

**Structure and decay constant of the  $\rho$  meson within the Bethe-Salpeter equation**Z. G. Wang<sup>1,\*</sup> and S. L. Wan<sup>2</sup><sup>1</sup>*Department of Physics, North China Electric Power University, Baoding 071003, People's Republic of China*<sup>2</sup>*Department of Modern Physics, University of Science and Technology of China, Hefei 230026, People's Republic of China*

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In this article, we study the structure of the  $\rho$  meson in the framework of the coupled rainbow Schwinger-Dyson equation and ladder Bethe-Salpeter equation with a confining effective potential. The  $u$  and  $d$  quark propagators get significantly modified, the mass poles are absent in the timelike region, which implements confinement naturally. The Bethe-Salpeter amplitudes of the  $\rho$  meson center around zero momentum and extend to the energy scale about  $q^2 = 1 \text{ GeV}^2$ , which happens to be the energy scale of chiral symmetry breaking, strong interactions in the infrared region result in bound state. The numerical results of the mass and decay constant of the  $\rho$  meson are in agreement with the experimental data.

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**I. INTRODUCTION**

Low energy nonperturbative properties of quantum chromodynamics (QCD) put forward a great challenge to physicists as the  $SU(3)$  gauge coupling at low energy scale does not allow perturbative calculations. Among the existing theoretical approaches, the coupled rainbow Schwinger-Dyson equation (SDE) and ladder Bethe-Salpeter equation (BSE) are typical. They have been very successful in describing the long distance properties of low energy QCD and the QCD vacuum (for reviews, one can consult Refs. [1–5]). The SDE can naturally embody dynamical chiral symmetry breaking and confinement, which are two crucial features of QCD, although they correspond to two very different energy scales [5,6]. The possible interplay of two dynamics lies in the following two facts: the first one is that at high energy scale the chiral massive quark dresses itself with a gluon cloud and quark-antiquark pairs, and creates a constituent quark mass; the second one is the double role of the light pseudoscalar mesons, as both Nambu-Goldstone bosons and  $q\bar{q}$  bound states. The BSE is a conventional approach in dealing with two-body relativistic bound state problems [7]. From solutions of the BSE, we can obtain useful information about the bound state structure of the mesons and obtain powerful tests of the quark theory.

Many analytical and numerical calculations indicate that the coupled rainbow SDE and ladder BSE with phenomenological potentials can give model independent results and satisfactory values [1–5]. The pseudoscalar mesons, especially the  $\pi$  and  $K$ , have been studied extensively to understand the bound state structures from nonperturbative QCD (for example, Ref. [8], for more literature, one can consult Refs. [1–5]). Chiral symmetry and axial Ward identity play an important role, in chiral limit, the dominating Bethe-Salpeter amplitude (BSA) of the pseudoscalar mesons with the structure  $\gamma_5$  is given by  $B(q^2)/f_P$ , where  $B$  is the scalar part of the quark self-energy and  $f_P$  is the decay constant of the pseudoscalar meson. We can perform many phenomenological analysis without solving the BSE explicitly.

The situation is much worse for the vector mesons. There is a conserved vector current and the corresponding Ward identity relates the longitudinal (not transverse) part of the vector vertex with the quark propagator. We cannot obtain dynamical insight into the relevant degrees of freedom without solving the BSE directly. Furthermore, it is more difficult to solve the vector BSE than the pseudoscalar BSE due to a larger number of covariants and higher masses, which require extrapolation into larger domain of the complex  $q^2$  plane [9].

The often used effective potential models are confining Dirac  $\delta$  function potential, Gaussian distribution potential, and flat bottom potential (FBP) [10–12]. The FBP is a sum of Yukawa potentials, which not only satisfies gauge invariance, chiral invariance and fully relativistic covariance, but also suppresses the singular point which the Yukawa potential has. It displays dynamical chiral symmetry breaking and confinement [13], and works well in understanding the QCD vacuum (for example, the quark condensate, mixed condensate and vacuum susceptibility) as well as the meson structures (for example, the electromagnetic form factors, radius and decay constants) [12,14,15].

