/or phenomenological theories [3,13,14].

To illustrate further the dependence of the symmetry energy on the temperature and to find the quantitative characteristic on this dependence, the values of $E_{\rm sym}(T;n)$ for various values of the density n are derived with the least-squares fit method and found to take the general form

$$E_{\rm sym}(T;n) = \frac{A}{1+(T/T_0)^c} + B. \tag{37}$$

The values of the density-dependent parameters A, B, T_0, and c, for $E_{\rm sym}^{\rm tot}(T;n)$, $E_{\rm sym}^{\rm kin}(T;n)$ and $E_{\rm sym}^{\rm int}(T;n)$ for $n = 0.1, 0.3, 0.5\ {\rm fm}^{-3}$ are presented in Table I. It is easy to find that in the case of low temperature limit ($T/T_0 \ll 1$) all kinds of the symmetry energy decrease approximately according to $E_{\rm sym}(T;n) \propto C_1 - C_2T^2$ (where C_1 and C_2 are density-dependent constants). In the high-density limit ($T/T_0 \gg 1$) the symmetry energy decreases approximately according to $E_{\rm sym}(T;n) \propto C_3T^{-2} + C_4$ (where also C_3 and C_4 are density-dependent constants). It is noted that the same behavior holds for $E_{\rm sym}^{\rm tot}(T;n)$ as well as for $E_{\rm sym}^{\rm kin}(T;n)$ and $E_{\rm sym}^{\rm int}(T;n)$. This behavior is well expected for the kinetic part of the symmetry energy (see also Ref. [3,25]), where analytical calculations are possible (see the proof in Appendix B). From the above study, it is concluded that there is a similar temperature dependence both for the kinetic and the interaction part of the symmetry energy and consequently for the total symmetry energy, in the case of momentum-dependent interaction. Recently, the temperature dependence of the kinetic and interaction part of the SE has been studied and illustrated in Ref. [14]. The results of the present work agree with those of Ref. [14] although different models have been employed to evaluate SE.

In Fig. 3, we plot $E_{\rm sym}^{\rm tot}(T;n)$ as a function of temperature for various low values of the baryon density. In the same figure we also include experimental data of the measured temperature-dependent symmetry energy from Texas A&M University (TAMU) [29] and the INDRA-ALADIN Collaboration at GSI [30]. The comparison then allows to estimate the required density of the fragment-emitting of the experiments. As pointed out by Li *et al.* [3] the experimentally observed evolution of the SE is mainly due to the change in density rather than temperature.

Figure 4 illustrates the behavior of the $E_{\rm sym}^{\rm tot}(n;T)$ (a), $E_{\rm sym}^{\rm kin}(n;T)$ (b), $E_{\rm sym}^{\rm int}(n;T)$ (c), as a function of the baryon density n for various fixed values of the temperature T. The case $T = 0$ corresponds to the fundamental expression of the present work, i.e.,

$$E_{\rm sym}^{\rm tot}(u;T=0) = 13u^{2/3} + 17u. \tag{38}$$

In any case, the trends of the various parts of the symmetry energy are similar. An increase in the temperature leads just to a shift to lower values for the symmetry energy. It is worth pointing out that the maximum decrease of $E_{\rm sym}^{\rm tot}(n;T)$, in the area under study (for $T = 0$ MeV up to $T = 50$ MeV), is between 40% (for $n = 0.1\ {\rm fm}^{-3}$) and 4% (for $n = 1\ {\rm fm}^{-3}$). Correspondingly, the decrease of $E_{\rm sym}^{\rm kin}(n;T)$ is between 57% (for $n = 0.1\ {\rm fm}^{-3}$) and 5% (for $n = 1\ {\rm fm}^{-3}$) and of the $E_{\rm sym}^{\rm int}(n;T)$ is between 22% (for $n = 0.1\ {\rm fm}^{-3}$) and 5% (for $n = 1\ {\rm fm}^{-3}$). It is obvious that the thermal effects are more pronounced on the kinetic part than in the interaction part of the symmetry energy and, in addition, more pronounced in lower values of the baryon density.

