

Updated analysis of NN elastic scattering to 3 GeV

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(Received 18 June 2007; published 16 August 2007)

A partial-wave analysis of NN elastic scattering data has been updated to include a number of recent measurements. Experiments carried out at the Cooler Synchrotron (COSY) by the EDDA Collaboration have had a significant impact above 1 GeV. Results are discussed in terms of the partial-wave and direct-reconstruction amplitudes.

DOI: [10.1103/PhysRevC.76.025209](https://doi.org/10.1103/PhysRevC.76.025209)

PACS number(s): 11.80.Et, 13.75.Cs, 25.40.Cm, 25.40.Dn

I. INTRODUCTION

In our previous analysis of nucleon-nucleon (NN) elastic scattering data [1], the upper energy limit was extended from 2.5 to 3.0 GeV in the laboratory kinetic energy, T_p , to accommodate higher-energy proton-proton (pp) scattering measurements from SATURNE II at Saclay and the EDDA Collaboration at COSY. Comparisons were made with the direct amplitude reconstruction (DAR) and partial-wave analysis (PWA) study of Ref. [2], which gave amplitudes to 2.7 GeV. In some cases, where the DAR allowed two distinct solutions, our energy-dependent and single-energy results were seen to follow different sets of DAR amplitudes. With additional precise measurements, it was expected that these discrepancies would be removed. Such data have now appeared and our fits have changed substantially as a result.

In the following, we list those data recently added to our database. After a brief reminder of the differences between the PWA and the DAR techniques, we give our latest fit results. Some of the changes have been quite large, and these are discussed in light of the constraints imposed by the earlier Saclay DAR results. Finally, we summarize the status of the NN problem and consider what further work may be expected.

II. DATABASE

The full database and a number of fits, from our group and others, are available through the on-line Scattering Analysis Interactive Dial-in (SAID) facility [3]. Here we concentrate only on those new measurements added since the SP00 (spring 2000) solution was published.

Table I lists recent (post-SP00) contributions to the pp database. A major contribution has come from the EDDA collaboration. From this source we have added unpolarized, single-polarized, and double-polarized cross sections covering a wide energy range. Final A_{yy} measurements from SATURNE II have also been added. The PNPI group has provided P and D_t at a single energy.

Table II lists post-SP00 contributions to the np database. Major contributions to the np (single and double) polarized data have come from PSI. The Uppsala facility has provided a detailed study of the angular shape of cross sections at 95 MeV. The IUCF group has done similar work at 194 MeV. Low-energy (below 20 MeV) cross sections have come from Ohio University [11] and the TUNL facility [10], with an additional medium-energy piece coming from JINR [22].

III. AMPLITUDE ANALYSIS

Here, we use the notation of Ref. [2] and write the scattering matrix, M , as

$$M(\vec{k}_f, \vec{k}_i) = \frac{1}{2}[(a+b) + (a-b)(\vec{\sigma}_1 \cdot \vec{n})(\vec{\sigma}_2 \cdot \vec{n}) + (c+d) \times (\vec{\sigma}_1 \cdot \vec{m})(\vec{\sigma}_2 \cdot \vec{m}) + (c-d)(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_2 \cdot \vec{l}) + e(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{n}], \quad (1)$$

where \vec{k}_f and \vec{k}_i are the scattered and incident momenta in the center-of-mass system and

$$\vec{n} = \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_i \times \vec{k}_f|}, \quad \vec{l} = \frac{\vec{k}_i + \vec{k}_f}{|\vec{k}_i + \vec{k}_f|}, \quad \vec{m} = \frac{\vec{k}_f - \vec{k}_i}{|\vec{k}_f - \vec{k}_i|}.$$

Writing the scattering matrix in this form, any pp observable can be expressed in terms of the five complex amplitudes a through e . If a sufficient number of independent observables are measured (precisely) at a given energy and angle, these amplitudes can be determined up to an overall undetermined phase. The advantage of this method is its model independence; nothing beyond the data is required to determine a solution. In addition, once the amplitudes are found, any further experimental quantity can be predicted at the energy-angle points of the DAR. There are, however, a number of disadvantages. The process gives amplitudes only at single energy-angle points, and no result is possible if an insufficient number of observables is available.

More standard is the PWA, which has observables constructed from a series of amplitudes or phase shifts with allowed combinations of spin, angular momentum, and isospin. This series must be cut off or augmented with a model for the high angular momentum states. It should be noted that, given a set of partial-wave amplitudes, the amplitudes a through e can be constructed, whereas the existence of amplitudes a through e , at single energy-angle points, is insufficient to construct partial-wave amplitudes.

At low energies, the PWA technique can generate a solution that is stable and requires less than a complete set of measurements. Inelasticity and a growing number of significant phase shifts make this method increasingly model dependent at higher energies.

