Scaling of v_2 in heavy ion collisions

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(Received 4 March 2007; published 8 August 2007)

We interpret the scaling of the corrected elliptic flow parameter with respect to the corrected multiplicity, observed to hold in heavy ion collisions for a wide variety of energies and system sizes. We use dimensional analysis and power-counting arguments to place constraints on the changes of initial conditions in systems with different center of mass energy $\sqrt{s}$. Specifically, we show that a large class of changes in the (initial) equation of state, mean free path, and longitudinal geometry over the observed $\sqrt{s}$ are likely to spoil the scaling in v_2 observed experimentally. We therefore argue that the system produced at most Super Proton Synchrotron (SPS) and Relativistic Heavy Ion Collider (RHIC) energies is fundamentally the same as far as the soft and approximately thermalized degrees of freedom are considered. The "sQGP" (Strongly interacting Quark-Gluon Plasma) phase, if it is there, is therefore not exclusive to RHIC. We suggest, as a goal for further low-energy heavy ion experiments, to search for a "transition" $\sqrt{s}$ where the observed scaling breaks.

DOI: 10.1103/PhysRevC.76.024903 PACS number(s): 25.75.Dw, 25.75.Nq

I. INTRODUCTION

The azimuthal anisotropy of mean particle momentum (parametrized by its second Fourier component v_2), thought of as originating from the azimuthal anisotropy in collective flow ("elliptic flow"), has long been regarded as an important observable in heavy ion collisions. The main reasons for this is that elliptic flow has long been understood to be "self-quenching" [1,2]: The azimuthal pressure gradient extinguishes itself soon after the start of the hydrodynamic evolution, so the final v_2 is insensitive to later stages of the fireball evolution and therefore allows us to probe the hottest, best thermalized, and possibly deconfined phase.

In addition, as has been shown in [3], the v_2 signature is highly sensitive to viscosity. The presence of even a small but non-negligible viscosity, therefore, can in principle be detected by a careful analysis of v_2 data.

Indeed, one of the most widely cited (in both the academic and popular press) news coming out of the heavy ion community concerns the discovery, at RHIC, of a "perfect fluid," also sometimes referred to as "sQGP" [4–7]. The evidence for this claim comes from the successful modeling of RHIC v_2 by boost-invariant hydrodynamics [13–15]. The scaling of v_2 according to the number of constituent quarks further suggests that the flow we are seeing is partonic, rather than hadronic, in origin [8,9], especially since the scaling applied to kinetic energy (rather than transverse momentum) holds for every known species up to the lowest momentum [10–12].

While hydrodynamics is a fully deterministic theory, it contains a crucial not very well understood assumption: initial conditions. While the degree of boost invariance is not currently well known experimentally, the transverse structure of the energy density should follow a Glauber model [16], based on the superposition of the initial nuclear densities. That allows us to characterize the collision in terms of a number less than one called the eccentricity ϵ, related to the impact parameter b and the radius R ($\sim A^{1/3}$ fm)

$$\epsilon = \frac{\sqrt{2R+b} - \sqrt{2R-b}}{\sqrt{2R+b}} \tag{1}$$

the total transverse area of the system S depends, similarly, on R and b

$$S = 2R^2 \cos^{-1}\left(\frac{b}{2R}\right) - b\sqrt{R^2 - \frac{b^2}{4}}\,. \tag{2}$$

Since the announcement of the discovery of the perfect fluid, a considerable amount of high quality experimental data has been collected. In particular, extension of RHIC beams to smaller colliding systems such as Cu-Cu have allowed us to compare systems of similar multiplicity but in very different energy regimes. The results have been remarkable: It seems that v_2/ϵ (where ϵ is the initial eccentricity), plotted against $\frac{dN}{Sdy}$ (where S is the area of the collision system and $\frac{dN}{dy}$ is the multiplicity rapidity density), fall on a "universal" curve, which links very different regimes, ranging from Alternating Gradient Synchrotron (AGS) to RHIC ([17,18], Fig. 1).

This scaling, albeit with a linear dependence rather than the slightly curved one observed, has been predicted previously [19,20] on the basis of a nearly "free streaming" calculation where the mean free path is comparable to the system size. Such a limit considerably under-predicts the observed v_2/ϵ, which is why a nearly-perfect hydrodynamic regime is thought to apply. As we will show, however, the same scaling in this regime is far from guaranteed.

We do not possess at present the tools, such as 3D viscous hydrodynamics, to quantitatively analyze this data. However, the extent of the scaling suggests that these very different systems vary somehow only in one scale, and that this scale is connected to the total entropy produced [21].

In this work, we shall perform a *qualitative* analysis, using elementary tools such as Taylor expansion, dimensional analysis, and power counting. We show that these tools, together with experimental data, allow us to place stringent

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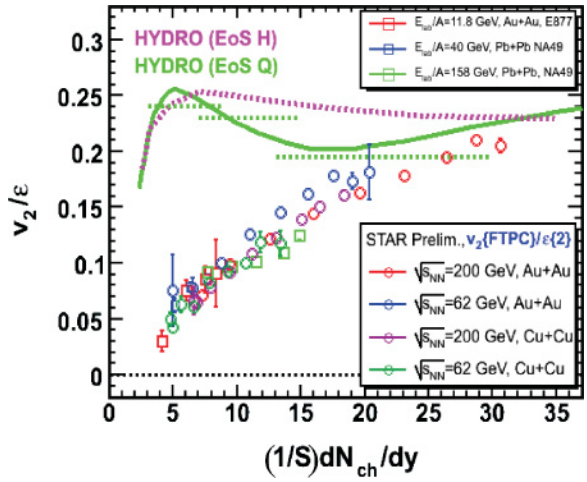

FIG. 1. (Color online) A compendium of evidence for the universal (for the observed energies) scaling of v_2/ϵ vs $\frac{1}{S}\frac{dN}{dy}$ [18]. Predictions from ideal boost-invariant hydro are also shown (lines, see references of [18]).

limits on the initial conditions of the system created in heavy ion collisions, from AGS to RHIC energies.