The total symmetry energy $E_{\rm sym}^{\rm tot}(u;T)$, for various values of the temperature T, was derived with the least-squares fit on

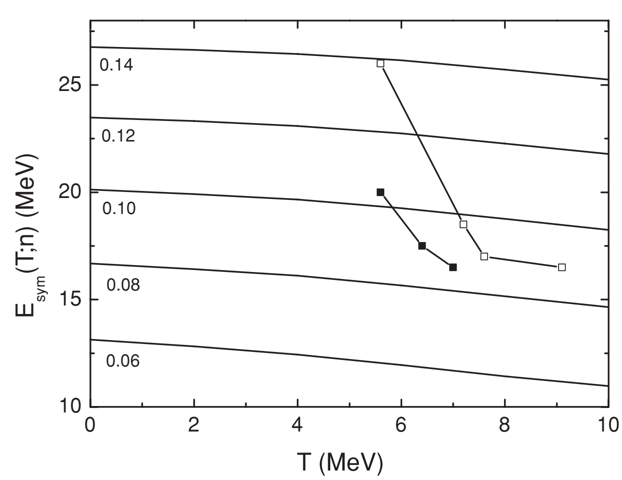

FIG. 3. Temperature dependence of the symmetry energy for low values of the baryon density ($n = 0.06, 0.08, 0.10, 0.12, 0.14\ {\rm fm}^{-3}$). The experimental data are from Ref. [29] (solid squares) and Ref. [30] (open squares) are included for comparison.

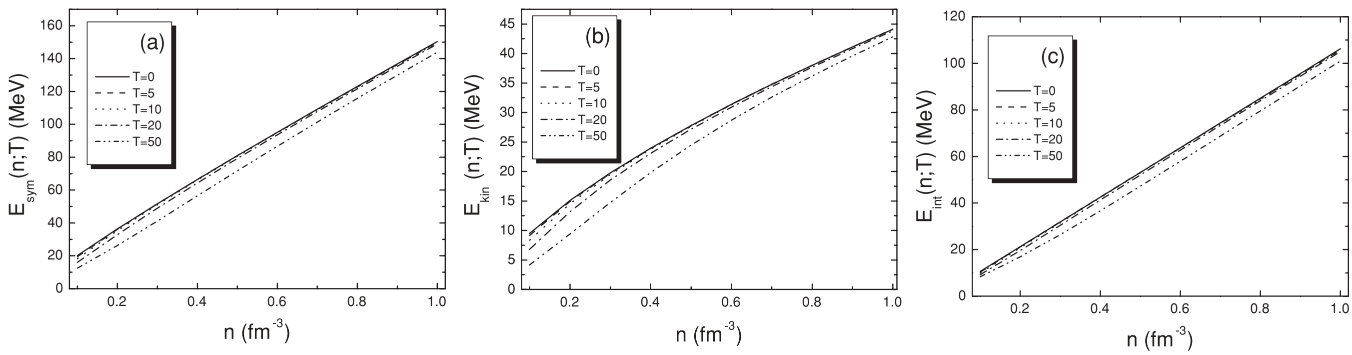

FIG. 4. Density dependence of the total nuclear symmetry energy $E_{\rm sym}^{\rm tot}(n,T)$ as well its kinetic $E_{\rm sym}^{\rm kin}(n,T)$ and interaction $E_{\rm sym}^{\rm int}(n,T)$ part for various values of the temperature T.

the numerical results taken from Eq. (20) and has the form

$$\begin{aligned} E_{\rm sym}^{\rm tot}(u;T=5) &= 1.676 + 29.711u - 2.110u^2 \\ &\quad + 0.275u^3 - 0.015u^4, \\ E_{\rm sym}^{\rm tot}(u;T=10) &= -0.118 + 30.863u - 2.455u^2 \\ &\quad + 0.325u^3 - 0.017u^4, \\ E_{\rm sym}^{\rm tot}(u;T=20) &= -1.910 + 29.470u - 1.466u^2 \\ &\quad + 0.120u^3 - 0.004u^4, \\ E_{\rm sym}^{\rm tot}(u;T=50) &= 0.099 + 18.172u + 2.9u^2 \\ &\quad - 0.548u^3 + 0.033u^4. \end{aligned} \tag{39}$$

It is also useful to record some relations for $E_{\rm sym}^{\rm tot}(u;T)$ derived by least-squares fit on the numerical results, in the case where SE is parametrized in a way similar to that one holding for $T=0$. In that case, the parametrization is the following [the case $E_{\rm sym}^{\rm tot}(u;T=0)$ is included also for comparison].