In this article, we take the point of view that the  $\rho$  meson is the  $q\bar{q}$  bound state, and study its structure and decay constant in the framework of the coupled rainbow SDE and ladder BSE with the infrared modified FBP, which take advantages of both the Gaussian distribution potential and the FBP.

The article is arranged as the following. We introduce the infrared modified FBP in Sec. II. In Secs. III, IV, and V, we solve the coupled rainbow SDE and ladder BSE, study analytical property of the quark propagators, finally obtain the mass and decay constant of the  $\rho$  meson. Section VI is reserved for conclusion.

**II. INFRARED MODIFIED FLAT BOTTOM POTENTIAL**

The present techniques in QCD calculation cannot give satisfactory analytical results for the infrared behavior of the two-point Green's function of the gluon. Infrared enhanced effective potential models have been phenomenologically quite successful [10–12].

\*Corresponding author; wangzgyiti@yahoo.com.cn

One can use a Gaussian distribution function to represent the infrared behavior of the two-point Green's function of the gluon,<sup>10</sup>

$$G_1(q^2) = \frac{\varpi^2}{\Delta^2} e^{-\frac{q^2}{\Delta}}, \quad (1)$$

which determines the quark-antiquark interaction through a strength parameter  $\varpi$  and a range parameter  $\Delta$  [11]. This form is inspired by the Dirac  $\delta$  function potential (in other words the infrared dominated potential) used in Ref. [10], which it approaches in limit  $\Delta \rightarrow 0$ . The integral  $\int d^4q q^{2n} G_1(q^2)$  is finite for spacelike squared momentum  $q^2$ . Such an infrared behavior can result in large dressing for the quark's Schwinger-Dyson functions (SDFs)  $A(q^2)$  and  $B(q^2)$  near  $q^2 = 0$ , the curves at the infrared region may be steep enough to forbid extrapolation to deep timelike region. When we introduce an extra factor,  $q^2/\Delta$ , the modified gaussian distribution can result in more flat curve near zero momentum. Furthermore, systematic studies with the coupled SDEs of the quark, gluon and ghost indicate this type of behavior at about  $q^2 = 0$  [16]. We use the following modified gaussian distribution to represent the infrared behavior of the two-point Green's function of the gluon [11],

$$\frac{\varpi^2}{\Delta^2} e^{-\frac{q^2}{\Delta}} \rightarrow \frac{\varpi^2}{\Delta^2} \frac{q^2}{\Delta} e^{-\frac{q^2}{\Delta}}. \quad (2)$$

In numerical calculation, the range parameter  $\Delta$  is taken to be  $\sqrt{\Delta} \approx 0.62$  GeV, the Gaussian type of function  $q^2 e^{-\frac{q^2}{\Delta}}$  centers around  $q = 0.6$  GeV, and extends to about  $q = 1.2$  GeV. Systematic studies with the coupled SDEs indicate that the nonperturbative gluon propagator is greatly enhanced at about  $q = 1$  GeV [16]. The value  $\sqrt{\Delta} \approx 0.62$  GeV is more reasonable than the value  $\sqrt{\Delta} = 0.3$  GeV taken from Ref. [9].

For intermediate momentum, we take the FBP as the best approximation and neglect the contribution from perturbative QCD calculations as the strong coupling constant at high energy scale is very small. The FBP is a sum of Yukawa potentials which is an analogy to exchange of a series of particles and ghosts with different masses,

$$G_2(q^2) = \sum_{j=0}^n \frac{a_j}{q^2 + (N + j\eta)^2}, \quad (3)$$

where  $N$  stands for the minimum value of the masses,  $\eta$  is their mass difference, and  $a_j$  is their relative coupling constant. Definition of momentum regions between infrared and intermediate momentum is about  $\Lambda_{\text{QCD}} = 200$  MeV, which is set up naturally by the minimum value of the masses  $N = 1\Lambda_{\text{QCD}}$ . Certainly, there are some overlaps between those regions, in this way, we can guarantee continuity for the momentum. The FBP at energy  $N + j\rho$  with  $j > 3$  extends to the perturbative region and exhibits some perturbative

characters. The infrared modified FBP is supposed to embody a great deal of physical information about all momentum regions.