In particular, we show that the observed scaling places very strong constraints on initial microscopic properties (entropy density, mean free path), as well as longitudinal structure. We argue that statistical and transport properties can not significantly vary between RHIC and observed lower energies. We therefore conclude that the "perfect fluid," if it is there, is a common characteristic of all experimentally studied systems so far.

It should be underlined that the "experimental result" on which we base our conclusions is, itself, somewhat theory-laden since ϵ is not an experimentally measured quantity. If the best physical description of the soft initial dynamics at RHIC is not the Glauber model but rather, for example, the color glass condensate [22], the scaling might need to be revisited [23]. Even within the Glauber model calculation, it is only when eccentricity fluctuations [24] are taken into account that the universal scaling is observed (one of the reasons why this scaling was not reported until recently, and missed for example in [25]). Yet, the fact that an improvement in the computation of ϵ results in improvement of the scaling is indicative that something physical must be behind it.

It should be noted that the result in Fig. 2 is based on p_T integrated v_2, and is therefore subject to systematic uncertainties due to the wide range of acceptance in the experiments summarized in Fig. 2. Recent investigations (Figs. 1 of [26,27]), however, seems to show that the scaling is *not* dependent on integrating over p_T, since a compatible scaling holds when separate p_T bins are considered. The universal scaling was found to hold in rapidity space also [27].

In the rest of the paper, we will regard the result as established because we wish to show that its consequences are profound. Finding simple scaling in a system as complicated as a heavy ion collisions is encouraging enough that it deserves exploration even through the scaling observation should be regarded as preliminary. We hope that the experimental

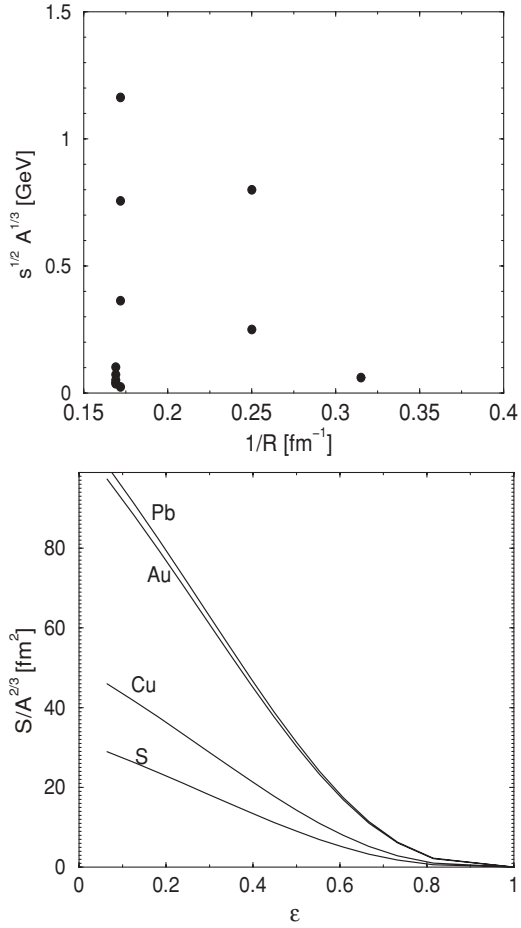

FIG. 2. Top panel (a): The range in $\sqrt{s}A^{1/3}$ vs $1/A^{1/3}$ for the available experimental data. Bottom panel (b): The range of S and ϵ covered by available data.

community will soon determine how universal this scaling really is.

II. HOW NATURAL IS THE OBSERVED SCALING WITHIN HYDRODYNAMICS?

For the system under consideration, three dimensionless parameters can safely be assumed to be significantly smaller than unity: The initial spatial eccentricity ϵ, the initial speed of sound c_s (that encodes information on the equation of state [28–30]), and the initial mean free path l_{mfp} divided by the initial size of the system, in general parametrized by the transverse radius R and the longitudinal size $\langle z\rangle$. The latter refers to the system's longitudinal size in configuration space, at midrapidity, *at the start* of the hydrodynamic evolution, and is a general definition. It is usually thought that $\langle z\rangle$ can interpolate from a Landau initial condition [31], where $\langle z\rangle$ is related to the nucleon mass m_N and the center of mass energy $\sqrt{s}$ by

$$\langle z\rangle \sim R\frac{\sqrt{s}}{m_N}, \tag{3}$$

to the Bjorken [32] initial condition, where, for a system with maximum rapidity y_L,

$$\langle z\rangle \sim 2\tau_0 \sinh(y_L)\,, \tag{4}$$

$$y_L = \frac{1}{2}\ln\left(\frac{\sqrt{s/4}+\sqrt{s/4-m_N^2}}{\sqrt{s/4}-\sqrt{s/4-m_N^2}}\right)\,, \tag{5}$$

where τ_0 is the thermalization timescale of the system.

Since hydrodynamic evolution is fully determined by the initial conditions and the equation of state,[1] any flow variable can be thought of as a function of the parameters characterizing these. If this function is integrable (i.e., if no turbulence occurs), than it is safe to expand this function around any dimensionless parameter less than one.