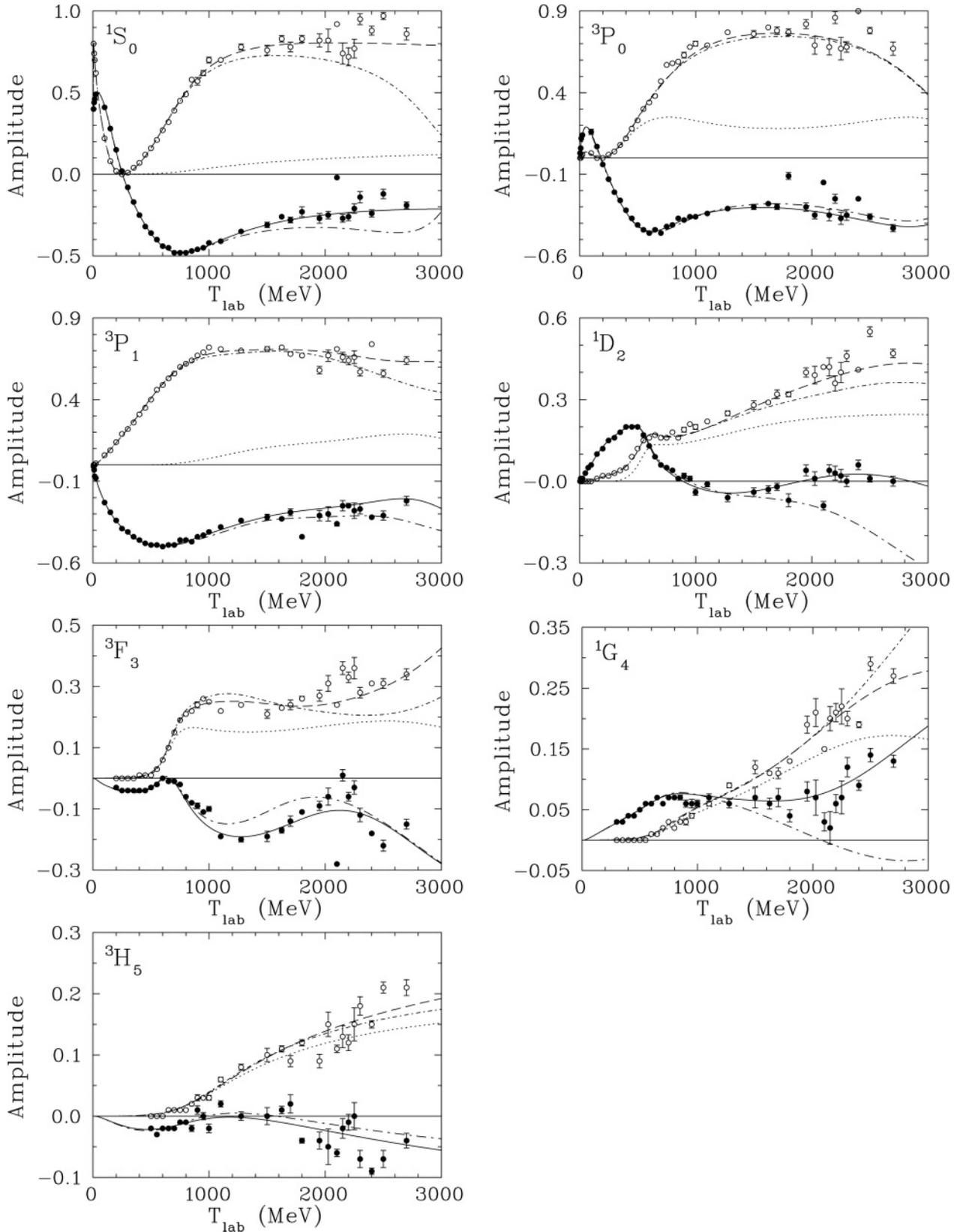


FIG. 1. Dominant isovector partial-wave amplitudes from threshold to $T_p = 3$ GeV. Solid (dashed) curves give the real (imaginary) parts of amplitudes corresponding to the recent SP07 solution. The real (imaginary) parts of single-energy solutions are plotted as filled (open) circles. The previous SP00 solution [1] is plotted with dash-dotted (short dash-dotted) lines for the real (imaginary) parts. The dotted curve gives the unitarity limit $ImT - T^2 - T_{sf}^2$ from SP07, where T_{sf} is the spin-flip amplitude. All amplitudes are dimensionless.