Due to the particular condition we take for the FBP, there is no divergence in solving the SDE. In its three dimensional form, the FBP takes the following form:

$$V(r) = - \sum_{j=0}^n a_j \frac{e^{-(N+j\eta)r}}{r}. \quad (4)$$

In order to suppress the singular point at  $r = 0$ , we take the following conditions:

$$\begin{aligned} V(0) &= \text{constant}, \\ \frac{dV(0)}{dr} &= \frac{d^2V(0)}{dr^2} = \dots = \frac{d^nV(0)}{dr^n} = 0. \end{aligned} \quad (5)$$

The  $a_j$  can be determined by solving equations inferred from the flat bottom condition. As in previous literature [12–15],  $n$  is set to be 9. The gluon propagator can be approximated by

$$G(q^2) = G_1(q^2) + G_2(q^2). \quad (6)$$

### III. SCHWINGER-DYSON EQUATION

The SDE, in effect the functional Euler-Lagrange equation of quantum field theory, provides a natural framework for studying the nonperturbative properties of quark and gluon Green's functions. By studying the evolution behavior and analytic structure of the dressed quark propagator, one can obtain valuable information about dynamical chiral symmetry breaking phenomenon and confinement. In the rainbow approximation, the SDE takes the following form:

$$\begin{aligned} S^{-1}(p) &= i\gamma \cdot p + \hat{m}_{u,d} + 4\pi \\ &\times \int \frac{d^4k}{(2\pi)^4} \gamma_\mu \frac{\lambda^a}{2} S(k) \gamma_\nu \frac{\lambda^a}{2} G_{\mu\nu}(k-p), \end{aligned} \quad (7)$$

where

$$\begin{aligned} S^{-1}(p) &= iA(p^2)\gamma \cdot p + B(p^2) \\ &\equiv A(p^2)[i\gamma \cdot p + m(p^2)], \end{aligned} \quad (8)$$

$$G_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) G(k^2). \quad (9)$$

In this article, we assume that a Wick rotation to Euclidean variables is allowed, and rotate  $p, k$  into the Euclidean region analytically. Alternatively, one can derive the SDE from Euclidean path-integral formulation of the theory, and avoid possible difficulties in performing the Wick rotation [17]. The analytical structures of quark propagators have interesting information about confinement, we will revisit this subject in Sec. V.

<sup>10</sup>In this article, we use the metric  $\delta_{\mu\nu} = (1, 1, 1, 1)$ ,  $\{\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu\} = 2\delta_{\mu\nu}$ , the coordinate  $x_\mu = (it, \vec{x})$ , the momentum  $p_\mu = (iE, \vec{p})$ .

#### IV. BETHE-SALPETER EQUATION

The BSE is a conventional approach in dealing with two-body relativistic bound state problems [7]. Precise knowledge about the quark structure of the  $\rho$  meson can result in better understanding of its property. In the following, we write down the ladder BSE of the  $\rho$  meson:

$$S^{-1} \left( q + \frac{P}{2} \right) \chi(q, P) S^{-1} \left( q - \frac{P}{2} \right) = -\frac{16\pi}{3} \int \frac{d^4 k}{(2\pi)^4} \gamma_\mu \chi(k, P) \gamma_\nu G_{\mu\nu}(q - k), \quad (10)$$

where  $P_\mu$  is four-momentum of the center of mass of the  $\rho$  meson,  $q_\mu$  is relative four-momentum between the two quarks,  $\gamma_\mu$  is the bare quark-gluon vertex, and  $\chi(q, P)$  is the BSA of the  $\rho$  meson.

We can perform the Wick rotation analytically and continue  $q, k$  into Euclidean region.<sup>2</sup> The Euclidean BSA of the  $\rho$  meson can be decomposed as

$$\chi(q, P) = \epsilon_\mu \chi_\mu(q, P), \quad (11)$$

$$\begin{aligned} \chi_\mu(q, P) = & \left\{ \gamma_\mu - \frac{P_\mu P}{P^2} \right\} \{ i F_0 + P F_1 - \not{q} F_2 + i [P, \not{q}] F_3 \} \\ & + \left\{ q_\mu - \frac{P_\mu q \cdot P}{P^2} \right\} \{ F_2 + 2i P F_3 \} \\ & + \left\{ q_\mu - \frac{P_\mu q \cdot P}{P^2} \right\} \{ F_4 + i P F_5 - i \not{q} F_6 + [P, \not{q}] F_7 \}, \end{aligned} \quad (12)$$