$$\begin{aligned} E_{\rm sym}^{\rm tot}(u;T=0) &= 13u^{2/3} + 17u, \\ E_{\rm sym}^{\rm tot}(u;T=5) &= E_{\rm sym}^{\rm tot}(u;T=0) - 0.374u^{-0.956}, \\ E_{\rm sym}^{\rm tot}(u;T=10) &= E_{\rm sym}^{\rm tot}(u;T=0) - 1.235u^{-0.804}, \\ E_{\rm sym}^{\rm tot}(u;T=20) &= E_{\rm sym}^{\rm tot}(u;T=0) - 3.420\,u^{-0.520}, \\ E_{\rm sym}^{\rm tot}(u;T=50) &= E_{\rm sym}^{\rm tot}(u;T=0) - 9.300\,u^{-0.097}. \end{aligned} \tag{40}$$

From Eq. (40), the decrease of the SE as a result of increasing T, is evident.

The interaction part of the symmetry energy $E_{\rm sym}^{\rm int}(u;T)$ for various values of the temperature T was derived by a least-squares fit on the numerical results taken from Eqs. (19) and (20) and has the form

$$\begin{aligned} E_{\rm sym}^{\rm int}(u;T=5) &= 17.041u^{0.997}, \\ E_{\rm sym}^{\rm int}(u;T=10) &= 16.782u^{1.005}, \\ E_{\rm sym}^{\rm int}(u;T=20) &= 16.022u^{1.028}, \\ E_{\rm sym}^{\rm int}(u;T=50) &= 13.404u^{1.104}. \end{aligned} \tag{41}$$

Similarly, for the kinetic part of the symmetry energy $E_{\rm sym}^{\rm kin}(u;T)$ we obtain

$$\begin{aligned} E_{\rm sym}^{\rm kin}(u;T=5) &= 12.856u^{0.674}, \\ E_{\rm sym}^{\rm kin}(u;T=10) &= 12.504u^{0.691}, \end{aligned}$$

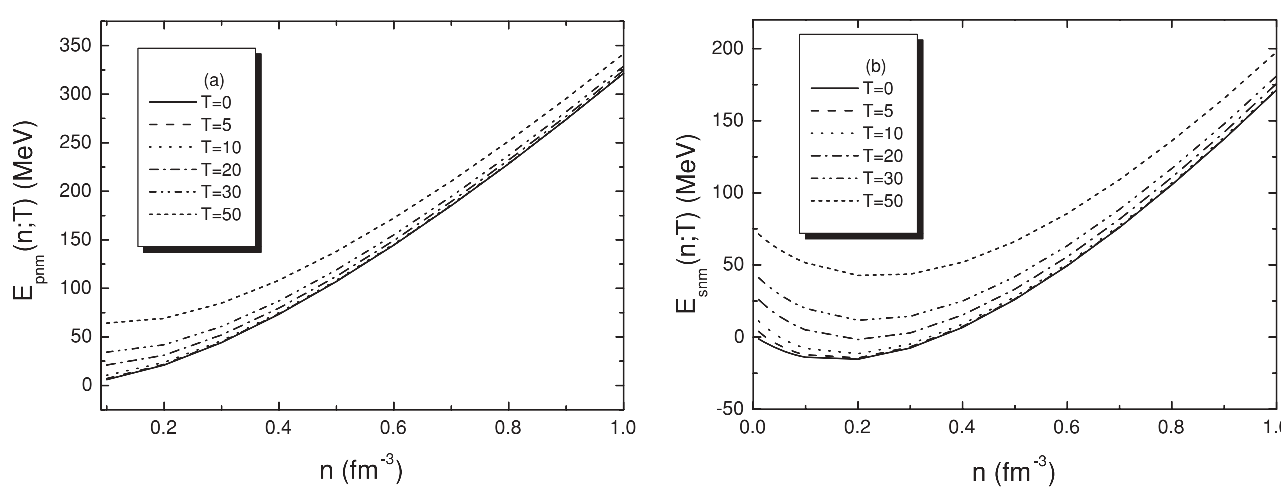

FIG. 5. (a) The energy per particle of pure neutron matter as a function of the baryon density for various values of the temperature T. (b) The energy per particle of symmetric nuclear matter as a function of the baryon density for various values of the temperature T.