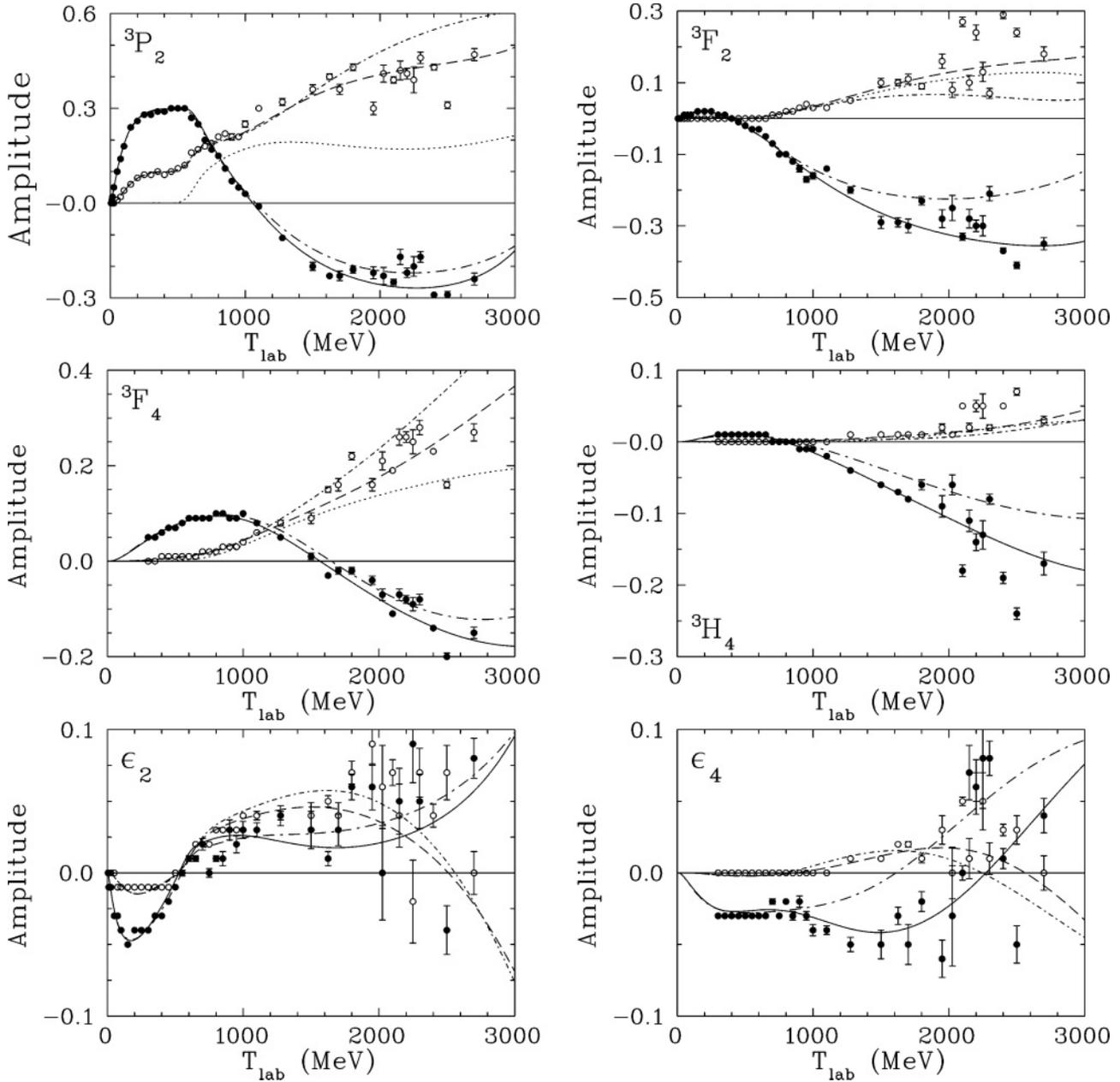


FIG. 2. Notation is as in Fig. 1.

IV. THE FIT TO 3 GeV

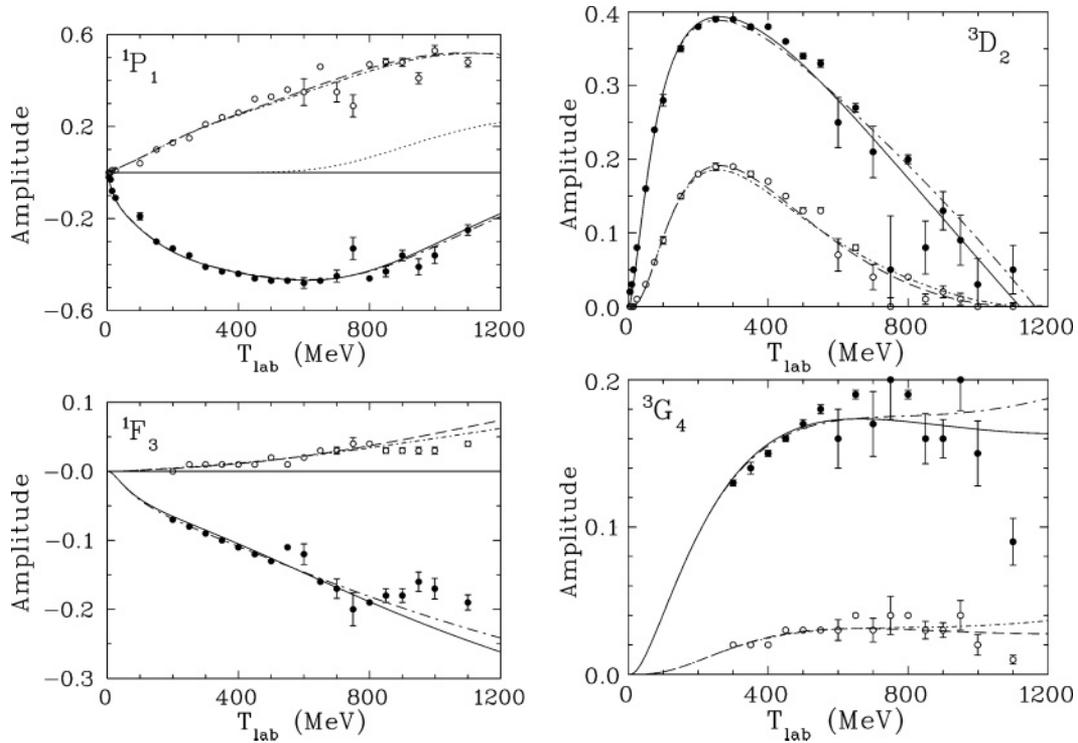
Table III charts the evolution of our NN elastic scattering analyses. The present solution (SP07) and previously published analysis (SP00) are compared, in terms of fit χ^2 , in Table IV. As in previous analyses, we used the systematic uncertainty as an overall normalization factor for angular distributions. The description of this procedure is given in our recent πN PWA article [28].

Below 1 GeV, where the χ^2/data are near unity, the PWA solution has changed little. However, above this energy, qualitative changes can be seen in some amplitudes, in particular, the 1D_2 and 1G_4 . The dominant isovector partial-wave amplitudes are compared in Figs. 1 and 2.

No similarly large changes are evident in the isoscalar waves, which extend to only 1.3 GeV. These amplitudes are displayed in Figs. 3 and 4.

Isovector phase shift parameters are given for the present energy-dependent and single-energy solutions, the energy-dependent SP00 solution, and the single-energy Saclay analysis [2,29] in Fig. 5. Note that a qualitative agreement between the Saclay and the SP00 results for 1D_2 and 1G_4 is absent in SP07.

