Subtleties of Lorentz invariance and shapes of the nucleon

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We study the effects of Lorentz invariance on relativistic constituent quark model wave functions. The model nucleon wave function of Gross *et al.* (nucl-th/0606029) is constructed such that there is no orbital angular momentum and that the spin-dependent density is spherical. This model wave function is claimed to be manifestly covariant. We consider two possible interpretations of the nucleon wave function in an arbitrary reference frame. In the first, the seeming covariance of the matrix elements of the electromagnetic current arises from using the Breit frame. Matrix elements have a different appearance in any other frame. In the second interpretation, the electromagnetic current is covariant yet it is not consistent with the general structure required by QFT (e.g., the wave function of the incoming nucleon depends on the momentum of the outgoing nucleon and vice versa).

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I. INTRODUCTION

Recent Jefferson Laboratory data on electromagnetic (em) form factors of the nucleon have created much theoretical interest. The key finding is that the ratio of the proton's G_E/G_M falls rapidly with increasing Q^2 [1,2]. But new results for the neutron electric and magnetic form factors have been or are about to be obtained; see the reviews [3–5].

It was argued [6] that reproducing the measured ratio G_E/G_M ratio requires a relativistic treatment that includes the effects of the quark's nonzero orbital angular momentum. Miller [7] introduced the idea of using the rest-frame to rest-frame matrix elements of a spin-dependent charge density operator to exhibit the influence of the orbital angular momentum. In particular, (for a model without explicit gluons) the probability for a quark to have a given momentum $\mathbf{K}$ and a given direction of spin, $\mathbf{n}$ is given by [7,8]

$$\widehat{\rho}_{\mathcal{O}}(\mathbf{K},\mathbf{n}) = \int \frac{d^3r}{(2\pi)^3} e^{i\mathbf{K}\cdot\mathbf{r}} \bar{\psi}(\mathbf{r})\mathcal{O}\frac{1}{2}(\gamma^0 + \boldsymbol{\gamma}\cdot\mathbf{n}\gamma_5)\psi(\mathbf{0}), \tag{1}$$

where $\mathcal{O}$ is $\widehat{Q}/e$, the quark charge operator in units of the proton charge for the spin-dependent charge density or $\mathcal{O} = 1$ for the spin-dependent matter density. The matrix element of the operator $\widehat{\rho}_{\mathcal{O}}(\mathbf{K},\mathbf{n})$ gives the spin-dependent matter densities. The quark field operators $\bar{\psi}(\mathbf{r})$, $\bar{\psi}(\mathbf{0})$ are evaluated at equal time. The rest-frame matrix element of this density operator in a nucleon state of definite total angular momentum defined by the unit vector $\mathbf{s}$, $|\Psi_\mathbf{s}\rangle$ is

$$\rho_{\mathcal{O}}(\mathbf{K},\mathbf{n},\mathbf{s}) \equiv \langle\Psi_\mathbf{s}|\widehat{\rho}_{\mathcal{O}}(\mathbf{K},\mathbf{n})|\Psi_\mathbf{s}\rangle, \tag{2}$$

where the subscript $\mathcal{O} = Q$ or $\mathcal{O} = 1$ specifies the operator used in Eq. (1). The most general shape of the proton, obtained if parity and rotational invariance are upheld, can be written as

$$\rho_{\mathcal{O}}(\mathbf{K},\mathbf{n},\mathbf{s}) = A_{\mathcal{O}}(\mathbf{K}^2) + B_{\mathcal{O}}(\mathbf{K}^2)\mathbf{n}\cdot\mathbf{s} + C_{\mathcal{O}}(\mathbf{K}^2)\big(\mathbf{n}\cdot\mathbf{K}\,\mathbf{s}\cdot\mathbf{K} - \tfrac{1}{3}\mathbf{n}\cdot\mathbf{K}\,\mathbf{K}^2\big), \tag{3}$$

with the last term generating the nonspherical shape. The effects of nonvanishing orbital angular momentum cause the matrix elements of the spin-dependent density operator, Eq. (2) to be nonspherical and yield a nonzero value of the coefficient $C_{\mathcal{O}}(\mathbf{K}^2)$. Although no experiment has been constructed to measure the spin-dependent density, this quantity can be evaluated by using the techniques of lattice QCD and has been measured in condensed matter physics [9].

Gross and Agbakpe [10] constructed a relativistic constituent quark model that was claimed to have a spherical shape. However, these authors did not consider the spin-dependent density operator. When we [8] used the wave function of Ref. [10] to evaluate the matrix element of the spin-dependent charge and matter density operators, a nonspherical nucleon shape was obtained. More recently, Gross *et al.* [11] claimed to find a covariant constituent quark-diquark model that describes all the available em form factors, but has no orbital angular momentum. In this case, the shape of the proton as determined by the rest-frame matrix element of the spin-dependent density matrix is indeed spherical. The question of whether or not it is possible to find a covariant model that is a pure S wave is an interesting one that we examine here.