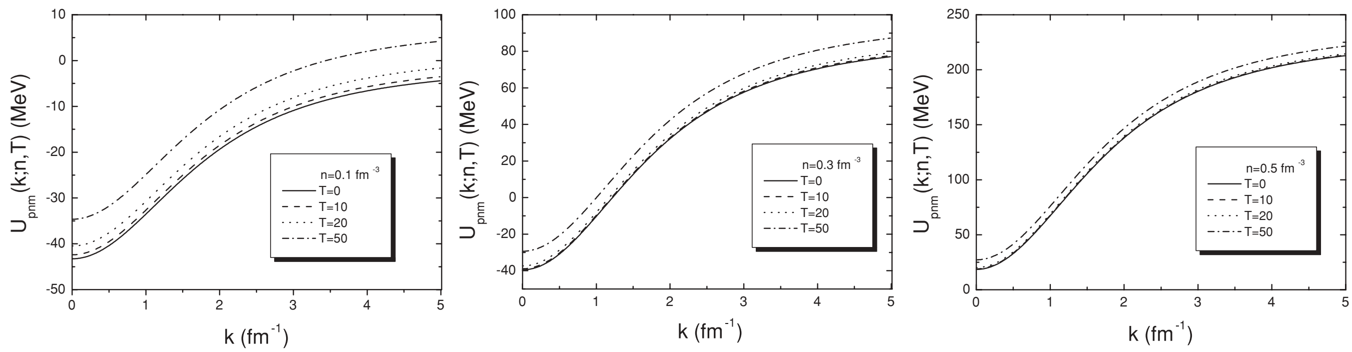

FIG. 6. The single-particle potential of the pure neutron matter as a function of the momentum k for various values of the temperature T and for $n = 0.1$, 0.3, and 0.5 fm^{-3}, respectively.

$$E_{\rm sym}^{\rm kin}(u; T = 20) = 11.518u^{0.736},$$
$$E_{\rm sym}^{\rm kin}(u; T = 50) = 8.577u^{0.891}. \qquad (42)$$

In Fig. 5 we plot the total energy per particle of the PNM (a) and of the SNM as a function of the density for various values of the temperature. In both cases it is concluded that the thermal effects become more pronounced when $T > 10$ MeV and for baryon densities $n < 0.5$ fm^{-3}.

Figure 6 displays the single-particle potential $U_{\rm pnm}(n, T, k)$ of the PNM as a function of the momentum k for various values of the density n and temperature T. An increase of T leads to corresponding increase of the values of the $U_{\rm pnm}(n, T, k)$, an effect, expected to be more pronounced for lower values of the baryon density ($n = 0.1$ fm^{-3}) compared to highest ($n = 0.5$ fm^{-3}). The same trend holds also for the single particle potential $U_{\rm snm}(n, T, k)$ of the SNM plotted in Fig. 7. Observing Figs. 6 and 7 one might expect that the change of T will affect slightly the nucleons with high momentum k. This could be seen by plotting the single-particle energy $e_\tau(n, k, T)$ [see Eq. (5)] as a function of k. However, the above effect cannot be seen in the present work, where we plot just the single-particle potential $U^\tau(n, k, T)$ as a function of k.

In Fig. 8 we display the single particle potential of neutron $U^n(n, T, k)$ [Figs. 8(a) and 8(b)] and proton $U^p(n, T, k)$ [Figs. 8(c) and 8(d)], in β-stable matter, as a function of the momentum k for various values of the temperature T for $n = 0.1$ and $n = 0.5$ fm^{-3}. The potential $U^\tau(n, T, k)$ is evaluated according to Eq. (22). The most striking feature of Fig. 8 is the reduced thermal effect for high values of the baryon density, especially in the case of the neutron single-particle potential. In the case of the proton, thermal effects are more pronounced.

In Fig. 9(a) the proton fraction x is displayed, calculated from Eq. (34) as a function of n for various values of T. Thermal effects increase the value of x between 57% (for $n = 0.1$ fm^{-3}) and 2% (for $n = 1$ fm^{-3}). This effect is directly related with the dependence of x on the symmetry energy. As discussed previously, the temperature influences slightly the symmetry energy at high values of the density and consequently this is reflected in the values of x. It is stressed that x depends on T in two ways, as one can see from Eq. (34). That is, it depends directly on T due the Dirac-Fermi distribution and also depends on the symmetry energy that is also temperature dependent.