Significant changes are also evident in comparisons with the Saclay DAR amplitudes. These are plotted in Figs. 6 and 7. Here we have compared both the energy-dependent and the single-energy results of SP00 and SP07 with the Saclay values. The Saclay results, for some amplitudes, show two branches


 FIG. 3. Dominant isoscalar partial-wave amplitudes from threshold to $T_p = 1.2$ GeV. Notation is as in Fig. 1.

for the DAR. In the SP00 publication, we noted that our single-energy and energy-dependent solutions were choosing different branches, particularly for the imaginary parts of a and b . This discrepancy has largely disappeared in SP07. Both the energy-dependent and the single-energy curves now follow a single branch of DAR results.

Some representative plots of the A_{xx} , A_{yy} , and A_{zx} data are given in Figs. 8 and 9. The SP00 solution fails to correctly predict the EDDA A_{xx} data above 1.5 GeV, and this discrepancy motivated the present study. The revised solution, SP07, provides a much improved fit to these data.

 TABLE I. Recent (since our previous publication [1]) pp elastic scattering data up to 3 GeV.

Observable	Energy (MeV)	Angle (deg)	Data	χ^2	Reference
$d\sigma/d\Omega$	240–2577	35–89	2888	3327	[4]
P	437–2492	32–88	1131	1984	[5]
A_{xx}	481–2492	32–87	403	1197	[6]
A_{yy}	481–2490	32–87	403	607	[6]
A_{zx}	481–2490	32–87	403	1333	[6]
A_{yy}	795–2795	47–105	477	671	[7]
P	1000	22–42	8	11	[8]
D_t	1000	22–42	4	13	[8]
A_{yy}	1795–2235	56–102	442	380	[9]

 TABLE II. Recent (since our previous publication [1]) np elastic scattering data up to 1.3 GeV.

Observable	Energy (MeV)	Angle (deg)	Data	χ^2	Reference
$\Delta\sigma_L$	5–20		6	11	[10]
$d\sigma/d\Omega$	10	60–180	6	2	[11]
$\Delta\sigma_T$	11–17		3	7	[10]
$d\sigma/d\Omega$	95	27–150	10	8	[12]
$d\sigma/d\Omega$	95	91–159	9	16	[13]
$d\sigma/d\Omega$	95	43–86	6	23	[14]
$d\sigma/d\Omega$	96	152–175	11	8	[15]
$d\sigma/d\Omega$	96	80–160	9	15	[16]
$d\sigma/d\Omega$	96	20–76	12	25	[17]
$d\sigma/d\Omega$	194	93–177	15	23	[18]
P	260–535	58–162	143	205	[19]
A_{yy}	260–535	58–162	143	351	[19]
A_{zz}	260–535	58–162	144	354	[19]
D	260–535	64–162	73	261	[20]
D_t	260–535	64–120	34	61	[20]
R_t	260–535	64–162	71	97	[20]
A_t	260–535	64–162	71	86	[20]
$N_{0s^{*}sn}$	260–535	64–162	71	52	[20]
$N_{0s^{*}kn}$	260–535	58–162	70	57	[20]
N_{0nkk}	260–535	104–162	40	38	[20]
$D_{0s^{*}0k}$	260–535	64–162	74	270	[20]
P	284–550	113–177	140	157	[21]
σ^{tot}	1300		1	6	[22]