We next describe the wave function of Ref. [11], using the notation of that reference. The nucleon wave function $\Psi_N(P,k)$, of total four-momentum P and diquark four-momentum k, is given by the expression

$$\Psi_N(P,k) = \frac{1}{\sqrt{2}}\psi_0(P,k)\phi_I^0 u(\mathbf{P},s) - \frac{1}{\sqrt{6}}\psi_1(P,k)\phi_I^1\gamma_5\, \not{\varepsilon}^{*}_P\, u(\mathbf{P},s), \tag{4}$$

which is a sum of contributions from a spin-isospin (0,0) diquark and a spin-isospin (1,1) diquark and $\psi_{0,1}$ are Lorentz scalar functions. The polarization vectors ε_P are given by the expression

$$\varepsilon_P = \mathcal{O}_P\epsilon_k, \tag{5}$$

where ϵ_k is a genuine relativistic polarization vector of a vector particle (diquark in the present case) $\epsilon_k \cdot k = 0$. This quantity is denoted by $\eta = \epsilon_k$ in Ref. [11]. The operator $\mathcal{O}_P$ is a Lorentz transformation, with

$$\mathcal{O}_P = B_P B_k^{-1} R_{\hat{k}}^{-1}. \tag{6}$$

The operator $R_{\hat{k}}^{-1}$ rotates $\mathbf{k}$ from a generic (θ, φ) direction to the positive z direction. B_k^{-1} boosts the four-momentum state $(E_s, 0, 0, k)$ to the diquark rest frame $(m_s, 0, 0, 0)$, and finally B_P boosts the vector $(M, 0, 0, 0)$ to the moving frame $(E_P, 0, 0, P)$. The wave function Ψ_N satisfies the Dirac equation because $\not{P}$ commutes with $\gamma_5 \not{\varepsilon}^*_P$. As stressed in Ref. [11], the essential difference between this model and the one introduced in Ref. [10] is that in the nucleon rest frame, the wave function (4) contains absolutely *no* angular dependence of any kind.

We discuss the general requirements for covariance and a proper treatment of a relativistic constituent (quark-diquark) model. We study two interpretations of Ref. [11] based on two different generalizations of the boost B_P in Eq. (4) to the case of the arbitrary P and find that, using the first (conventional) interpretation, the model wave function of Ref. [11] does not satisfy these requirements because it is not covariant, and as a result it produces an em form factor that is not Lorentz invariant. Using the second (unconventional) interpretation leads indeed to a Lorentz-invariant em form factor. However, the use of the second interpretation does not yield a model that satisfies the basic requirements (that any composite quark model must satisfy) discussed in Sec. IV. We summarize in Sec. V.

II. COVARIANT VECTOR DIQUARK WAVE FUNCTION

Let us denote the vector diquark wave function as $\Psi_{P,s}(k, \epsilon)$, defined as

$$\bar{\Psi}_{P,s}(k, \epsilon) = \langle P, s|\bar{q}(0)|k, \epsilon\rangle, \tag{7}$$

where $\langle P, s|$ and $|k, \epsilon\rangle$ are nucleon and diquark eigenstates and $\bar{q}(0)$ represents a quantized quark field operator. Note that the dependence on the polarization vector ϵ and nucleon spin is made explicit. Lorentz invariance requires that

$$\bar{\Psi}_{P,s}(k, \epsilon) \sim \bar{U}(P, s)\Gamma_\mu(P, k)\epsilon_k^\mu, \tag{8}$$

where $\Gamma_\mu(P, k)$ is a *covariant* vector given by

$$\Gamma_\mu(P, k) = A\gamma_5\gamma_\mu + B\gamma_5 k_\mu + C\gamma_5 P_\mu + \cdots, \tag{9}$$

where A, B, and C are Lorentz scalar functions built from the four-vectors P and k. The forms [Eqs. (8) and (9)] have been known for a long time [12] and have been applied recently [13,14].

To see how this Lorentz invariance of Eq. (8) works in practice, consider the relevant particular example of the matrix element of the em current [15]:

$$\begin{aligned}\langle P_+, s'|J^\alpha(0)|P_-, s\rangle &= \mathcal{M}^\alpha \\ &= \int d^4k \sum_\epsilon \bar{\Psi}_{P_+,s'}(k, \epsilon)\gamma^\alpha \Psi_{P_-,s}(k, \epsilon) \\ &\sim \int d^4k \cdots \sum_\epsilon \bar{U}(P_+, s')\Gamma_\mu(P_+, k) \\ &\quad \times \epsilon_k^\mu \gamma^\alpha \epsilon_k^\nu \Gamma_\nu(P_-, k)U(P_-, s),\end{aligned} \tag{10}$$

where initial and final nucleon four-momentum are denoted as P_- and P_+. In Ref. [11] their Eq. (11)] $P_\pm$ are explicitly chosen in the Breit frame. Here the only restriction is that $P_+ = P_- + q$ and $P_\pm^2 = M^2$, where q is the four-momentum of the virtual photon and M is the nucleon mass. The quantity $\mathcal{M}^\alpha$ should be explicitly Lorentz invariant. The sum over polarizations is performed as

$$\sum_\epsilon \epsilon_k^\mu \epsilon_k^\nu = \frac{k^\mu k^\nu}{m^2} - g^{\mu\nu}, \tag{11}$$

where m is the diquark mass. Thus one finds

$$\begin{aligned}\mathcal{M}^\alpha \sim \int d^4k \cdots \bar{U}(P_+, s')\Gamma_\mu(P_+, k)\gamma^\alpha \left(\frac{k^\mu k^\nu}{m^2} - g^{\mu\nu}\right) \\ \times \Gamma_\nu(P_-, k)U(P_-, s).\end{aligned} \tag{12}$$

The result Eq. (12) has a manifestly covariant form as a Lorentz four-vector that results from the use of Eq. (11).