Both the dimensionless v_2 and the dimensionful $\frac{dN}{Sdy}$ ($\sim \mathrm{fm}^{-2}$) can therefore be Taylor-expanded around these quantities. For v_2, we know that the zeroth term is 0 (perfectly central collisions have no v_2), so

$$\begin{aligned} v_2 \sim\ & a_{100}\epsilon + a_{200}\epsilon^2 + \epsilon\left(a^R_{110}\frac{l_{mfp}}{R} + a^z_{110}\frac{l_{mfp}}{\langle z\rangle} + \right) \\ & + \epsilon c_s\left(a^R_{111}\frac{l_{mfp}}{R} + a^z_{111}\frac{l_{mfp}}{\langle z\rangle}\right) + \dots\,, \end{aligned} \tag{6}$$

a_{ijk} are in general (probably transcendental) functions of an arbitrary number of dimensionless quantities constructed out of $\langle z\rangle, T, \mu_B, R$ (in general $\zeta = \sum_{mnl}\zeta_{mnl}$, where $\zeta_{mnl} = \langle z\rangle^m T^n \mu_B^l R^{l+n-m}$, all m, n, l). The exact form of a_{ijk} can be obtained integrating the hydrodynamic equation from the initial time to freeze-out time (through they are expected to be insensitive to the latter, and equivalently to $\frac{T_{\text{freeze-out}}}{T_{\text{initial}}}$). Ideal boost-invariant hydrodynamics, with a bag model or lattice-inspired equation of state [13–15], predicts that $a_{20j} \ll a_{10j}$, and hence v_2/ϵ is approximately constant (as the lines in Fig. 1 show).

The experimentally observed rise of v_2/ϵ with multiplicity, and encounter with the hydrodynamic calculation (see Fig. 1), can therefore be interpreted as RHIC energy being the only point where the system reaches the "ideal hydrodynamics limit".

It is however unclear to what extent is such a conclusion an artifact of the models being used to perform this comparison assuming exact [13–15] or approximate [33,34] boost-invariance as an initial condition at all energies. While there are good physical arguments for why such an initial condition is appropriate for heavy ion collisions at midrapidity [32], the fact that some experimental data are more compatible with Landau hydrodynamics even at RHIC highest energies [35] suggests the need to question this assumption, and in particular to evaluate its effect on our estimate of the transport properties *and their energy dependence*. While no viscous calculation using Landau initial conditions has so far appeared in the literature, it is reasonable to suppose that the slower cooling from a Landau initial condition would leave more time for v_2 to form. Hence, the limits on viscosity/mean free path/thermalization time inferred from v_2 data are strongly correlated with the degree of assumed boost invariance.

$\frac{dN}{Sdy}$ contains information about both the longitudinal structure at freeze-out and the *final* particle number density (a function of the initial T and μ_B). In an ideal (isenthropic) expansion, the final entropy is equal to the initial entropy content of the system [$\sim$ the initial particle density $n(T, \mu_B)$], so

$$\frac{1}{S}\frac{dN}{dy} \sim \langle z\rangle n(T, \mu_B)\,. \tag{7}$$

Collective evolution, if the system has a non-negligible mean free path, can however create additional entropy.

The first correction to isenthropic expansion should therefore be proportional to the entropy creation due to viscous processes. This is given by [36]

$$\Delta S \sim \int dt\,\eta\langle\partial_\mu u_\nu\rangle^2\frac{V(t)}{T}\,, \tag{8}$$

where η is the viscosity, u_ν the flow field, and $V(t)$ the volume of the fluid.

Viscosity is in turn proportional to the mean free path, density n, and mean momentum current $\langle p\rangle$ [31]

$$\eta \sim l_{mfp} n(T, \mu_B)\langle p\rangle(T, \mu_B) \tag{9}$$

since the initial volume is, to zeroth order in eccentricity, $\sim\langle z\rangle R^2$, we get

$$\begin{aligned} \frac{1}{S}\frac{dN}{dy} \sim\ & \langle z\rangle n(T, \mu_B)\left(1 + b^R_{010}\frac{l_{mfp}}{R} + b^{\langle z\rangle}_{010}\frac{l_{mfp}}{\langle z\rangle}\right. \\ & + \epsilon\left(b^R_{110}\frac{l_{mfp}}{R} + b^{\langle z\rangle}_{110}\frac{l_{mfp}}{\langle z\rangle}\right) \\ & \left. + c_s\left(b^R_{011}\frac{l_{mfp}}{R} + b^{\langle z\rangle}_{011}\frac{l_{mfp}}{\langle z\rangle}\right) + \cdots\right)\,, \end{aligned} \tag{10}$$

where, once again, the coefficients b are functions of any arbitrary sum $\sum_{m,n,l}\langle z\rangle^m T^n \mu_B^l R^{n+l-m}$, that have to be calculated by integrating the viscous hydrodynamic equations. Since entropy is predominantly produced in the initial collisions [37], we can again disregard $\frac{T_{\text{freeze-out}}}{T_{\text{initial}}}$.

It is immediately clear that several of the parameters used in the previous expansions (in particular the c_s, l_{mfp}, $\langle z\rangle$) cannot, by causality, depend on the *transverse* system size A, and have to depend only on the local energy density *only*. If soft observables scale with the number of participants, therefore, these parameter's dependence on energy and system size can only be a function of σ, where

$$\sigma = f(\sqrt{s})A^{1/3} \sim \sqrt{s}A^{1/3}\,. \tag{11}$$

Not all parameters, however, have this dependence: The initial transverse system size R and eccentricity ϵ exhibit no energy dependence. Rapidity [27] provides a further independent direction.