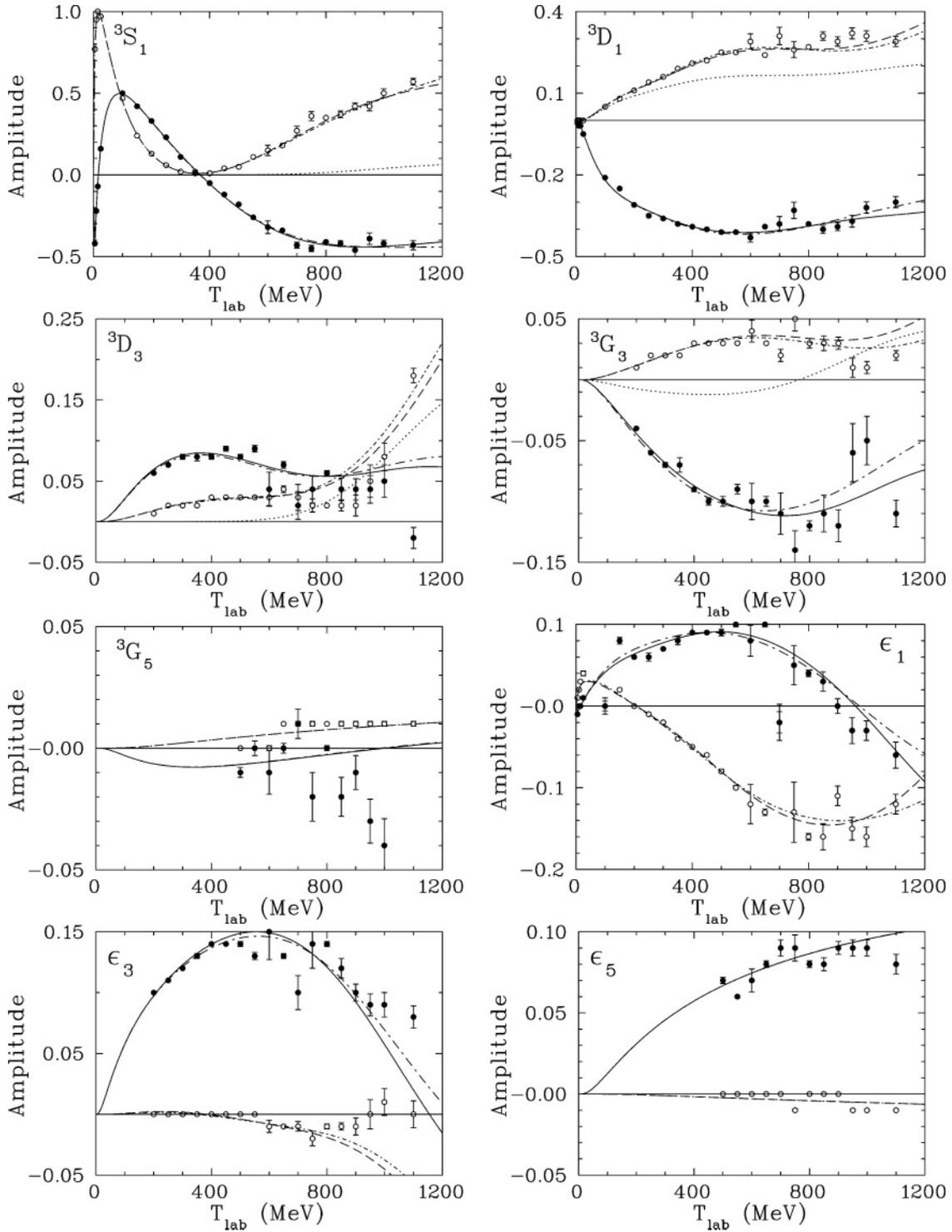


FIG. 4. Notation as in Fig. 3.

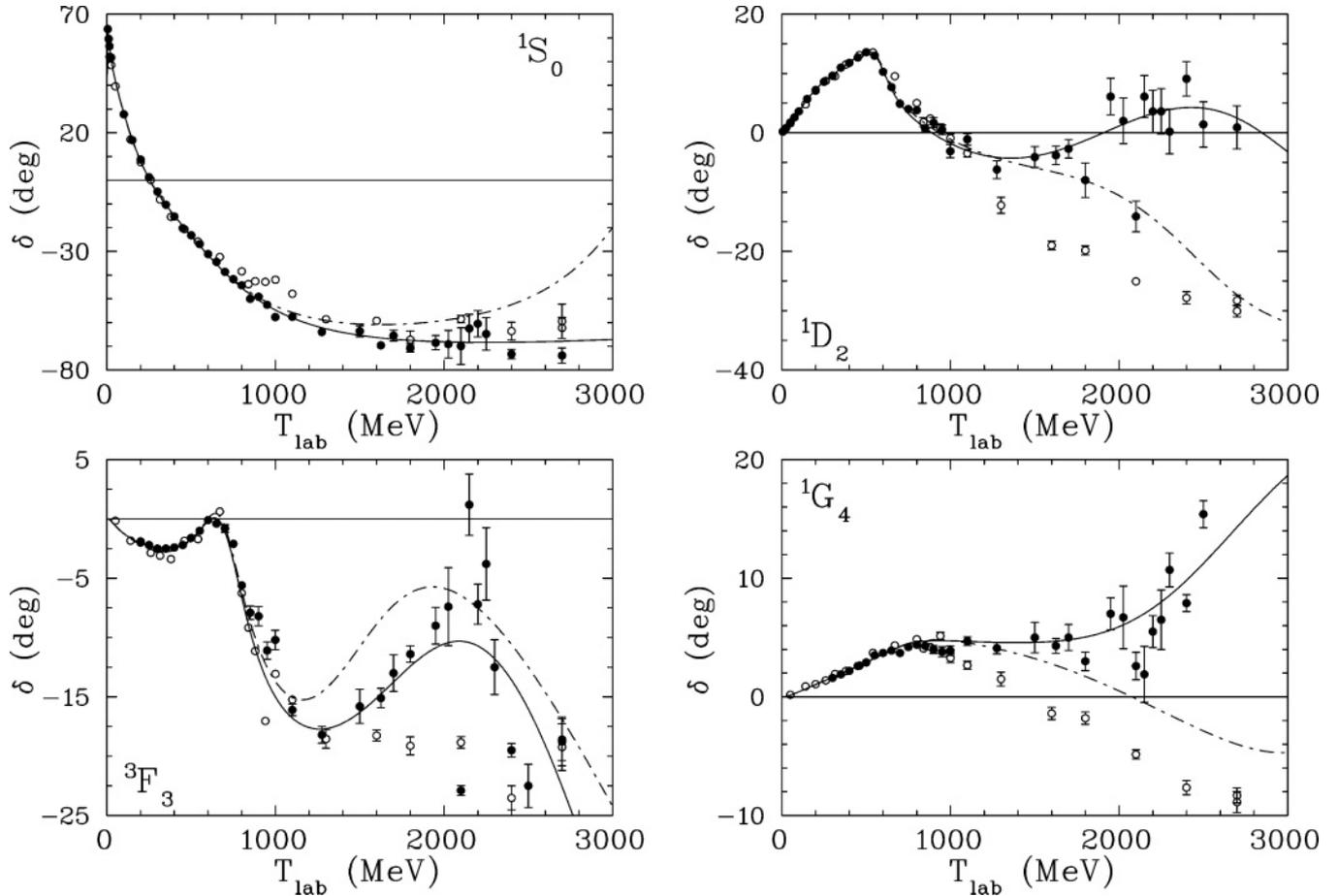


FIG. 5. Phase-shift parameters for dominant isovector partial-wave amplitudes from threshold to $T_p = 3$ GeV. The SP07 and SP00 [1] solutions are plotted as solid and dash-dotted curves, respectively. The GW single-energy solutions and those from Saclay [2,29] are given by filled and open circles, respectively.