III. NONCOVARIANT WAVE FUNCTION OF GROSS *et al.*

We need the nucleon wave function Eq. (4) in an arbitrary reference frame. For this we use first the straightforward (conventional) definition of the boost $\mathcal{L}_P$, which is explicitly stated in Ref. [11], between Eqs. (6) and (7): "The spin states are analogous of (2) and (3), and their form in an arbitrary frame is obtained by boosting the nucleon to momentum $|\mathbf{P}| = P$ along the z direction and then rotating." This is equivalent to the statement that the z direction cannot be favored among others in Eq. (4). We will show that such a model corresponding to the "first interpretation" of Ref. [11] is not consistent with Lorentz invariance. We note in advance that the essential point will be that different polarization vectors ε_{P_+} and ε_{P_-} enter into the sum over polarization vectors.

Here consider the following (first interpretation) generalization of the vector-diquark part of the nucleon wave function [Eq. (4)] to an arbitrary reference frame:

$$\begin{aligned}&\bar{\Psi}_P(k) \sim \bar{U}(P, s)\gamma_5\gamma_\mu\varepsilon_P^\mu = \bar{U}(P, s)\gamma_5\gamma_\mu\left(\mathcal{L}_P\mathcal{L}_k^{-1}\epsilon_k\right)^\mu, \\ &\mathcal{L}_k^{-1}\epsilon_k = (0, \vec{\epsilon}) \equiv \epsilon_0,\end{aligned} \tag{13}$$

where P and k are the nucleon and diquark on-mass-shell momenta, $\mathcal{L}_k^{-1}$ is the boost transformation $\mathcal{L}_k^{-1}k = (\sqrt{k^2}, \mathbf{0})$, ϵ_k is a genuine relativistic polarization vector of a vector particle (diquark in the present case), and $\epsilon_k \cdot k = 0$, which is denoted by $\eta = \epsilon_k$ in Ref. [11]. Furthermore, ϵ_0 is the diquark polarization four-vector in the diquark rest frame. Our notation here differs slightly from that of Ref. [11] because we use the notation $\mathcal{L}_k$ instead of $R_k B_k$ and because the quantity $\mathcal{L}_k^{-1}$ is not exactly the same as $B_k^{-1} R_k^{-1}$ of Eq. (6): For the sake of simplicity we not make the effects of the rotation explicit. This simplification does not affect our conclusions [16].

Let us emphasize that the first interpretation consists in replacing the boost B_P (with z-directed P) in Eq. (4) by the boost $\mathcal{L}_P$ (with arbitrary P). As noted in Eq. (9), Lorentz invariance requires $\Gamma_\mu(P,k)$ to be a covariant four-vector in any quark-diquark wave function $\bar{\Psi}_P(k) \sim \bar{U}(P,s)\Gamma_\mu(P,k)\epsilon_k^\mu$. The result [Eq. (13)] is not consistent with this requirement because the quantity $\gamma_5\gamma_\nu(\mathcal{L}_P\mathcal{L}_k^{-1})^\nu_\mu$ is not an (axial) four-vector. In particular, the explicit appearance of the product of boosts, $\mathcal{L}_P, \mathcal{L}_k^{-1}$, breaks covariance. Neither $\mathcal{L}_P$ nor $\mathcal{L}_k^{-1}$ is a covariant tensor. To see this we derive the boost tensor from the expression for the boost (Eq. (2.8) of [17], where a sign misprint in that equation is fixed here):

$$\left(\mathcal{L}_k^{-1}\right)^\mu_\nu = \delta^\mu_\nu - \frac{1}{(k^0+m)m}k_\nu k^\mu - \frac{1}{k^0+m}k_\nu\delta^\mu_0 + \frac{2k^0+m}{(k^0+m)m}\delta_{\nu 0}k^\mu - \frac{m}{k^0+m}\delta_{\nu 0}\delta^\mu_0. \tag{14}$$

The result, with the explicit presence of the index 0, makes it clear that $(\mathcal{L}_k^{-1})^\mu_\nu$ is not a covariant tensor. Similarly,

$$\begin{aligned}(\mathcal{L}_P)^\mu_\nu = \delta^\mu_\nu &- \frac{1}{(P^0+M)M}\left(2\delta^0_\nu P^0 - P_\nu\right)\left(2\delta^\mu_0 P^0 - P^\mu\right) \\ &- \frac{1}{P^0+M}\left(2\delta^0_\nu P^0 - P_\nu\right)\delta^\mu_0 + \frac{2P^0+M}{(P^0+M)M} \\ &\times \delta_{\nu 0}\left(2\delta^{\mu_0}P^0 - P^\mu\right) - \frac{M}{P^0+M}\delta_{\nu 0}\delta^\mu_0,\end{aligned} \tag{15}$$

where we have used $P_\mu = 2\delta^\mu_0 P^0 - P^\mu$.