due to Lorentz covariance. Here  $\epsilon_\mu$  is the polarization vector of the  $\rho$  meson [18]. The BSAs  $F_i(q, q \cdot P, P)$  can be expanded in terms of Tchebychev polynomials  $T_n^{\frac{1}{2}}(\cos \theta)$ ,

$$F_i(q, q \cdot P, P) = \sum_0^\infty i^n F_i^n(q, P) q^n P^n T_n^{\frac{1}{2}}(\cos \theta), \quad (13)$$

where  $\theta$  is the included angle between  $q_\mu$  and  $P_\mu$ . It is impossible to solve an infinite series of coupled equations of the  $F_i^n$ , we have to make truncation in one or the other ways. Numerical calculations indicate that taking only some terms

<sup>2</sup>To avoid any possible difficulties in performing the Wick rotation, we can derive both the SDE and BSE from Euclidean path-integral formulation of the theory directly, then continue the four-momentum of the center of mass of the  $\rho$  meson into Minkowski spacetime analytically,  $P^2 = -m_\rho^2$ . As far as only the numerical values are concerned, the two approaches are equal.

with  $n = 0, 1, 2$  can give satisfactory results,<sup>3</sup>

$$\begin{aligned} \chi_\mu(q, P) = & \left\{ \gamma_\mu - \frac{P_\mu P}{P^2} \right\} \{ i F_0^0 + i [4(q \cdot P)^2 - q^2 P^2] \\ & \times F_0^2 + P F_1^0 - \not{q} q \cdot P F_2^1 + i [P, \not{q}] F_3^0 \} \\ & + \left\{ q_\mu - \frac{P_\mu q \cdot P}{P^2} \right\} \{ q \cdot P F_2^1 + 2i P F_3^0 \} \\ & + \left\{ q_\mu - \frac{P_\mu q \cdot P}{P^2} \right\} \{ F_4^0 + i P q \cdot P F_5^1 \\ & - i \not{q} F_6^0 + [P, \not{q}] F_7^0 \}. \end{aligned} \quad (14)$$

In solving the BSE, it is important to translate the BSAs  $F_i^n$  into the same dimension of mass to facilitate the calculation

$$\begin{aligned} F_0^0 & \rightarrow \Lambda^0 F_0^0, & F_0^2 & \rightarrow \Lambda^2 F_0^2, & F_1^0 & \rightarrow \Lambda^1 F_1^0, \\ F_2^1 & \rightarrow \Lambda^3 F_2^1, & F_3^0 & \rightarrow \Lambda^2 F_3^0, & F_4^0 & \rightarrow \Lambda^1 F_4^0, \\ F_5^1 & \rightarrow \Lambda^4 F_5^1, & F_6^0 & \rightarrow \Lambda^2 F_6^0, & F_7^0 & \rightarrow \Lambda^3 F_7^0, \\ q & \rightarrow q/\Lambda, & P & \rightarrow P/\Lambda, \end{aligned}$$

where  $\Lambda$  is a quantity with dimension of mass. The ladder BSE of the  $\rho$  meson can be projected into the following nine coupled integral equations:

$$\sum_j H(i, j) F_j^{0,1,2}(q, P) = \sum_j \int d^4 k K(i, j), \quad (15)$$

where  $H(i, j)$  and  $K(i, j)$  are  $9 \times 9$  matrices, the corresponding ones for the pseudoscalar mesons are  $4 \times 4$  matrices [14]. The analytical expressions of the matrix elements  $H(i, j)$  and  $K(i, j)$  are cumbersome and will take up more than seven pages, and not shown explicitly for simplicity.