In Fig. 9(b) we present the electron chemical potential μ_e as a function of the density n for various T. An increase of T decreases μ_e. The effect is more pronounced when $T > 20$ MeV. We mention that the rate of electron capture on both free and bound protons depends in a very sensitive way on the difference $\hat{\mu} = \mu_n - \mu_p = \mu_e$ between neutron and proton

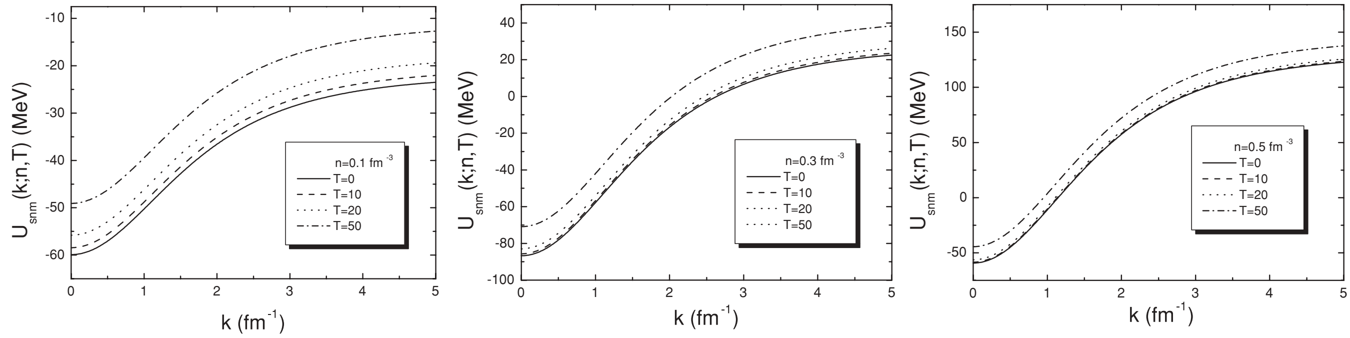

FIG. 7. The single-particle potential of the symmetric nuclear matter as a function of the momentum k for various values of the temperature T and for $n = 0.1$, 0.3, and 0.5 fm^{-3}, respectively.