Presently available heavy ion experiments have explored a significant range in ϵ, σ and R, shown in Fig. 2. Had v_2/ϵ scaled in a different way from $\frac{1}{S}\frac{dN}{dy}$ with respect to these variables, the currently available experimental data should

[1] An additional variable experimental observables can depend on is a *freeze-out criterion*, encoded by, for example, $\frac{T_{\text{freeze-out}}}{T_{\text{initial}}}$. We disregard this variable in the subsequent discussion as the observables we discuss should not strongly depend on it.

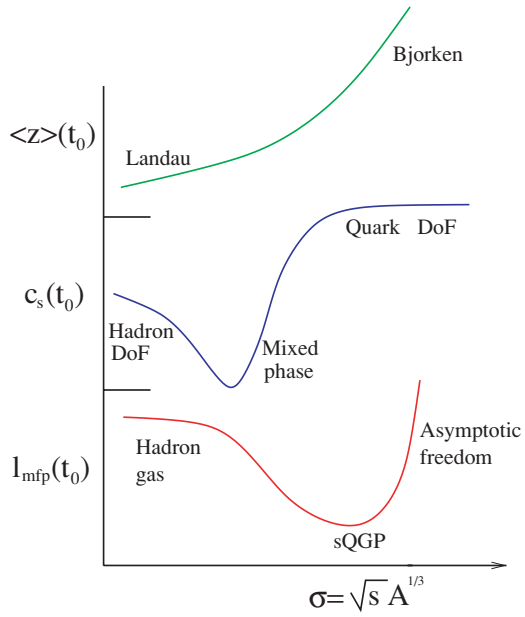

FIG. 3. (Color online) The expected dependence, with energy of the initial $\langle z\rangle$, c_s, and l_{mfp}. These quantities, by definition, will be independent of system size.

have signaled it by the appearance of "branches", systems with same $\frac{1}{S}\frac{dN}{dy}$ but different v_2/ϵ (or vice versa). Hence, the absence of such branches is a probe capable of constraining the nature of the system created in heavy ion collisions. In the rest of the paper, we will use this probe, as well as Eqs. (6) and (10), to constrain the system as much as possible.

Figure 3 sketches the dependence of c_s, l_{mfp}, and $\langle z\rangle$ as conventionally thought. As the energy increases, c_s rapidly dips in the "mixed phase"[2] and than increases as the system moves from a hadron gas to a quark gluon plasma. An increase in c_s, if other variables were fixed, would *increase* the v_2/ϵ (since there is more pressure buildup) and, since it is an equilibrium process, either maintain $\frac{dN}{Sdy}$ constant or decrease it (if longitudinal flow is significant).

l_{mfp} is sensitive to the phase structure in a somewhat different manner: the system should first enter the strongly coupled low viscosity sQGP regime, and then, when σ is very high ($\sqrt{s} \sim$ TeV), the higher viscosity high-temperature regime where the QGP becomes asymptotically free. A higher viscosity means a lower v_2 [3], but a higher $\frac{dN}{Sdy}$, as more entropy is created in the system and collective energy is transformed, by microscopic interactions, into thermal energy.

Finally, the stopping power of the system should decrease as the initial condition goes from Landau [31] to the Bjorken [32] limit. This decreases the pressure buildup needed to create v_2 as well as $\frac{1}{S}\frac{dN}{dy}$, since entropy is redistributed in a wider rapidity space.

The combined effect of these three scalings is difficult to evaluate without a solver of 3D viscous hydrodynamic equations. It is however likely to be non-negligible, and depend *only* on σ, *not* on A or centrality/ϵ. The total size of the system, and its lifetime, however, should depends strongly on *both* σ and A, as well as the eccentricity.

Hence, terms $\sim\frac{l_{mfp}}{R,\epsilon R,\langle z\rangle}$, present in both Eqs. (6) and (10) should vary in a nontrivial way as energy and system size are changed. In addition, for systems with a short lifetime, or a slow formation of v_2, terms $\sim\epsilon^{i>1}$ in Eq. (6) should become non-negligible. Thus, the appearance of a common scaling between systems with different σ, A, ϵ, such as the one observed appears highly unlikely *a priori*.

We shall further explore the naturalness of the observed scaling using *natural values* of a_{ijk}, b_{ijk}, together with parametrizations of c_s and l_{mfp}

$$\rho = \frac{\rho_{HG}}{2}(1+\tanh(T_c - T)) + \frac{\rho_{QGP}}{2}(1+\tanh(T - T_c)), \tag{12}$$

$$c_s = \frac{0.1}{2}(1+0.9\tanh(T_c - T)) + \frac{c_s^{ideal}}{2}(1+\tanh(T - T_c)), \tag{13}$$

$$\frac{l_{mfp}}{fm} = \frac{5}{2}(1+\tanh(T_c - T)) + \frac{0.1}{2}(1+\tanh(T - T_c))^* \log\left(1+\frac{T}{T_c}\right). \tag{14}$$

We then use the Bjorken formula to get the initial longitudinal and energy density distribution

$$\frac{1}{S}\frac{dN}{dy} \sim \frac{1}{\pi A^2}\frac{dN}{dy} = \tau_0\rho(T,\mu_B), \tag{15}$$

where $\frac{dN}{dy}$ can be obtained from experiment by a phenomenological formula [38].

$$\frac{dN}{dy} = \frac{N_{\text{participants}}}{1.48}\ln\left(\frac{\sqrt{s}}{1.48\text{ GeV}}\right). \tag{16}$$

The temperature is found by solving the resulting conservation of energy equations, using the equation of state given in Eq. (12). The resulting temperature is then plugged into Eqs. (14) and (13) to calculate c_s and l_{mfp}. These, together with "typical" coefficients a_{ijk}, b_{ijk} and $R \sim A^{1/3}, \langle z\rangle \sim \tau_0\sinh(y_L)$ are then used to investigate how v_2/ϵ and $\frac{1}{S}\frac{dN}{dy}$ depend on each other.