TABLE III. Comparison of present SP07 and previous SP00 [1], SM97 [23], SM94 [24], FA91 [25], SM86 [26], and SP82 [27] energy-dependent partial-wave analyses. The χ^2 values for the previous solutions correspond to our published results.

Solution	Range (MeV)	χ^2/pp data	Range (MeV)	χ^2/np data
SP07	0–3000	44463/24916	0–1300	21496/12693
SP00	0–3000	36617/21796	0–1300	18693/11472
SM97	0–2500	28686/16994	0–1300	17437/10854
SM94	0–1600	22371/12838	0–1300	17516/10918
FA91	0–1600	20600/11880	0–1100	13711/7572
SM86	0–1200	11900/7223	0–1100	8871/5474
SP82	0–1200	9199/5207	0–1100	9103/5283

TABLE IV. Comparison of χ^2/data for normalized (Norm) and unnormalized (Unnorm) pp elastic scattering data for the present SP07 and previous SP00 [1] solutions.

Range (MeV)	SP07	SP00
	Norm/Unnorm	Norm/Unnorm
0–500	1.4/4.5	1.3/4.3
500–1000	1.4/9.9	1.3/8.9
1000–1500	2.0/6.7	2.2/6.3
1500–2000	2.0/7.0	2.6/6.4
2000–2500	3.0/8.3	3.7/8.1
2500–3000	3.3/29.6	3.6/50.5

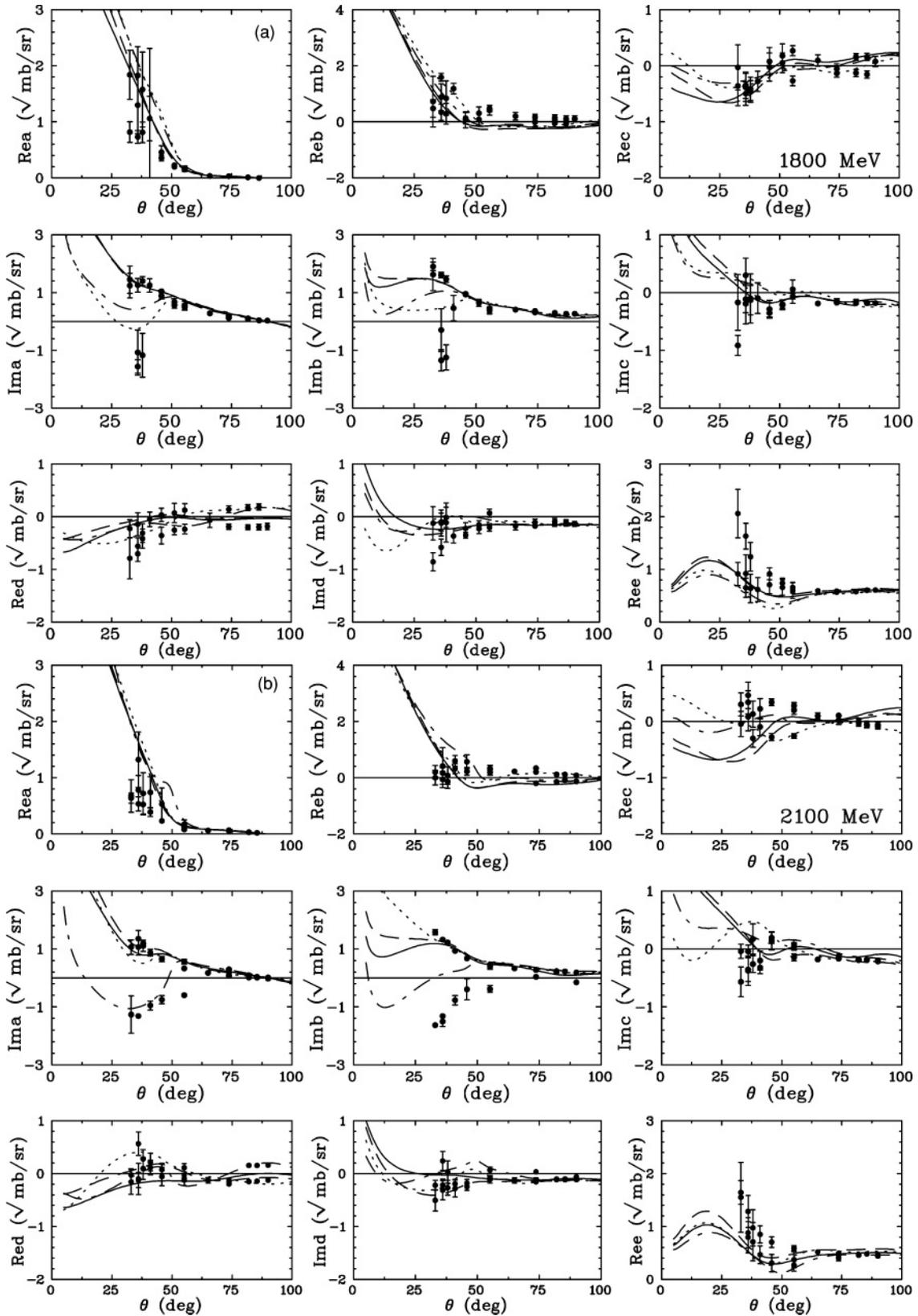


FIG. 6. Direct-reconstruction amplitudes a to e for pp elastic scattering at (a) $T_p = 1.8$ GeV and (b) 2.1 GeV as a function of c.m. scattering angle. The real and imaginary parts of amplitudes [2] are shown. Our SP07 (single-energy) solution is plotted with solid (dashed) lines. Our previous SP00 (single-energy) [1] results are shown with dash-dotted (dotted) lines.