Lorentz invariance is lost if one uses the wave function of Ref. [11], $\bar{\Psi}_P(k) \sim \bar{U}(P,s)\Gamma_\mu(P,k)\epsilon^\mu_P$, because the sum over diquark polarization, $\sum_\epsilon \epsilon^\mu_{P_+}\epsilon^\nu_{P_-}$, that enters in the matrix element of the em current is not Lorentz invariant. Let us calculate the diquark polarization sum $D^{\mu\nu}$. We find

$$\begin{aligned}D_{\mu\nu} &= \sum_\epsilon (\epsilon_{P_+})_\mu(\epsilon_{P_-})_\nu = \sum_{\alpha\beta}(\mathcal{L}_{P_+})^\alpha_\mu\epsilon_{0\alpha}(\mathcal{L}_{P_-})^\beta_\nu\epsilon_{0\beta} \\ &= \sum_{i=1,2,3}(\mathcal{L}_{P_+})^i_\mu(\mathcal{L}_{P_-})^i_\nu.\end{aligned} \tag{16}$$

Note that here the quantity $D_{\mu\nu}(P^+,P^-)$ is seen to be a sum of product functions, with one function depending only on P^+ and the other depending only on P^-.

We use the expression for the boost [Eq. (15)] to evaluate Eq. (16), with the result

$$\begin{aligned}D_{\mu\nu} = \delta_{\mu 0}\delta_{\nu 0}&\frac{M^2 - P_+\cdot P_-}{(P^0_+ + M)(P^0_- + M)} - g_{\mu\nu} + \frac{P^+_\mu P^+_\nu}{(P^0_+ + M)M} \\ &+ \frac{P^-_\nu P^-_\mu}{(P^0_- + M)M} + \frac{P^+_\mu P^-_\nu \mathbf{P}_+\cdot\mathbf{P}_-}{(P^0_+ + M)(P^0_- + M)M^2} \\ &+ \frac{\delta^0_\mu P^+_\nu}{P^0_+ + M} + \frac{P^-_\mu\delta^0_\nu}{P^0_- + M} \\ &- \frac{P^+_\mu\delta_{\nu 0}(P_+P_- + P^0_+M) + \delta^0_\mu P^-_\nu(P_+P_- + P^0_-M)}{(P^0_+ + M)(P^0_- + M)M}.\end{aligned} \tag{17}$$

A brief inspection shows that $D_{\mu\nu}$, as obtained in a general reference frame, involves the noncovariant expressions $\delta_{\mu 0}$ as well as explicit three-vectors and therefore is not a covariant tensor. This result means that the wave function of Ref. [11] is not covariant and that the expressions for matrix elements of the em current that result from using Eq. (17) are not covariant.

However, one can be fooled by using one particular frame—the Breit frame. In this case, the four-vectors $P^\mu_\pm$ are given by $P_+ = (E,0,0,Q/2)$, $P_+ = (E,0,0,-Q/2)$. It is also useful to note that the noncovariant expression $\delta^\mu_0 = (1,0,0,0)$ can be written in an apparently covariant form

$$\delta^\mu_0 = \frac{(P_+ + P_-)^\mu}{\sqrt{4M^2 - (P_+ - P_-)^2}}. \tag{18}$$

We proceed by evaluating Eq. (17) in the Breit frame. Use Eq. (18) and $\mathbf{P}_+ = -\mathbf{P}_- = \mathbf{P}$, $P_+\cdot P_- = 2P_0^2 - M^2$ in Eq. (17) to obtain

$$\begin{aligned}D_{\mu\nu} &= (P^+_\mu + P^-_\mu)(P^+_\nu + P^-_\nu)\frac{2M^2 - 2P_0^2}{4(P^0+M)^2P_0^2} - g_{\mu\nu} + \frac{P^+_\mu P^+_\nu}{(P^0+M)M} + \frac{P^-_\nu P^-_\mu}{(P^0+M)M} + \frac{P^+_\mu P^-_\nu\left(M^2 - P_0^2\right)}{(P^0+M)^2M^2} + \frac{(P^+_\mu + P^-_\mu)P^+_\nu}{2P^0(P^0+M)} \\ &\quad + \frac{P^-_\mu(P^+_\nu + P^-_\nu)}{2P^0(P^0+M)} - \frac{[P^+_\mu(P^+_\nu + P^-_\nu) + (P^+_\mu + P^-_\mu)P^-_\nu]\left[2P_0^2 - M^2 + P^0M = (P^0+M)(2P^0 - M)\right]}{2(P^0+M)^2P^0M} \\ &= (P^+_\mu + P^-_\mu)(P^+_\nu + P^-_\nu)\frac{1}{2P_0^2} - g_{\mu\nu} - \frac{P^+_\mu P^-_\nu}{M^2} = (P^+_\mu + P^-_\mu)(P^+_\nu + P^-_\nu)\frac{1}{M^2 + P_+\cdot P_-} - g_{\mu\nu} - \frac{P^+_\mu P^-_\nu}{M^2}.\end{aligned} \tag{19}$$

This result, obtained previously in Ref. [11], has a illusory covariant appearance, resulting from the explicit use of the Breit frame. Equation (19) would not be correct in any frame other than the frame where 3D parts of P_+ and P_- are collinear. In particular, the factor $M^2 + P_+\cdot P_-$ that appears in the denominator of Eq. (19) violates the sum of product functions form of Eq. (16).