Since all "small" dimensionless parameters are encoded in $c_s, \epsilon, \frac{l_{mfp}}{R,\langle z\rangle}$, all coefficients $a_{ijk} \sim v_2|_{exp}, b_{ijk} \sim 1$. Figure 4 shows what kind of scaling is to be expected from Eqs. (5) and (9) if the approximate equalities hold exactly. The branch structure is clearly seen across the experimentally studied system sizes. It is of course possible to eliminate and produce universal scaling, but, in the absence of a deeper principle why that should be so, the coefficients a, b would need to be carefully *fine-tuned*.

Thus, the experimentally observed scaling of Fig. 1 places very profound constraints on how the microscopic properties, and the global longitudinal structure, can vary between AGS and RHIC energies.

[2]In a first order phase transition, it dips to zero. In a crossover, it merely decreases [30].

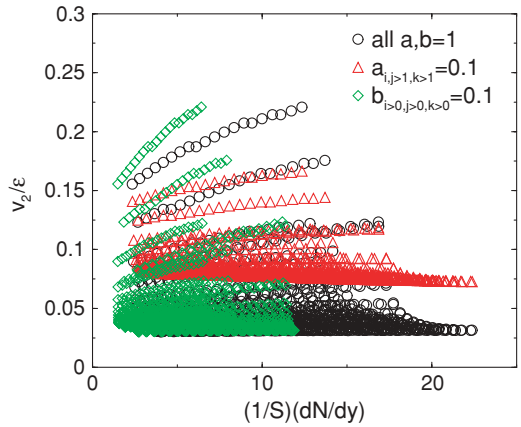

FIG. 4. (Color online) Scaling expected with for "typical" values of a_{ijk}, b_{ijk} and the formula in [38] for multiplicity. The bands, lower to upper, correspond to different classes in centrality Eq. (10) and different system sizes (Pb, Au, Cu, S).

III. WHAT DRIVES THE SCALING?

The point made in the previous section is actually straightforward to understand: As illustrated in Fig. 5 a universal scaling means that the two quantities that scale are functions of a *common* variable (that can be, in general, a function of still further variables).

Thus, the systems from AGS to RHIC appear to be controlled by a common scale, related to the total multiplicity, which varies smoothly and drives both v_2/ϵ and $\frac{1}{S}\frac{dN}{dy}$. This conclusion is a strong indication that *microscopic* properties of the system (equation of state and mean free path) are unchanged, up to a shift related to this scale, in the experimentally accessed energy range. It also raises the question of the exact nature of the variable that drives the scaling.

It has been suggested, in [13,39], that the system, even at its initial stage, is not entirely in the nearly inviscid QGP phase, but a fraction of it is in a highly viscous hadron gas. In this case, Eqs. (6) and (10) should be updated with new parameters $c_s \Rightarrow c_s^{QGP,HG}$, $l_{mfp} \Rightarrow l_{mfp}^{QGP,HG}$ and an additional parameter

$$\alpha \sim \frac{(dN/dy)_{QGP}}{(dN/dy)_{total}} \tag{17}$$

should be added.

Could it be that α is what moves from zero to unity in the curve of Fig. 1? If we assume a Glauber model and a Woods-Saxon distribution for initial density, and a critical "transition" energy density ρ_c (independent of energy and system size), α becomes

$$\alpha = \frac{\int \rho(x,y,z,A,\sqrt{s})\Theta(\rho(x,y,z,A,\sqrt{s}) - \rho_c)d^3x}{\int \rho(x,y,z,A,\sqrt{s})d^3x}, \tag{18}$$

$$\rho \sim \frac{\sqrt{s}A^{1/3}}{\langle z\rangle}\left[T_A(\sqrt{(x+b)^2+y^2}) + T_B(\sqrt{(x-b)^2+y^2})\right], \tag{19}$$