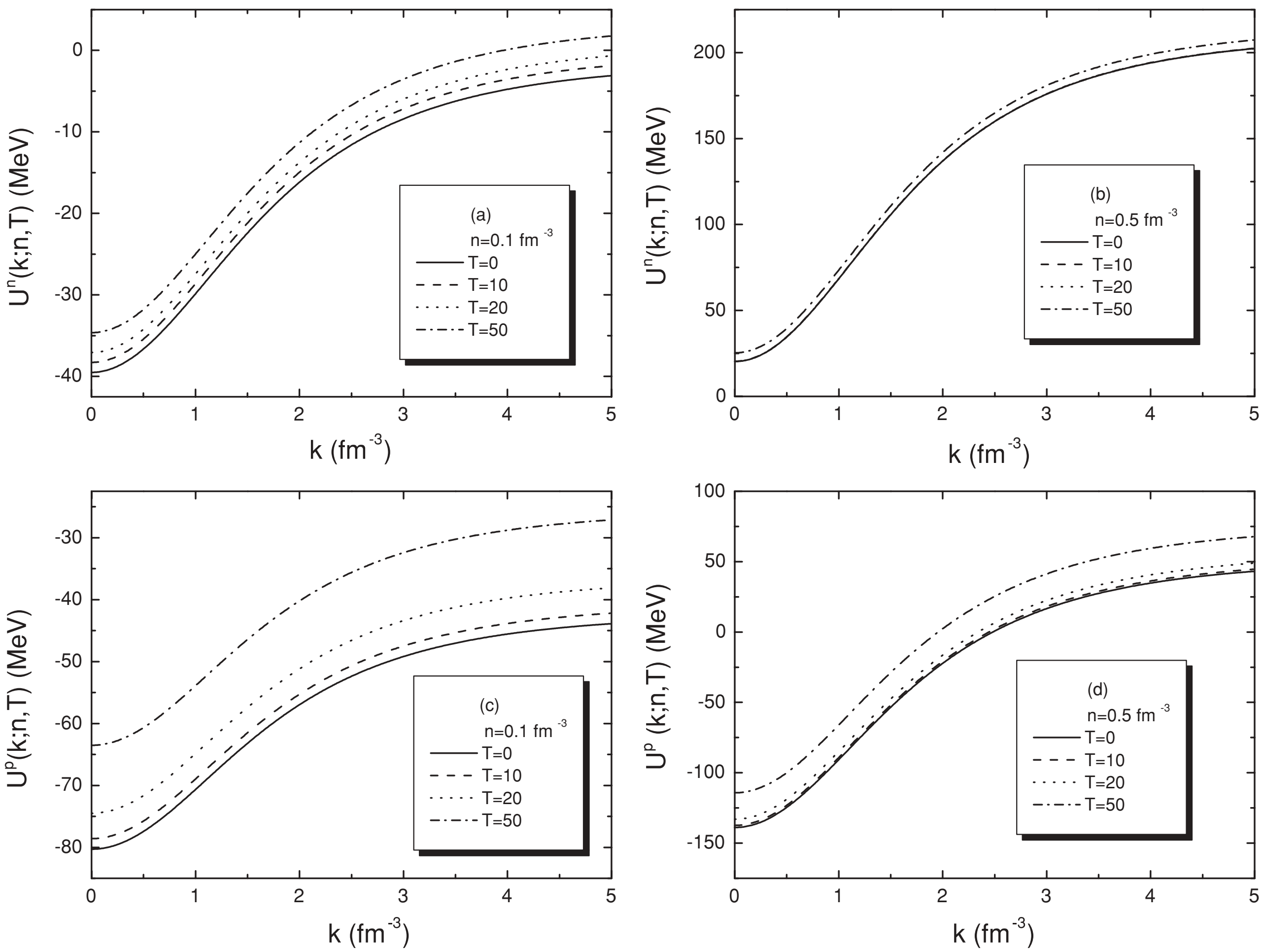

FIG. 8. The single-particle potential of β-stable matter for neutron [(a) and (b)] and for proton [(c) and (d)] as a function of the momentum k for various values of the temperature T for $n = 0.1$ and $0.5\,\mathrm{fm}^{-3}$.

chemical potentials [9]. Larger values of $\hat{\mu} = \mu_e$ inhibit the neutronization process, because it becomes more difficult to transform a proton into a neutron.

Finally, in Fig. 10 we present the equation of state of β-stable matter constructed by applying the present momentum-dependent interaction model for various values of the temperature T. It is obvious that the thermal effects are enhanced when $T > 20\,\mathrm{MeV}$. The above EOS is very important for the calculation of the neutron stars properties and also in combination with the calculated proton fraction and electron chemical potentials for the thermal evaluation of the neutron stars.

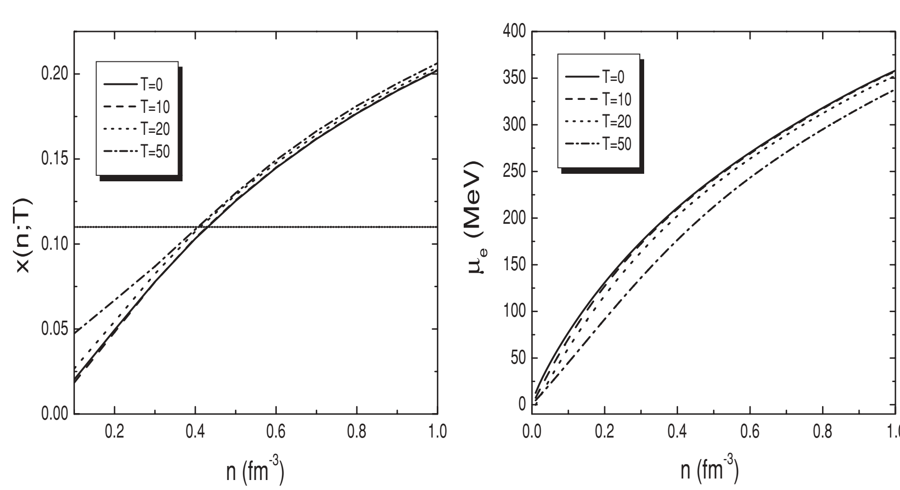

FIG. 9. (a) The proton fraction x in β-stable matter as a function of the density n for various values of the temperature T. The straight line corresponds to the case $x = 11\%$. (b) The electron chemical potential $\mu_e = \hat{\mu} = \mu_n - \mu_p$ as a function of the density n for various values of the temperature T.