IV. WAVE FUNCTION WITH UNCONVENTIONAL POLARIZATION VECTORS

We found out in the previous section that the first interpretation wave function of Ref. [11] is not covariant, and as a result it fails to produce a Lorentz-invariant em form factor. Then we can ask the following question: What should the wave function of Ref. [11] be in an arbitrary reference frame to produce Lorentz invariant form factors, given that the wave function of a nucleon moving in the z direction is described by Eq. (4)? The unambiguous solution to this question is to use the "unconventional" diquark polarization vectors $\xi(P_\pm)$[1] in the covariant wave function (8) and correspondingly in Eq. (10) instead of the usual ϵ_k:

$$\xi(P_+) = \Lambda\epsilon(Z_+) = \Lambda\mathcal{L}_{Z_+}\epsilon_0, \quad \xi^T(P_-) = \epsilon^T(Z_-)\Lambda^T = \epsilon_0^T\mathcal{L}_{Z_-}^T\Lambda^T, \tag{20}$$

where $\Lambda = \mathcal{L}_{P_+ + P_-}$ is the boost transformation defined as

$$\Lambda^{-1}(P_+ + P_-) = (\sqrt{(P_+ + P_-)^2}, \mathbf{0}) = Z_+ + Z_-, \quad Z_\pm = \Lambda^{-1}P_\pm. \tag{21}$$

So the four-vectors Z_+ and Z_- are collinear: $\vec{Z}_- = -\vec{Z}_+$. Indeed Lorentz invariance of the em current implies that it transforms under the Lorentz transformation L as[2]

$$\langle P_+, s'|J^\alpha(0)|P_-, s\rangle = \mathcal{M}^\alpha = \bar{U}(P_+, s') \times M^\alpha(P_+, P_-)U(P_-, s), \tag{22}$$
$$M^\alpha(LP_+, LP_-) = L^\alpha_\beta S_L M^\beta(P_+, P_-)S_L^+,$$

where

$$S_L\gamma^\alpha S_L^+ = (L^{-1})^\alpha_\beta\gamma^\beta.$$

Therefore the em current can be boosted to the arbitrary frame as follows:

$$M^\alpha(P_+, P_-) = M^\alpha(\Lambda Z_+, \Lambda Z_-) = \Lambda^\alpha_\beta S_\Lambda M^\beta(Z_+, Z_-)S_\Lambda^+, \tag{23}$$

where

$$M^\alpha(Z_+, Z_-) \sim \sum_i \int d^4k\delta^+(k^2 - m^2)\psi_1(Z_+, k)\epsilon_i^\mu(Z_+) \times \epsilon_i^\nu(Z_-)\gamma_\mu\gamma_5\gamma^\alpha\gamma_5\gamma_\nu\psi_1(Z_-, k) \tag{24}$$

is expressed in terms of the wave functions [Eq. (4)] of nucleons moving along the z direction. Simple algebra leads unambiguously to the unconventional polarization vectors of Eq. (20):

$$\begin{aligned}
M^\alpha(P_+, P_-) &\sim \Lambda^\alpha_\beta \sum_i \int d^4k\delta^+(k^2 - m^2)\psi_1(Z_+, k)\epsilon_i^\mu(Z_+)\epsilon_i^\nu(Z_-) \\
&\quad \times S_\Lambda\gamma_\mu\gamma^\beta\gamma_\nu S_\Lambda^+\psi_1(Z_-, k) \\
&= \sum_i \int d^4k\delta^+(k^2 - m^2)\psi_1(P_+, \Lambda k)[\Lambda\epsilon_i(Z_+)]^\mu \\
&\quad \times [\Lambda\epsilon_i(Z_-)]^\nu\gamma_\mu\gamma^\alpha\gamma_\nu\psi_1(P_-, \Lambda k) \\
&= \sum_i \int d^4k\delta^+(k^2 - m^2)\psi_1(P_+, k)\xi_i^\mu(P_+)\xi_i^\nu(P_-) \\
&\quad \times \gamma_\mu\gamma^\alpha\gamma_\nu\psi_1(P_-, k),
\end{aligned} \tag{25}$$

where Lorentz invariance for the scalar functions was used $[\psi_1(Z_\pm, k) = \psi_1(P_\pm, \Lambda k)]$. Now, by using Eq. (25) in the first line of Eq. (22), the expression for the em form factor in terms of the new (unconventional) polarization vectors reads

$$\begin{aligned}
\mathcal{M}^\alpha &\sim \sum_i \int d^4k\delta^+(k^2 - m^2)\bar{U}(P_+)\bar{\Gamma}'_\mu(P_+, k)\xi^\mu(P_+, i) \\
&\quad \times \bar{\xi}^\nu(P_-, i)\gamma^\alpha\Gamma'_\nu(P_-, k)U(P_-) \\
&= \int d^4k\delta^+(k^2 - m^2)\bar{U}(P_+)\bar{\Gamma}'_\mu(P_+, k)D^{\mu\nu}(P_+, P_-) \\
&\quad \times \gamma^\alpha\Gamma'_\nu(P_-, k)U(P_-),
\end{aligned} \tag{26}$$

