Nuclear matrix elements of $0\nu\beta\beta$ decay with improved short-range correlations

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Nuclear matrix elements of the neutrinoless double beta $(0\nu\beta\beta)$ decays of 96 Zr, 100 Mo, 116 Cd, 128 Te, 130 Te, and 136 Xe are calculated for the light-neutrino exchange mechanism by using the proton-neutron quasiparticle random-phase approximation (pnQRPA) with a realistic nucleon-nucleon force. The particle-particle strength parameter g_{pp} of the pnQRPA is fixed by the data on the two-neutrino double β and single β decays. The finite size of a nucleon, the higher-order terms of nucleonic weak currents, and the nucleon-nucleon short-range correlations (s.r.c) are taken into account. The s.r.c. are computed by the traditional Jastrow method and by the more advanced unitary correlation operator method (UCOM). A comparison of the results obtained by the two methods reveals that the UCOM computed matrix elements are considerably larger than the Jastrow computed ones. This result is important to the assessment of the neutrino-mass sensitivity of present and future double β experiments.

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I. INTRODUCTION

The neutrinoless double beta $(0\nu\beta\beta)$ decay plays a key role in the search for massive Majorana neutrinos and their mass scale. The experimental search for $0\nu\beta\beta$ decay has become front-line physics because of the verification of the existence of neutrino mass by oscillation experiments [1] and the claimed discovery of the $0\nu\beta\beta$ decay [2,3]. At present, two important experiments are providing data: the NEMO 3 [4] and CUORICINO [5] experiments. A host of important future experiments are under research and development planning and construction [6]. For all these expensive experiments, the computed values of the involved nuclear matrix elements have become an important issue [7–10]. They are essential when one starts to extract quantitative neutrino properties from the measured data.

Many nuclear models of different types have been devised to compute the nuclear matrix elements of the $0\nu\beta\beta$ decay [11]. Essentially two complementary families of nuclear models are on the market: the nuclear shell model [12-14] and the proton-neutron quasiparticle random-phase approximation (pnQRPA) [11,15,16]. The pnQRPA is constructed to describe the energy levels of odd-odd nuclei and their beta decays to the neighboring even-even nuclei [17]. Also its derivative, renormalized pnQRPA [18], has been used [19,20] to compute double-beta matrix elements. The pnQRPA (and the renormalized pnQRPA) calculations can be fine tuned by the so-called particle-particle strength parameter g_{pp} , which controls the magnitude of the proton-neutron two-body interaction for the 1^+ intermediate states in double β decay [15,16]. The value of this parameter can be fixed by using either the data on $2\nu\beta\beta$ decays [20] or the data on single β decays [21,22]. In this work, we use the $2\nu\beta\beta$ data to fix the possible values of g_{pp} and cross-check the results against data on single β decays wherever possible.

In the mass mode of the $0\nu\beta\beta$ decay, a light virtual Majorana neutrino is exchanged by the two decaying neutrons of the initial nucleus. The average exchanged momentum is large, so the two neutrons tend to overlap. To prevent this, a Jastrow type of correlation function was introduced in Refs. [12,23] following the parametrization by Miller and Spencer [24]. This method, although microscopically inspired, is just a phenomenological way to introduce *short-range correlations* into the two-nucleon relative wave function. The Jastrow function simply cuts off the wave function of the two nucleons at short relative distances r leading to a violation of the norm of the wave function.

In the present calculations, we improve on the Jastrow method by engaging the more sophisticated microscopic approach of the unitary correlation operator method (UCOM) [25]. In the UCOM one obtains the correlated many-particle state from the uncorrelated one by a unitary transformation, and thus the norm of the correlated state is conserved and no amplitude is lost in the relative wave function. In the $0\nu\beta\beta$ calculations this leads to a more complete description of the relative wave function for small r, as was demonstrated in Ref. [26] for the decays of ⁴⁸Ca and ⁷⁶Ge. In this work and in Ref. [27], where the decays of ⁷⁶Ge and ⁸²Se were analyzed, it is demonstrated that the Jastrow procedure leads to the excessive reduction of 25–40% in the magnitudes of the $0\nu\beta\beta$ nuclear matrix elements. At the same time, the UCOM reduces the magnitudes of the matrix elements only by 4-16%. The magnitude of the short-range corrections affects the magnitudes of the nuclear matrix elements which, in turn, dictate the neutrino-mass sensitivity of the potentially successful future double β experiments. The notable differences between the Jastrow and UCOM corrections influence the cost estimates of large-scale experiments if a given neutrino-mass sensitivity is wanted.