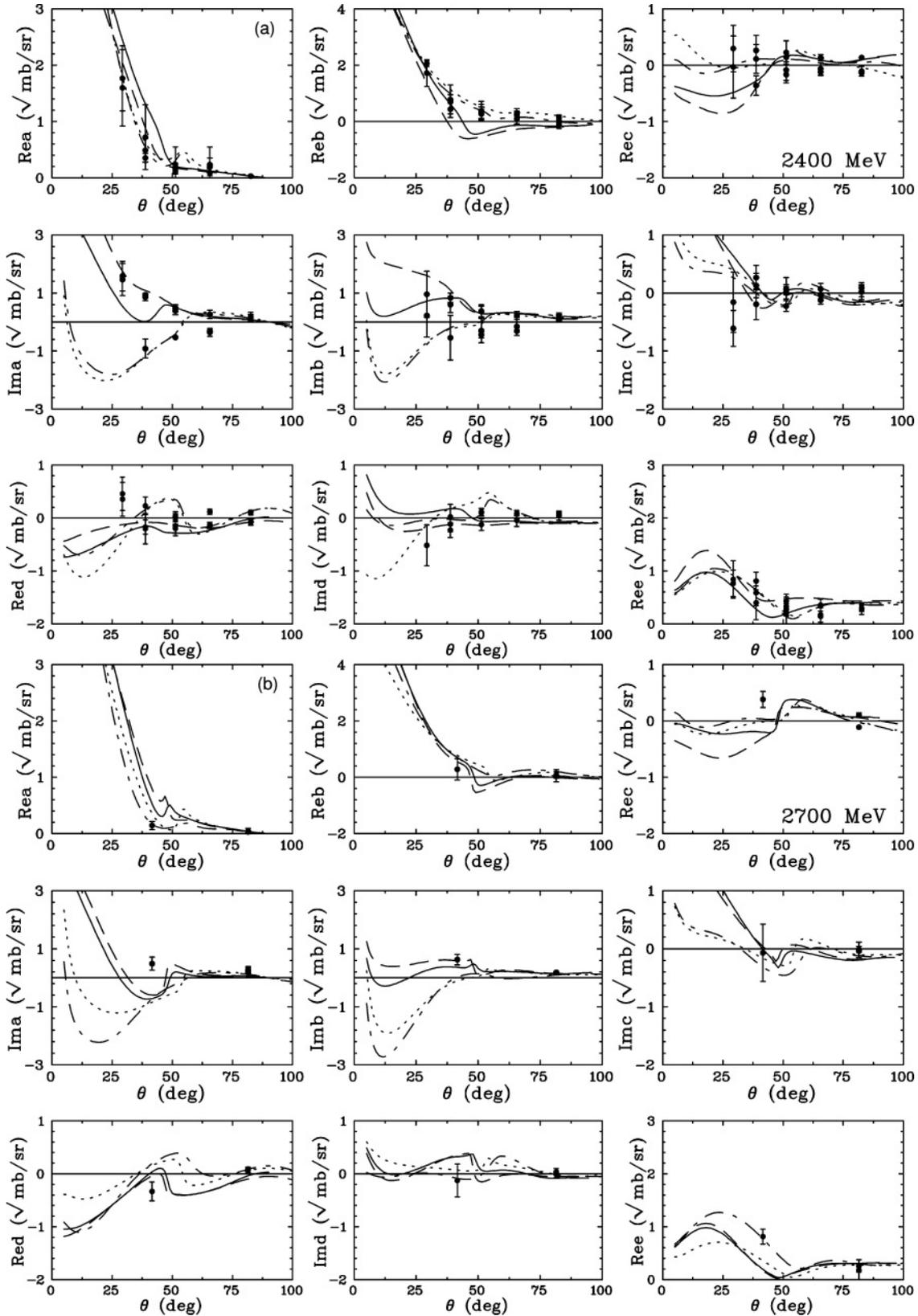


FIG. 7. Direct-reconstruction a to e amplitudes for pp elastic scattering at (a) $T_p = 2.4$ GeV and (b) 2.7 GeV as a function of c.m. scattering angle. Notation is as in Fig. 6.