where $\Gamma'_\nu(P_-, k) = \gamma_5\gamma_\nu\psi_1(P_-, k)$ corresponds to the wave function [Eq. (4)]. In the following the primed $\Gamma'_\nu(P_-, k)$ denote more general matrices used in Eq. (32) (i.e., in the wave functions of Ref. [11] corresponding to the round spin-dependent density). They should not be confused with the (unprimed) $\Gamma_\nu(P_-, k)$ used in the covariant wave functions [Eq. (8)], which according Eq. (9) are strictly covariant and do not depend on the momentum of other nucleons to be consistent with the well-known principles.

To assess Eq. (26) we need to compare it with Eq. (12), the most general expression for the nucleon current, $\langle P_+, s'|J^\alpha|P_-, s\rangle$, in the relativistic quark-spectator diquark model (with diquark on mass shell) with the photon interacting only with the quark (using a point-like photon-quark interaction and ignoring explicit factors of charge), where $\bar{\Gamma}_\mu(P_+, k)$ and $\Gamma_\nu(P_-, k)$ are *covariant* functions of Dirac γ matrices and the momentum variables. The factor $\frac{k^\mu k^\nu}{m^2} - g^{\mu\nu}$ arises from the axial-vector diquark propagator and must present in all quark spectator axial-vector diquark models.

We stress that Eq. (12) must hold independently of the choice of the particular form of the diquark polarization vectors. A given model is defined only by the choice of a specific form for $\bar{\Gamma}_\mu(P_+, k)$ and $\Gamma_\nu(P_-, k)$; the rest is fixed by Eq. (12), and there is nothing left to choose. The results of Ref. [11] or any other model *must* be consistent with Eq. (12) and should correspond to particular choices of a *covariant* $\Gamma_\nu(P_\pm, k)$.

It is useful to discuss the result [Eq. (12)] in the operator formalism of quantum field theory. We compute the matrix element $\langle P_+|\bar{q}\gamma^\alpha q|P_-\rangle$ by inserting a sum over a complete set of states:

$$\mathcal{M}^\alpha = \langle P_+, s'|\bar{q}\gamma^\alpha q|P_-, s\rangle \sim \sum_n \langle P_+, s'|\bar{q}|n\rangle\gamma^\alpha\langle n|q|P_-, s\rangle. \tag{27}$$

Truncating the sum to the term of the valence quark model, $\sum|n\rangle\langle n| \to \int\sum|k, i\rangle\langle k, i|$ (where $|k, i\rangle$ are diquark states), gives

$$\mathcal{M}^\alpha \sim \int d^4k\delta^+(k^2 - m^2)\sum_i \bar{U}(P_+, s')\bar{\Gamma}_\mu(P_+, k) \times \gamma^\alpha\epsilon^\mu(k, i)\bar{\epsilon}^\nu(k, i)\Gamma_\nu(P_-, k)U(P_-, s), \tag{28}$$

[1] We understand that these polarization vectors are suggested (and named "unconventional") by F. Gross [18] for the construction of the pure S-wave wave function.

[2] The same arguments of Lorentz invariance for the quark-diquark wave function [Eq. (7)] unambiguously lead to the *covariant* form [Eq. (8)]. This paper addresses possible violations of general principles if forms inconsistent with Eq. (8) are used.

where the diquark "conventional" polarization vectors, $\epsilon^{\mu}(k,i)$, $\bar{\epsilon}^{\nu}(k,i)$, are exposed. The integrand of Eq. (28) is a sum of contributions factorized in the nucleon momenta, P_+, P_-; each terms is of the form of a product of scalar functions, $F_1(P_+,k)F_2(P_-,k)$. This property—that the integrand is a sum of products—is a signature property of a valence quark model (covariant and noncovariant).

Thus the relevant requirements for obtaining a correct evaluation of a relativistic valence quark spectator axial diquark model are summarized in Eq. (12) and Eq. (28).

We find that $D^{\mu\nu}(P_+,P_-)$ in Eq. (26) is indeed the covariant tensor given in Eq. (28) of Ref. [11]:

$$\begin{aligned} D^{\mu\nu}(P_+,P_-) &= \sum \xi(P_+)\times\xi^T(P_-) = \Lambda\epsilon(Z_+)\times\epsilon^T(Z_-)\Lambda^T \\ &= \Lambda\left\{(Z^+_\mu + Z^-_\mu)(Z^+_\nu + Z^-_\nu)\frac{1}{M^2+Z_+\cdot Z_-} - g_{\mu\nu} - \frac{Z^+_\mu Z^-_\nu}{M^2}\right\}\Lambda^T \\ &= (P^+_\mu + P^-_\mu)(P^+_\nu + P^-_\nu)\frac{1}{M^2+P_+\cdot P_-} - g_{\mu\nu} - \frac{P^+_\mu P^-_\nu}{M^2}. \end{aligned} \tag{29}$$

But a brief inspection shows that the result [Eq. (26)] is not consistent with the *general* form [Eq. (12)]. The integrand of the general requirement [Eq. (12)] is the sum of factorized terms of the form $F_1(P_+,k)F_2(P_-,k)$ in the nucleon momenta, P_+, P_-; the integrand of Eq. (26) is not factorizable owing to the presence of the denominator $M^2+P.P_-$ in $D^{\mu\nu}(P_+,P_-)$. There is no way to derive the integrand of Eq. (26) from Eq. (12).