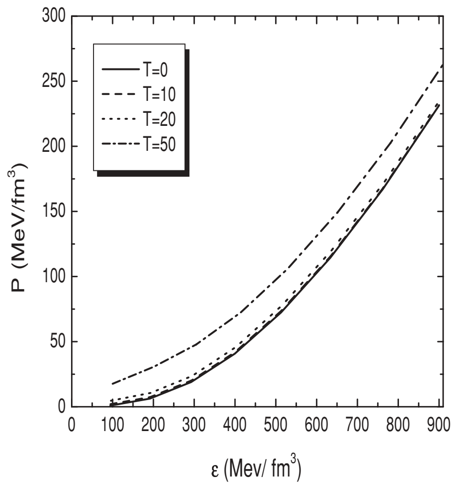

FIG. 10. The equation of state $P = P(\epsilon)$ of β-stable matter corresponding to the present momentum-dependent effective interaction model for various values of the temperature T.

IV. SUMMARY

The knowledge of the nuclear symmetry energy of hot neutron-rich matter is important for understanding the dynamical evolution of massive stars and the supernova explosion mechanisms. In view of the above statement, we investigate, in the present work, the thermal effects on the nuclear symmetry energy. To perform the above investigation we apply a model with a momentum-dependent effective interaction. In that way, we are able to study the thermal effect not only on the kinetic part of the symmetry energy but also on the interaction part which, in turn, due to a momentum dependence, is affected by the variation of the temperature. It is concluded that, in general, by increasing T we obtain a decreasing SE. Our finding that both kinetic and interaction parts exhibit the same trend both for low and high values of the temperature is an interesting result. Analytical relations, derived by the method of least-squares fit are given also for the above quantities. Temperature effects on the pure neutron matter and also on symmetric nuclear matter are also investigated and presented. The single-particle potential of proton and neutron is of interest in heavy-ion collision experiments, is calculated also for pure neutron matter, symmetric nuclear matter and β-stable matter for various values of the baryon density and fixed values of T. It is concluded that thermal effects are more pronounced for low values of the density n, where for high values of n the effects are almost negligible. Quantities, which are of great interest for the thermal evaluation of supernova and neutron stars, i.e., the proton fraction $x = x(n, T)$ and the electron chemical potential $\mu_e = \mu_e(n, T)$, are calculated and their temperature and density dependence is investigated. Thermal effects are larger for low values of the density and high values of T.

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APPENDIX A

The energy density of the SNM as well as of the PNM, at zero temperature are easily calculated from Eqs. (12) and (15), respectively, by setting $f_\tau = \theta(k_{F_\tau} - k)$ [where $\theta(k_{F_\tau} - k)$ is the *theta* function and k_{F_τ} is the Fermi momentum of the nucleon τ] and takes the following forms

$$\begin{aligned}\epsilon_{\rm snm}(n, k; T = 0) &= \frac{3}{5} E_F^0 n_0 u^{5/3} + \frac{1}{2} A n_0 u^2 + \frac{B n_0 u^{\sigma+1}}{1 + B' u^{\sigma-1}} \\ &+ 3 n_0 u \sum_{i=1,2} C_i \left(\frac{\Lambda_i}{k_F^0}\right)^3 \left(\frac{u^{1/3}}{\frac{\Lambda_i}{k_F^0}} - \tan^{-1}\frac{u^{1/3}}{\frac{\Lambda_i}{k_F^0}}\right), \qquad \text{(A1)}\end{aligned}$$

$$\begin{aligned}\epsilon_{\rm pnm}(n, k; T = 0) &= 2^{2/3}\frac{3}{5} E_F^0 n_0 u^{5/3} + \frac{1}{3} A n_0 (1 - x_0) u^2 \\ &+ \frac{\frac{2}{3} B n_0 (1 - x_3) u^{\sigma+1}}{1 + \frac{2}{3} B'(1 - x_3) u^{\sigma-1}} + \frac{3}{5} n_0 u \sum_{i=1,2} (3C_i - 4Z_i) \\ &\times \left(\frac{\Lambda_i}{k_F^0}\right)^3 \left[\frac{(2u)^{1/3}}{\frac{\Lambda_i}{k_F^0}} - \tan^{-1}\frac{(2u)^{1/3}}{\frac{\Lambda_i}{k_F^0}}\right], \qquad \text{(A2)}\end{aligned}$$

where $E_F^0 = \hbar^2 {k_F^0}^2/2m$ is the Fermi energy of nuclear matter at the equilibrium density.