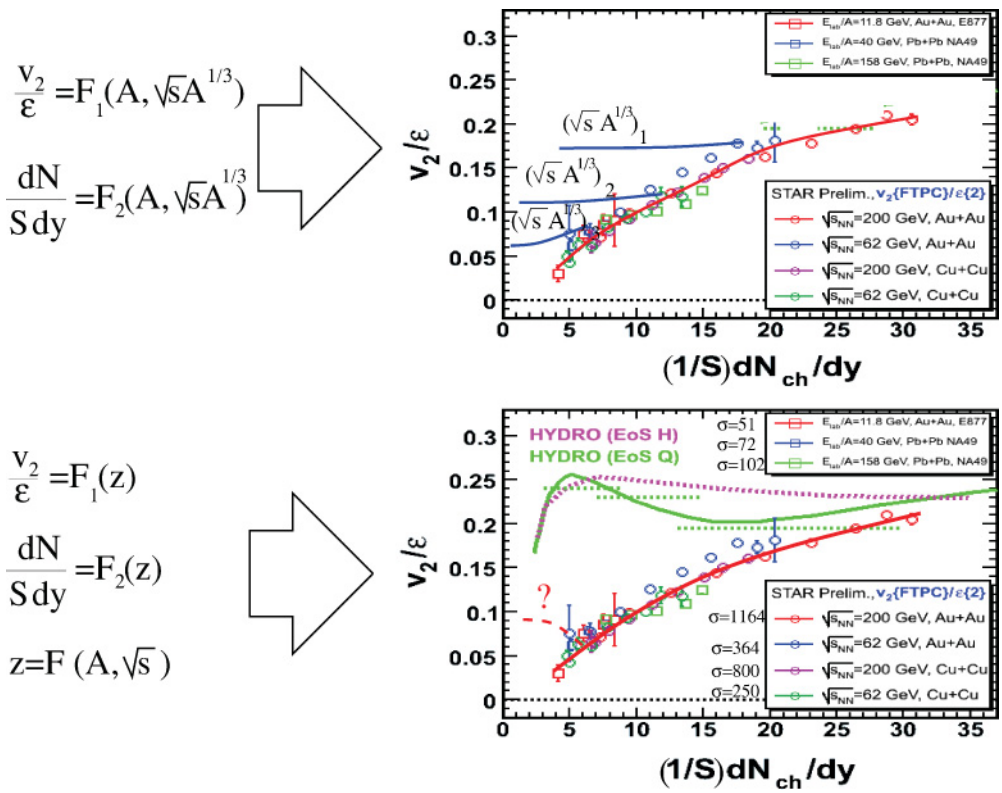

FIG. 5. (Color online) An illustration of the mathematical implications of universal scaling. A possible breaking of the scaling is shown in the bottom panel.

and $T_{A,B}(r)$ is the usual transverse participant density of the target nuclei,

$$T(x, y) = \int dz \rho_N\left(\sqrt{x^2 + y^2 + z^2}\right). \tag{20}$$

Such scaling is very nontrivial to model exactly, but it seems to obey an approximate error function dependence in σ

$$\alpha \sim \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{\sigma f(b) - \rho_c}{\Delta_\sigma}\right)\right), \tag{21}$$

it is difficult to see how such a scaling can be compatible with the observed universal curve. In particular, there will be limiting values in $\sqrt{s}$ at which α should go at 0 and 1 independently of A and b. above/below these scales, the system reduces to a one-component system and branches should start appearing.

Similarly, at low energies there will be a regime where $\alpha = 0$ below a certain impact parameter, and $\alpha > 0$ above this impact parameter. In the first class of centralities, the large mean free path in the "corona" would introduce a dependence on A that does not conform to the universal curve. While RHIC experiments have isolated several low centrality bins where v_2 is significantly below ideal hydro predictions (the corona dominates), such a critical centrality has so far not been observed. (although this can be explored further with collisions of very heavy nuclei, such as uranium [40,41]).

We conclude that, if the scaling parameter is indeed the QGP fraction of the system, the energies explored so far are far from the regime where this fraction is either close to unity or to zero. While this is possible, given the large variation in energy within the systems explored so to date, it would be somewhat surprising.

An alternative ansatz to reproduce the observed scaling is by postulating a universality in initial conditions, up to a scale parameter, that also smoothly controls the initial temperature (and hence c_s and l_{mfp}). The basic constraint that the universal scaling imposes on a set of uniform initial conditions is the requirement that a single dimensionful scale $\langle\tau\rangle$ exists, and varying *either* the energy or the system size only shifts the system up and down the scale $\langle\tau\rangle$. Both $\langle z\rangle n(T, \mu_B)$ and v_2 are then functions of only $\langle\tau\rangle$ as well as constants independent of energy and system size. A natural interpretation for $\langle\tau\rangle$ is the system's lifetime in the co-moving frame. In units of the mean free path, this corresponds to the inverse of the Knudsen number [42], the number of collisions between the system's degrees of freedom.

Because of the leading dependence of $\frac{1}{S}\frac{dN}{dy}$ on $\langle z\rangle n(T, \mu_B)$ (Eq. (10)), it follows that $\langle\tau\rangle$ can only scale with A, $\sqrt{s}$, σ in the same way as $\langle z\rangle n(T, \mu_B)$,

$$\langle\tau\rangle = F^{-1}(\langle z\rangle n(T, \mu_B)), \tag{22}$$

where $F(\langle\tau\rangle)$ is the same for all energies. A different dependence would lead to two different $\langle\tau\rangle$s corresponding to the same $\langle z\rangle n(T, \mu_B)$, which would again break the observed scaling.

Equations (6) and (10) then simplify to

$$\begin{aligned} v_2 \sim{}& \epsilon\left(a_{100} + a_{101}c_s + a_{102}c_s^2 + \ldots\right) \\ &+ \frac{l_{mfp}}{\langle\tau\rangle}\epsilon(a_{110} + a_{111}c_s + \ldots) \\ &+ \left(\frac{l_{mfp}}{\langle\tau\rangle}\right)^2 \epsilon(a_{120} + a_{121}c_s + \ldots) + \ldots, \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{1}{S}\frac{dN}{dy} \sim{}& F(\langle\tau\rangle)\left(1 + b_{010}\frac{l_{mfp}}{\langle\tau\rangle} + b_{011}\frac{c_s l_{mfp}}{\langle\tau\rangle}\right. \\ &\left. + b_{200}\left(\frac{l_{mfp}}{\langle\tau\rangle}\right)^2 + \ldots\right), \end{aligned} \tag{24}$$

where l_{mfp}, b, a depend only on $\langle\tau\rangle$, presumably via a simple scaling between $\langle\tau\rangle$ and initial temperature. This scaling will most probably be monotonic

$$\langle\tau\rangle = F\left(\frac{T_{freeze\text{-}out}}{T_{initial}}\right) \sim \sigma^n. \tag{25}$$

More complicated scalings, with minima and sharp transitions, will in general lead to a violation of the universal scaling, since events with similar final multiplicities could in this case have different v_2/ϵ.