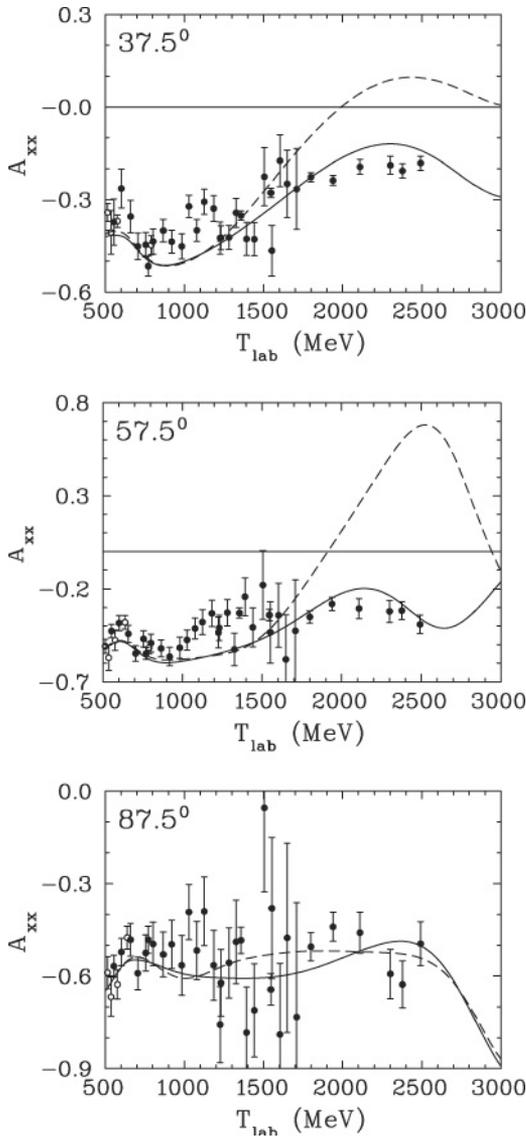


FIG. 8. Excitation function A_{xx} for pp elastic scattering at three c.m. scattering angles. The EDDA Collaboration data (filled circles) are from Ref. [6]. Other previous measurements (for references see SAID database [3]) within a 5° bin are shown as open circles. The SP07 (SP00 [1]) solution is plotted as a solid (dashed) curve.

V. SUMMARY AND CONCLUSIONS

We have generated a new fit to the full database of pp and np elastic scattering data to 3 GeV (1.3 GeV for np data). This updated PWA provides a much improved fit to recent polarized data measured by the EDDA collaboration at COSY. The new fit (SP07) has resolved some ambiguities found in comparing the previous SP00 energy-dependent and single-energy fits to DAR results from Saclay. However, the resulting partial-wave amplitudes, in particular 1D_2 and 1G_4 , have changed dramatically above 1 GeV. Given the impact of these data, and their absence above 2.5 GeV, our solution should be considered at best qualitative between 2.5 and 3 GeV.

Our agreement with the Saclay DAR amplitudes does not imply that agreement is necessary at the PWA level. As we have noted, a PWA requires some model input, whereas the DAR method requires only precise experimental data. As a test, we generated a second solution having 1D_2 and 1G_4 amplitudes initially set to the SP00 values. After the data were fitted, the original SP07 behavior was regained. A fit having the SP00 behavior for 1D_2 and 1G_4 (a second χ^2 minimum) was not found.

Further progress on np scattering will require a program to extend measurements above 1.3 GeV. Such a program has been

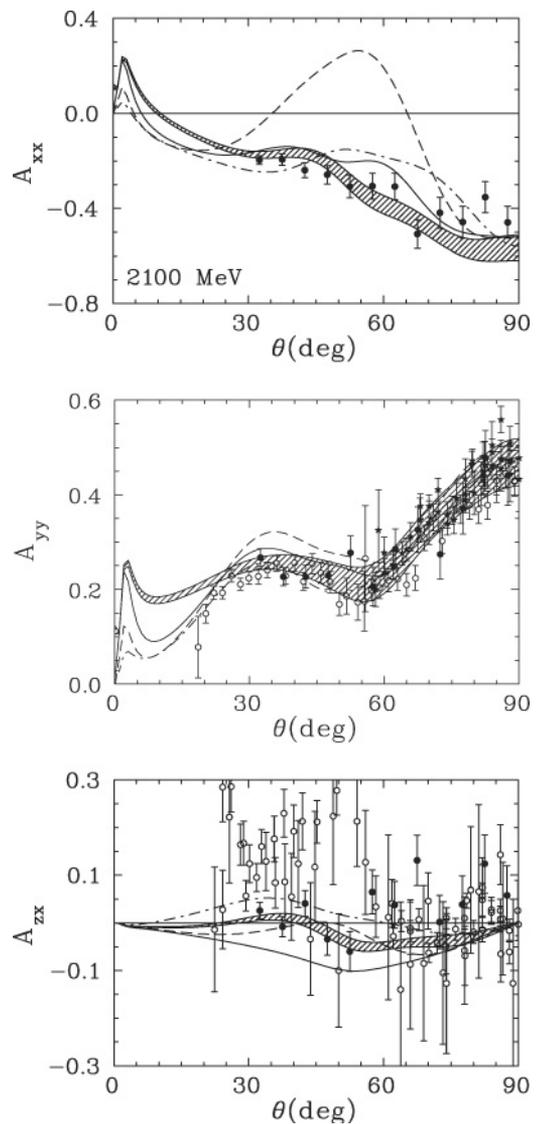


FIG. 9. Angular distributions for pp elastic scattering at $T_p = 2100$ MeV. The EDDA Collaboration data (filled circles) are from Ref. [6]. Recent SATURNE II A_{yy} measurements [7,9] are shown as stars. Other previous measurements (for references see SAID database [3]) within a 10 MeV bin are shown as open circles. The SP07 (SP00 [1]) solution is plotted as a solid (dashed) curve. Saclay direct-reconstruction results [2] are shown as dash-dotted lines. Our single-energy solution is given by a shaded band.

proposed for the Nuclotron at JINR, Dubna, using a polarized deuteron beam and a polarized proton target [30].

ACKNOWLEDGMENTS

The authors express their gratitude to C. E. Allgower, F. Bauer, J. Blomgren, A. D. Carlson, J. Franz, F. Hinterberger, H. Lacker, A. B. Laptev, F. Lehar, P. Mermod, N. Olsson,

W. Scobel, V. I. Sharov, H. Spinka, W. Tornow, S. Vigdor, and V. G. Vovchenko for providing experimental data prior to publication or for clarification of information already published. This work was supported in part by the U.S. Department of Energy under Grant DE-FG02-99ER41110. The authors acknowledge partial support from Jefferson Lab, by the Southeastern Universities Research Association under DOE Contract DE-AC05-84ER40150.

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