We explain this in more detail by explicitly using the model of Ref. [11] in Eq. (26). This is to illustrate that the Dirac operators do not yield a factor of $M^2+P_+\cdot P_-$ in the numerator that cancels the one in the denominator. The model takes the form $\Gamma'_\mu(P_-,k)=\gamma_\mu\gamma^5\psi_1(P_-,k)$, with ψ_1 a scalar wave function. Use these Γ'_μs and Eq. (19) in Eq. (26) to obtain

$$\begin{aligned} \mathcal{M}^\alpha &= \int d^4k\delta^+(k^2-m^2)\psi_1(P_+,k)\psi_1(P_-,k)\bar{U}(P_+,s') \\ &\quad\times\gamma_\mu D^{\mu\nu}(P_+,P_-)\gamma^\alpha\gamma_\nu U(P_-,s) \\ &= \int d^4k\delta^+(k^2-m^2)\psi_1(P_+,k)\psi_1(P_-,k)\bar{U}(P_+,s') \\ &\quad\times\left[\frac{4M\,(P^\alpha_+ + P^\alpha_-)}{M^2+P_+\cdot P_-}-\gamma^\alpha\right]U(P_-,s). \end{aligned} \tag{30}$$

The explicit appearance of a term inversely proportional to $M^2+P_+\cdot P_-$ shows that the model of Ref. [11] violates the general requirements expressed in Secs. II and IV. The use of the covariant expression [Eq. (19)] for $D^{\mu\nu}$ leads to a contradiction with well-known principles.

It is important to see that the transformation Λ and vectors Z_+ and Z_- depend on both momenta P_+ and P_- and thereby so do the unconventional polarization vectors (and hence the notation $\xi(P_\pm)$ with only one momentum argument is misleading). This fact is not emphasized in Ref. [11]. It might be interesting to show that this dependence on two momenta is completely contained in Wigner rotations, $W^i_j(P_\pm,Z_\pm)$, usually involved in the Lorentz transformation of polarization vectors

$$\begin{aligned} \xi(P_+,i) = \Lambda\epsilon^i(Z_+) &= \sum_j \epsilon^j(\Lambda Z_+)W^i_j(\Lambda Z_+,Z_+) \\ &= \sum_j \epsilon^j(P_+)W^i_j(P_+,Z_+), \end{aligned} \tag{31}$$

where the polarization vector, $\epsilon^j(P_+)=\mathcal{L}_{P_+}\epsilon^j_0$, depends only on P_+ just in the same way as the genuine polarization vector ϵ^j_k from Eq. (8) depends on k. The whole dependence on the two momenta P_+ and P_- is contained in the Wigner (purely 3D) rotations, $W^i_j(P_+,Z_+)$, through $Z_+=\Lambda^{-1}P_+$.

As a result the wave function of the initial (final) nucleon in Eq. (26),

$$\begin{aligned} \Psi_{P_-} &\sim \bar{\xi}^\nu(P_-,i)\gamma^\alpha\Gamma'_\nu(P_-,k)U(P_-), \\ \bar{\Psi}_{P_+} &\sim \bar{U}(P_+)\bar{\Gamma}'_\mu(P_+,k)\xi^\mu(P_+,i), \end{aligned} \tag{32}$$

depends not only on the momentum of the initial P_- (final P_+) nucleon but on both P_+ and P_-, which is beyond the very idea of quark models. This dependence on both momenta comes via the "unconventional" diquark polarization vectors.

Having shown that the wave function of Ref. [11] corresponding to Eq. (4) is not consistent with well-known principles, here we can discuss restrictions these principles impose on the general form of $\bar{\Gamma}'_\mu(P_+,k)$ if one is to use the wave functions of Eq. (32).

The minimal consistency condition is

$$\begin{aligned} &\sum_i \bar{U}(P_+,s')\bar{\Gamma}_\mu(P_+,k)\gamma^\alpha\epsilon^\mu(k,i)\bar{\epsilon}^\nu(k,i) \\ &\quad\times\Gamma_\nu(P_-,k)U(P_-,s) \\ &\quad= \sum \bar{U}(P_+s')\bar{\Gamma}'_\mu(P_+,k)\xi^\mu(P_+,i)\bar{\xi}^\nu(P_-,i)\gamma^\alpha \\ &\quad\times\Gamma'_\nu(P_-,k)U(P_-s), \end{aligned} \tag{33}$$

which restricts Γ'_ν because $\Gamma_\nu(P_-,k)$ is restricted to being covariant and independent of P_+. This leads to

$$\begin{aligned} &\bar{U}(P_+,s')\bar{\Gamma}_\mu(P_+,k)\gamma^\alpha\left(\frac{k^\mu k^\nu}{m^2}-g^{\mu\nu}\right)\Gamma_\nu(P_-,k)U(P_-,s) \\ &\quad=\bar{U}(P_+s')\bar{\Gamma}'_\mu(P_+,k)D^{\mu\nu}(P_+,P_-)\gamma^\alpha\Gamma'_\nu(P_-,k)U(P_-s). \end{aligned} \tag{34}$$