To fully appreciate these constraints, it must be remembered that the lifetime strongly depends on the system's longitudinal initial conditions, and in particular Landau and Bjorken type initial conditions, with the same equation of state and transport coefficients, will lead to very different $\langle\tau\rangle$s [43].

Thus, the scaling of v_2 rules out a transition of the system from the Landau to the Bjorken limit such as the one seen in the top panel of Fig. 3, in the considered range of energies and system sizes: Such a transition would mean that two events with the same $\frac{1}{S}\frac{dN}{dy}$, one high-energy noncentral, the other low-energy central, would correspond to two different lifetimes (the first close to the Bjorken limit, the second to the Landau limit), and hence, in general, to two v_2/ϵ.

The monotonic increase of v_2/ϵ further constrains either the initial conditions to be far away from the Bjorken limit, or the mean free path to be negligible, at all energies: As initially inferred in [1], and explicitly shown in [13–15] v_2 is a *self-quenching* signature, which saturates after a finite time τ_{v2}, with $\tau_{v2} \ll \langle\tau\rangle$ in the Bjorken limit [13–15]. The v_2 scaling than implies that the system never reaches τ_{v2}. If it did, systems with different $\langle\tau\rangle_{1,2} > \tau_{v2}$ but the same ϵ would have the same v_2/ϵ. It is clear that such systems would, in general, have very different $\frac{1}{S}\frac{dN}{dy}$, breaking the scaling.

The universal scaling of v_2 in pseudo-rapidity space observed in [27] adds a further layer of constraints to Eq. (23). It appears that the σ is also a uniquely determined universal function of the relative position of the volume element in rapidity space. This rules out a "Landau"/fire-streak model where the system is closely localized in rapidity. Instead, the fireball evolves in a way that is both local in rapidity, and strongly rapidity-dependent. Perhaps the BGK initial condition [46], could provide such an ansatz, although it would imply that such a geometry holds, to a good approximation, up to AGS energies.

The picture suggested by the "universal" scaling of v_2 is then considerably different from the "RHIC reaches the perfect fluid" scenario: The "fluid" produced in heavy ion collisions, at all energies and system sizes where the scaling holds, should have a comparable l_{mfp}, equation of state (c_s) and longitudinal structure of the initial condition. All that varies is lifetime $\langle\tau\rangle$, which is uniquely determined by the initial density multiplied by longitudinal size, $\langle z\rangle n(T, \mu_B)$. This universal structure is robust inasmuch the scaling experimentally observed.

The alternative is that "we all have got it wrong" and the picture quantitatively analyzed in [19], of a very weakly interacting system, is the appropriate one for describing heavy ion collisions, from AGS to RHIC. But, aside from the difficulty in modeling such a large v_2 in this picture, the conclusions in the previous paragraph would actually *not* be changed: For the scaling shown in [19] to hold, it is necessary for $\langle v_{ij}\rangle\sigma_{ij}$, where v_{ij} is the relative speed and σ_{ij} the cross-sectional area of the system's microscopic degrees of freedom, not to change significantly with energy and system size. This is equivalent to requiring the microscopic properties of the system, such as viscosity and equation of state, to remain the same.

Experimentally it will be very interesting to see at what point, in *low* energy collisions, is the universal scaling observed here broken. This point could well be the critical σ that produces a deconfined system.

Perhaps a greater energy and system size exploration around the region of the so-called "kink," "horn," "step" [44] anomaly can yield discoveries. As seen in Fig. 5 (lower panel, $\frac{1}{S}\frac{dN}{dy} \sim 6\ \mathrm{fm}^{-2}$), there might be a hint of splitting in the scaling curve; While the error bars abundantly drown out any firm evidence at this point, the approximate coincidence with the features highlighted in [44], as well as the breaking of HBT radii scaling with multiplicity and $\sqrt{s}$ [45] are suggestive. Likewise, it will be interesting to see if peripheral LHC collisions, where the initial temperature should lie in the asymptotic freedom regime, can be related to central top energy RHIC collisions.

The observation that the scaling could apply not just with integrated p_T but within p_T bins [26,27] also deserves further exploration. One could object that, since $\langle p_T\rangle$ is energy dependent, a meaningful comparison across different energy regimes cannot be obtained. However, in the hydrodynamic picture $\langle p_T\rangle$ is also strongly system-size dependent because of the growth of transverse flow, yet [26,27] finds a p_T specific scaling to hold when Cu-Cu and Au-Au are compared. We await further, energy dependent results in this direction, but remark that the breaking of this scaling could signal the energy scale at which equilibration stops applying.

IV. CONCLUSION

In conclusion, we have shown that the scaling of v_2/ϵ with $\frac{1}{S}\frac{dN}{dy}$ places tight constraints on the hydrodynamic initial conditions in heavy ion collisions. It imposes an energy-independent relationship between initial energy density and longitudinal size, and makes it likely that longitudinal structure and microscopic parameters, such as the initial temperature, equation of state and viscosity are comparable in the considered range of energies and system sizes. We have suggested that looking for when the given scaling breaks might yield information about the critical energy and system size at which we can speak of a deconfined collective phase.