<sup>3</sup>We can borrow some ideas from the twist-2 light-cone distribution amplitudes  $\phi(\mu, u)$  of the pseudoscalar mesons  $\pi$  and  $K$ , where  $u$  is momentum fraction of the quark, and  $\mu$  is the energy scale. The  $\phi(\mu, u)$  is always expanded in terms of Gegenbauer polynomials,  $\phi(\mu, u) = 6u(1-u)\{1 + a_1(\mu)C_1^{3/2}(2u-1) + a_2(\mu)C_2^{3/2}(2u-1) + a_4(\mu)C_4^{3/2}(2u-1) + \dots\}$ , where  $C_1^{3/2}(2u-1)$ ,  $C_2^{3/2}(2u-1)$  and  $C_4^{3/2}(2u-1)$  are Gegenbauer polynomials, and  $a_i(\mu)$  are nonperturbative coefficients. The coefficients  $a_i(\mu)$  can be estimated with the QCD sum rules approach, for large  $i$ , the  $a_i(\mu)$  involve high dimension vacuum condensates which are known poorly. In general, one can retain only the first few terms and fit them with experimental data, and the truncated light-cone distribution amplitude  $\phi(\mu, u)$  always gives satisfactory results [19]. In this article, we retain only some terms with  $n = 0, 1$  and  $2$  in expansion with Tchebychev polynomials  $T_n^{1/2}(\cos \theta)$ , the contributions from other terms are supposed to be small and neglected here. If the contributions from the neglected terms are large, they cannot decouple approximately from the BSEs and warrant the BSEs have reasonable solution. The numerical results indicate that the dominating contribution comes from the  $F_0^0$ , the sub-dominating contributions come from the  $F_1^0$  and  $F_4^0$ , and  $F_0^0 \gg q^2 m_\rho^2 F_0^2$ , we expect the truncation is reasonable. It is obvious that if we take into account more terms in expansion, more accurate values can be obtained. However, the analytical expressions of the coupled BSEs become very clumsy and are beyond capability of our computer in numerical calculations.

We can introduce a parameter  $\lambda(P^2)$  and solve above equations as an eigenvalue problem. If there really exist a bound state in the vector channel, the mass of the  $\rho$  meson can be determined by the condition  $\lambda(P^2 = -m_\rho^2) = 1$ ,

$$\sum_j H(i, j) F_j^{0,1,2}(q, P) = \lambda(P^2) \sum_j \int d^4k K(i, j). \quad (16)$$

The matrix elements  $H(i, j)$  are functions of the quark's SDFs  $A(q^2 + \frac{P^2}{4} \pm q \cdot P)$  and  $B(q^2 + \frac{P^2}{4} \pm q \cdot P)$ . The relative four-momentum  $q_\mu$  is a quantity in Euclidean spacetime, while the center of mass four-momentum  $P_\mu$  is a quantity in Minkowski spacetime,  $q \cdot P$  varies throughout a complex domain. We can expand the  $A$  and  $B$  in terms of Taylor series of  $q \cdot P$  to avoid solving the SDE with complex values of quark momentum, for example,

$$A\left(q^2 + \frac{P^2}{4} \pm q \cdot P\right) = A\left(q^2 + \frac{P^2}{4}\right) \pm A\left(q^2 + \frac{P^2}{4}\right)' q \cdot P + \dots$$

The other problem is that we cannot solve the SDE in the timelike region. The two-point Green's function of the gluon cannot be exactly inferred from the  $SU(3)$  gauge theory even in small spacelike momentum region. We can extrapolate the values of the  $A(q^2)$  and  $B(q^2)$  from the spacelike region smoothly to the timelike region  $q^2 = -\frac{m_\rho^2}{4} \approx -0.15 \text{ GeV}^2$  with suitable polynomial. The masses of the vector mesons are larger than the pseudoscalar mesons. So it is very difficult to extrapolate the values to the deep timelike region. We must be careful in choosing the polynomial to avoid possible violation of confinement in sense of appearance of pole masses  $q^2 = -m^2(q^2)$  in the timelike region [11,14]. This requires a certain amount of fine tuning. Furthermore, if the curves of the  $A(q^2)$  and  $B(q^2)$  are very steep near  $q^2 = 0$ , very large values of the  $A(q^2 + \frac{P^2}{4})$  and  $B(q^2 + \frac{P^2}{4})$  are obtained in the timelike region ( $q^2 + \frac{P^2}{4} < 0$ ), the solution of the BSE in the small spacelike region ( $0 \leq q^2 < \frac{m_\rho^2}{4}$ ) is not reasonable. In this article, we use the modified Gaussian distribution rather than the Gaussian distribution to modify the infrared behavior of the two-point Green's function of the gluon to outcome the above difficulties.