Although Eq. (34) admits a covariant solution for $\Gamma'_\nu(P_-,k)$ it requires $\Gamma'_\nu(P_-,k)$ to depend also on P_+. A solution to Eq. (34) is

$$\begin{aligned} \Gamma'_\nu(P_-,k) &= \left(\mathcal{L}_k R\mathcal{L}^{-1}_{Z_-}\Lambda^{-1}\right)^\beta_\nu\Gamma_\beta(P_-,k), \\ \bar{\Gamma}'_\mu(P_+,k) &= \bar{\Gamma}_\alpha(P_+,k)\left(\mathcal{L}_k R\mathcal{L}^{-1}_{Z_+}\Lambda^{-1}\right)^\alpha_\mu, \end{aligned} \tag{35}$$

where R is an arbitrary 3D rotation. The mentioned dependence on the momenta of both nucleons comes from $\mathcal{L}_k R\mathcal{L}^{-1}_{Z_+}\Lambda^{-1}$. The solution [Eqs. (35)] can be verified by using the identity

$$\frac{k^\alpha k^\beta}{m^2}-g^{\alpha\beta} = \left(\mathcal{L}_k R\mathcal{L}^{-1}_{Z_+}\Lambda^{-1}\right)^\alpha_\mu D^{\mu\nu}(P_+,P_-)\left(\mathcal{L}_k R\mathcal{L}^{-1}_{Z_-}\Lambda^{-1}\right)^\beta_\nu. \tag{36}$$

The arbitrariness of R follows from the identity

$$R^{\alpha}_{\mu}(\delta^{0\mu}\delta^{0\nu} - g^{\mu\nu})R^{\beta}_{\nu} = \delta^{0\alpha}\delta^{0\beta} - g^{\alpha\beta}. \quad (37)$$

The expressions in Eq. (34) are closely related to the generalized parton distributions (GPD) corresponding to the parton (quark) momentum $P_- - k$. We would like to note that for any given model of the nucleon one can construct such a GPD-related function [19] that reproduces exactly the em current of the given model and at the same time corresponds to the round spin-dependent matter density. The problem is that in GPD the initial and final nucleon momenta are not factorized and therefore this GPD cannot be obtained in the framework of a valence quark model.

V. ASSESSMENT

Using the first of two interpretations discussed here (conventional polarization vectors), we have shown that the seemingly covariant appearance of the expressions of Ref. [11] results from the explicit use of the Breit frame. This failure to maintain covariance results from using the polarization vectors ε_P instead of ϵ_k to describe the vector diquark wave function. However, this is a very important point in the present context because it is exactly the use of ε_P that allows the construction of a model wave function without orbital angular momentum. As noted in Ref. [11], the result [Eq. (19)] has no angular dependence, so the evaluation of the matrix element of the spin-dependent density operator would yield a spherical shape. However, this roundness is caused solely by the lack of Lorentz invariance. Using the polarization vector ϵ_k would lead to a model much like that of Ref. [10], which does have a nonspherical shape, as measured by the spin-dependent matter density.

Given the importance of Lorentz invariance, we have derived the general form of the wave function that would produce a Lorentz-invariant em form factor. This derivation led us unambiguously to the "unconventional" diquark polarization vectors $\xi(P_\pm)$, suggested in Ref. [18], to interpret the wave functions used in Ref. [11]. These yield covariant results. Unfortunately, neither the derived wave functions nor the expression for the em current satisfy well-known principles discussed in Sec. IV. For example, the wave function of the incoming nucleon depends on the momentum of the outgoing nucleon too and vice versa, so the em current cannot be written in the traditional form of the convolution of two proper wave functions.

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$$D_{\mu\nu}(P_+, P_-) \to R^{\alpha}_{\mu}(P_+)D_{\alpha\beta}(Z_+, Z_-)R^{\beta}_{\nu}(P_-), \quad (38)$$

where $Z_\pm = (P^0_\pm, 0_\perp, |P_\pm|)$. To evaluate Eq. (38) we use

$$\begin{aligned} R(P_\pm)Z_\pm &= P_\pm \\ (R(P_+)Z_-)_\mu &= [R(P_+)(P^0_-, 0_\perp, |P_-|)]_\mu = \left(P^0_-, \frac{|P_-|}{|P_+|}\vec{P}_+\right)_\mu \\ &= \frac{|P_-|}{|P_+|}P^+_\mu + \left(P^0_- - \frac{|P_-|}{|P_+|}P^0_+\right)\delta^0_\mu \end{aligned} \quad (39)$$

The result of this exercise would lead to a $D_{\mu\nu}$ that contains even more factors that violate covariance than the $D_{\mu\nu}$ obtained in Eq. (17).
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