ACKNOWLEDGMENTS

The author would like to thank the Von Humboldt foundation and Frankfurt university for the support provided, the institute for Nuclear Physics in Krakow for hospitality in the days most of this paper was written, and Miklos Gyulassy, Sangyong Jeon, Sergei Voloshin Wojciech Florkowski, Wojciech Broniowski, and Piotr Bozek for useful discussions.

[1] J. Y. Ollitrault, Phys. Rev. D **46**, 229 (1992).
[2] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C **58**, 1671 (1998).
[3] D. Teaney, Phys. Rev. C **68**, 034913 (2003).
[4] I. Arsene *et al.* (BRAHMS Collaboration), Nucl. Phys. **A757**, 1 (2005).
[5] B. B. Back *et al.*, Nucl. Phys. **A757**, 28 (2005).
[6] J. Adams *et al.* (STAR Collaboration), Nucl. Phys. **A757**, 102 (2005).
[7] K. Adcox *et al.* (PHENIX Collaboration), Nucl. Phys. **A757**, 184 (2005).
[8] D. Molnar and S. A. Voloshin, Phys. Rev. Lett. **91**, 092301 (2003).
[9] R. J. Fries, B. Muller, C. Nonaka, and S. A. Bass, Phys. Rev. C **68**, 044902 (2003).
[10] S. Afanasiev *et al.* (PHENIX Collaboration), arXiv:nucl-ex/0703024.
[11] B. I. Abelev *et al.* (the STAR Collaboration), Phys. Rev. C **75**, 054906 (2007).
[12] R. A. Lacey and A. Taranenko, to be published in PoS CFRNC2006:021, 2006.
[13] D. Teaney, J. Lauret, and E. V. Shuryak, Phys. Rev. Lett. **86**, 4783 (2001).
[14] P. F. Kolb and U. W. Heinz, in *Invited review for 'Quark Gluon Plasma 3'*, edited by R. C. Hwa and X. N. Wang (World Scientific, Singapore), pp. 634-714.
[15] P. F. Kolb, U. W. Heinz, P. Huovinen, K. J. Eskola, and K. Tuominen, Nucl. Phys. **A696**, 197 (2001).
[16] R. J. Glauber, Phys. Rev. **100**, 242 (1955).
[17] D. J. Hofman (the PHOBOS Collaboration), arXiv:nucl-ex/0701051.
[18] S. A. Voloshin, arXiv:nucl-ex/0701038.
[19] H. Heiselberg and A. M. Levy, Phys. Rev. C **59**, 2716 (1999).
[20] S. A. Voloshin and A. M. Poskanzer, Phys. Lett. **B474**, 27 (2000).
[21] H. Caines, Eur. Phys. J. C **49**, 297 (2007); E. K. G. Sarkisyan and A. S. Sakharov, AIP Conf. Proc. **828**, 35 (2006).
[22] D. Kharzeev and M. Nardi, Phys. Lett. **B507**, 121 (2001).
[23] H. J. Drescher and Y. Nara, Phys. Rev. C **75**, 034905 (2007).
[24] R. S. Bhalerao and J. Y. Ollitrault, Phys. Lett. **B641**, 260 (2006).

[25] C. Adler *et al*. (STAR Collaboration), Phys. Rev. Lett. **87**, 182301 (2001).
[26] A. Adare *et al*. (PHENIX Collaboration), Phys. Rev. Lett. **98**, 162301 (2007).
[27] R. Nouicer (PHOBOS Collaboration), presented at 19th International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions: Quark Matter 2006 (QM2006), Shanghai, China, Nov. 14–20, 2006.
[28] G. Baym, B. L. Friman, J. P. Blaizot, M. Soyeur, and W. Czyz, Nucl. Phys. **A407**, 541 (1983).
[29] M. Chojnacki and W. Florkowski, Phys. Rev. C **74**, 034905 (2006).
[30] M. Chojnacki and W. Florkowski, arXiv:nucl-th/0702030.
[31] L. D. Landau, Izv. Akad. Nauk Ser. Fiz. **17**, 51 (1953).
[32] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[33] T. Hirano, Nucl. Phys. **A774**, 531 (2006); Eur. Phys. J. A **29**, 19 (2006).
[34] Y. Hama, R. P. G. Andrade, F. Grassi, O. J. Socolowski, T. Kodama, B. Tavares, and S. S. Padula, Nucl. Phys. **A774**, 169 (2006).
[35] P. Steinberg, Acta Phys. Hung. A **24**, 51 (2005).
[36] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon Press, New York, 1987).
[37] B. Muller and A. Schafer, Phys. Rev. C **73**, 054905 (2006).
[38] A. Milov (PHENIX Collaboration), J. Phys. Conf. Ser. **5**, 17 (2005).
[39] T. Hirano and M. Gyulassy, Nucl. Phys. **A769**, 71 (2006).
[40] U. W. Heinz and A. Kuhlman, Phys. Rev. Lett. **94**, 132301 (2005).
[41] C. Nepali, G. Fai, and D. Keane, Phys. Rev. C **73**, 034911 (2006).
[42] R. S. Bhalerao, J. P. Blaizot, N. Borghini, and J. Y. Ollitrault, Phys. Lett. **B627**, 49 (2005).
[43] M. Gyulassy and D. H. Rischke, Heavy Ion Phys. **17**, 261 (2003).
[44] P. Seyboth *et al*. (NA49 Collaboration), Acta Phys. Pol. B **36**, 565 (2005).
[45] M. Lisa, AIP Conf. Proc. **828**, 226 (2006).
[46] S. J. Brodsky, J. F. Gunion, and J. H. Kuhn, Phys. Rev. Lett. **39**, 1120 (1977).