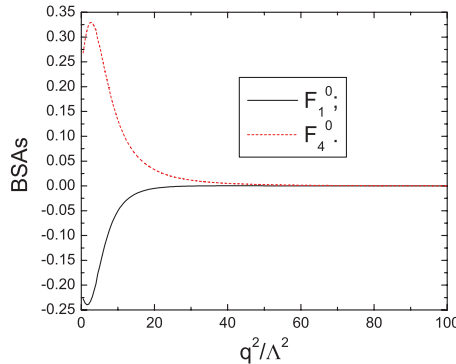
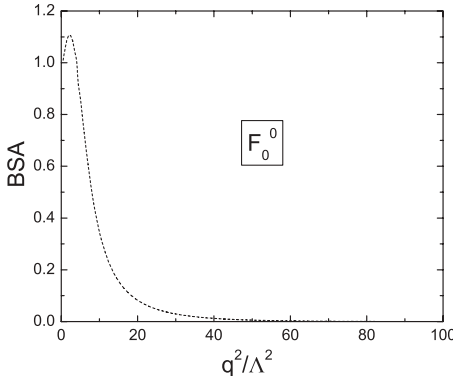


FIG. 1. (Color online) The dominating and subdominating components of the BSAs.

Finally we write down normalization condition for the BSAs of the  $\rho$  meson,

$$\frac{N_c}{3} \int \frac{d^4q}{(2\pi)^4} \text{Tr} \left\{ \bar{\chi} \frac{\partial S_+^{-1}}{\partial P_\mu} \chi(q, P) S_-^{-1} + \bar{\chi} S_+^{-1} \chi(q, P) \frac{\partial S_-^{-1}}{\partial P_\mu} \right\} = 2P_\mu, \quad (17)$$

where  $\bar{\chi} = \gamma_4 \chi^\dagger \gamma_4$ ,  $S_+ = S(q + \frac{P}{2})$  and  $S_- = S(q - \frac{P}{2})$ .

## V. COUPLED RAINBOW SDE AND LADDER BSE

Now we study the coupled equations of the rainbow SDE and ladder BSE of the  $\rho$  meson.

In order to demonstrate confinement of quarks, we take the Fourier transform with respect to Euclidean time  $T$  for the scalar part of the quark propagator [1,3,20],

$$S_s^*(T) = \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4 T} \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)} \Big|_{\vec{q}=0}, \quad (18)$$

where the three-vector part of  $q_\mu$  is set to zero. If the  $S(q)$  has a mass pole at  $q^2 = -m^2(q^2)$  in the real timelike region, the Fourier transformed  $S_s^*(T)$  would fall off as  $e^{-mT}$  for large  $T$ . In numerical calculations, for small  $T$ , the values of  $S_s^*$  are positive, and decrease rapidly to zero with increase of  $T$ , which are compatible with the result (tendency of curve with respect to  $T$ ) from lattice simulations [21]. For large  $T$ , the values of  $S_s^*$  are negative, except for occasionally a very small fraction of positive values. The negative values of  $S_s^*$  indicate an explicit violation of axiom of reflection positivity [22], the quarks are not physical observable.

The  $u$  and  $d$  quarks have small current masses, the dressing or renormalization is large and the curves of the SDFs are steep, which indicates dynamical chiral symmetry breaking phenomenon. At zero momentum,  $m_u(0) = m_d(0) = 0.51 \text{ GeV}$ , the Euclidean constituent quark masses are  $m_u(m_u) = m_d(m_d) = 0.42 \text{ GeV}$ , which are compatible with the constituent quark masses in literature. From the solutions of BSEs of the  $\rho$  meson as an eigenvalue problem, we obtain the mass

$$m_\rho = 770 \text{ MeV}. \quad (19)$$

It is obvious

$$m_u(m_u) + m_d(m_d) > m_\rho. \quad (20)$$

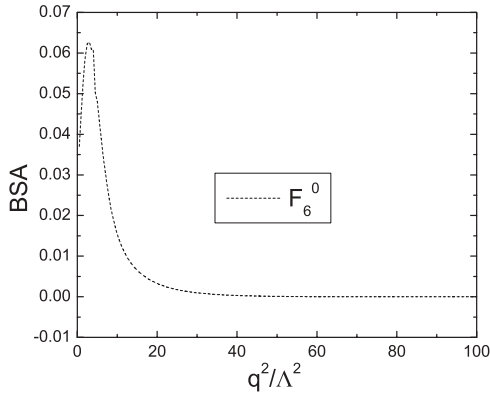


FIG. 2. One of the small components of the BSAs.