APPENDIX B

To compare the numerical results obtained from the kinetic part of the symmetry energy $E_{\rm sym}^{\rm kin}(n, T)$ with those predicted from analytical calculations, we calculate $E_{\rm sym}^{\rm tot}(n, T)$ in the low and in the hight temperature limit as follows

1. Low temperature limit

The kinetic energy per nucleon $E_{\rm kin}^\tau(n, T)$ at low temperature ($T \ll E_F$) has the form [31–33]

$$E_{\rm kin}^\tau(n, T) = \frac{3}{5} E_F^\tau \left[1 + \frac{5}{12}\pi^2 \left(\frac{T}{E_F^\tau}\right)^2\right], \qquad \text{(B1)}$$

where $E_F^\tau = (\hbar k_F^\tau)^2/2m = \hbar^2(3\pi^2 n_\tau)^{2/3}/2m$. Considering that $\delta = 1 - 2x = (n_n - n_p)/(n_n + n_p)$ after some algebra we found that the $E_{\rm kin}(n, T, \delta)$ of a two-component Fermi gas has the form

$$\begin{aligned}E_{\rm kin}(n, \delta, T) &= \frac{\langle E_F \rangle}{2}[(1 + \delta)^{5/3} + (1 - \delta)^{5/3}] + \frac{3}{10}\frac{1}{\langle E_F \rangle} \\ &\times \left(\frac{\pi}{2} T\right)^2 [(1 + \delta)^{1/3} + (1 - \delta)^{1/3}], \qquad \text{(B2)}\end{aligned}$$

where $\langle E_F \rangle = 3/5 E_F^0$. Expanding expression (B2) around the symmetric point $\delta = 0$ or $x = 1/2$ the kinetic energy takes the approximated form

$$E_{\rm kin}(n,T) = \langle E_F\rangle + \frac{3}{20}\frac{\pi^2}{\langle E_F\rangle}T^2 + (1-2x)^2 \underbrace{\left(\frac{5}{9}\langle E_F\rangle - \frac{1}{60}\frac{\pi^2}{\langle E_F\rangle}T^2\right)}_{E_{\rm sym}^{\rm kin}(n,T)}, \quad \text{(B3)}$$

with the contribution of the symmetry energy written explicitly. It is obvious that in the low-temperature limit $E_{\rm sym}^{\rm kin}(n,T)$ behaves as $E_{\rm sym}^{\rm kin}(n,T) \propto C_1 - C_2T^2$.

2. High temperature limit

The kinetic energy per nucleon $E_{\rm kin}(n,T,\delta)$ of a two-component Fermi gas at high temperature ($T \gg E_F$) is replaced by a virial expansion in $n\lambda^3$, where $\lambda = \sqrt{2\pi\hbar^2/mT}$ is the quantum wavelength. $E_{\rm kin}(n,T)$ is given by the relation [25,32]

$$E_{\rm kin}(n,\delta,T) = \frac{3}{2}T + \frac{3}{4}T\sum_\nu C_\nu\left(\frac{\lambda^3 n}{4}\right)^\nu \times [(1-\delta)^{\nu+1} + (1+\delta)^{\nu+1}]. \quad \text{(B4)}$$

Expanding expression (B4) around the symmetric point $\delta = 0$ or $x = 1/2$ the kinetic energy takes the approximated form

$$E_{\rm kin}(n,T,\delta) = \frac{3}{2}T\left[1+\sum_\nu C_\nu\left(\frac{\lambda^3 n}{4}\right)^\nu\right] + (1-2x)^2 \underbrace{\frac{3}{2}T\sum_\nu C_\nu\left(\frac{\lambda^3 n}{4}\right)^\nu \frac{\nu(\nu+1)}{2}}_{E_{\rm sym}^{\rm kin}(n,T)}. \quad \text{(B5)}$$

It is seen that in the high-temperature limit $E_{\rm sym}^{\rm kin}(n,T)$ behaves as $E_{\rm sym}^{\rm kin}(n,T) \propto C_1T^{-1/2} + C_2T^{-2} + \cdots$.

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