The attractive interaction between the quark and antiquark in the infrared region can result in bound state.

The dominating and subdominating components of the BSAs are the  $F_0^0$ ,  $F_1^0$ , and  $F_4^0$ , the six components  $F_0^2$ ,  $F_2^1$ ,  $F_3^0$ ,  $F_5^1$ ,  $F_6^0$ , and  $F_7^0$  are very small and of minor importance. From plotted BSAs (see Figs. 1 and 2 as examples), we can see that the BSAs of the  $\rho$  meson have analogical momentum dependence, while the quantitative values are different from each other. Just like the  $\bar{q}q$ ,  $\bar{q}Q$ , and  $\bar{Q}Q$  pseudoscalar mesons [14], the BSAs of the  $\rho$  meson center around zero momentum and extend to the energy scale about  $q^2 = 1 \text{ GeV}^2$ , which happens to be the energy scale of chiral symmetry breaking. The strong interactions in the infrared region result in bound state. The BSAs of the  $\rho$  meson can give satisfactory value for the decay constant, which is defined by

$$f_\rho m_\rho \epsilon_\mu = \langle 0 | \bar{q} \gamma_\mu q | \rho(P) \rangle,$$

$$= N_c \int \text{Tr}[\gamma_\mu \chi(k, P)] \frac{d^4 k}{(2\pi)^4}. \quad (21)$$

Carrying out trace explicitly, we can see that only the BSAs  $F_0^0$  and  $F_6^0$  are relevant to the decay constant. The  $F_6^0$  is numerically very small (see Fig. 2), the dominating contribution comes from the  $F_0^0$ . Finally we obtain the value

of the decay constant

$$f_\rho = 223 \text{ MeV}, \quad (22)$$

which is in agreement with the experimental data.

In calculation, the input parameters are taken as  $N = 1.0\Lambda$ ,  $V(0) = -14.0\Lambda$ ,  $\eta = 5.0\Lambda$ ,  $\hat{m}_u = \hat{m}_d = 6 \text{ MeV}$ ,  $\Lambda = 200 \text{ MeV}$ ,  $\varpi = 1.3 \text{ GeV}$ , and  $\Delta = 0.39 \text{ GeV}^2$ .

## VI. CONCLUSION

In this article, we study structure of the  $\rho$  meson in the framework of the coupled rainbow SDE and ladder BSE with the confining effective potential. By solving the coupled rainbow SDE and ladder BSE as an eigenvalue problem numerically, we obtain the SDFs, BSAs, mass, and decay constant of the  $\rho$  meson.

The dressing (or renormalization) for the SDFs of the  $u$  and  $d$  quarks is large and the curves are steep, which indicate dynamical chiral symmetry breaking phenomenon. The mass poles are absent in the timelike region, which implements confinement naturally. The BSAs of the  $\rho$  meson have analogical momentum dependence, while the quantitative values are different from each other, where the dominating and subdominating components are the  $F_0^0$ ,  $F_1^0$ , and  $F_4^0$ , other six components  $F_0^2$ ,  $F_2^1$ ,  $F_3^0$ ,  $F_5^1$ ,  $F_6^0$ ,  $F_7^0$  are of minor importance. The BSAs center around zero momentum and extend to the energy scale about  $q^2 = 1 \text{ GeV}^2$ , which happens to be the energy scale of chiral symmetry breaking. Strong interactions in the infrared region result in bound state.

The numerical results of the mass and decay constant of the  $\rho$  meson are in agreement with the experimental data. Once satisfactory SDFs and BSAs of the  $\rho$  meson are obtained, we can use them in other phenomenological analysis.

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