In this article, we continue the work of Ref. [27], in which the $0\nu\beta\beta$ nuclear matrix elements of ⁷⁶Ge and ⁸²Se were derived. We apply the UCOM and Jastrow short-range correlations on matrix elements derived by the pnQRPA method and corrected for the *higher-order terms of nucleonic weak current* and the *nucleon's finite size* using the recipes of Refs. [20,28]. We analyze the $0\nu\beta\beta$ decay matrix elements of ⁹⁶Zr, ¹⁰⁰Mo, ¹¹⁶Cd, ¹²⁸Te, ¹³⁰Te, and ¹³⁶Xe for all the mentioned corrections. The necessary theoretical background is briefly described in Sec. II, and the numerical application is

reviewed in Sec. III. The results are discussed in Sec. IV, and the summary and conclusions are presented in Sec. V.

II. THEORETICAL BACKGROUND

We start this short review of the theory by presenting the expression for the half-life of the $2\nu\beta\beta$ decay:

$$\left[t_{1/2}^{(2\nu)}(0_i^+ \to 0_f^+)\right]^{-1} = G^{(2\nu)} \left|M_{\text{DGT}}^{(2\nu)}\right|^2.$$
 (1)

The transition proceeds from the initial ground state 0_i^+ to the final ground state 0_f^+ . Here $G^{(2\nu)}$ is an integral over the phase space of the leptonic variables [11,29]. The involved double Gamow–Teller matrix element $M_{\text{DGT}}^{(2\nu)}$ can be written as

$$M_{\text{DGT}}^{(2\nu)} = \sum_{n} \frac{\left(0_{f}^{+} \| \sum_{j} \sigma(j)t_{j}^{-} \| 1_{n}^{+}\right)}{\left(\frac{1}{2}Q_{\beta\beta} + E_{n} - M_{i}\right)/m_{e} + 1} \times \left(1_{n}^{+} \left\| \sum_{j} \sigma(j)t_{j}^{-} \right\| 0_{i}^{+}\right), \qquad (2)$$

where the transition operators are the usual Gamow-Teller operators for β^- transitions, $Q_{\beta\beta}$ is the $2\nu\beta\beta Q$ value, E_n is the energy of the *n*th intermediate state, M_i is the mass energy of the initial nucleus, and m_e is the rest mass of the electron. It has to be noted here that the expression (2) is scaled by the electron rest mass to yield a dimensionless matrix element. This definition deviates from that of some other authors, such as in Ref. [20], where the scaling is not done.

The $0\nu\beta\beta$ decay can proceed via the exchange of a light virtual Majorana neutrino. Assuming this neutrino-mass mechanism to be the dominant one, the inverse of the $0\nu\beta\beta$ half-life can be written as

$$\begin{bmatrix} t_{1/2}^{(0\nu)}(0_i^+ \to 0_f^+) \end{bmatrix}^{-1} = G_1^{(0\nu)} \left(\frac{\langle m_\nu \rangle}{m_e} \right)^2 (M^{(0\nu)})^2,$$

$$M^{(0\nu)} = M_{\rm GT}^{(0\nu)} - \left(\frac{g_V}{g_A} \right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}.$$
(3)

In the above expression, $M^{(0\nu)}$ is the total nuclear matrix element consisting of the Fermi, Gamow-Teller, and tensor contributions. The effective mass of the neutrino is given by

$$\langle m_{\nu} \rangle = \sum_{j} \lambda_{j}^{\rm CP} m_{j} |U_{ej}|^{2}, \qquad (4)$$

where λ_j^{CP} is the CP phase, and U_{ej} is a component of the neutrino mixing matrix. The definition of the leptonic phase-space factor $G_1^{(0\nu)}$ can be found in Ref. [11].

The nuclear matrix elements involved in the mass mode of the $0\nu\beta\beta$ decay are defined as

$$M_F^{(0\nu)} = \sum_a (0_f^+ || h_F(r_{mn}, E_a) || 0_i^+), \tag{5}$$

$$M_{\rm GT}^{(0\nu)} = \sum_{a} (0_f^+ || h_{\rm GT}(r_{mn}, E_a) \sigma_m \cdot \sigma_n || 0_i^+), \tag{6}$$

where the summation runs over all the intermediate states and the integration is taken over the relative coordinate $r_{mn} =$ $|\mathbf{r}_m - \mathbf{r}_n|$ between the nucleons *m* and *n*. The neutrino potential $h_K(r_{mn}, E_a)$, where K = F, GT, is defined as

$$h_K(r_{mn}, E_a) = \frac{2}{\pi} R_A \int dq \frac{q h_K(q^2)}{q + E_a - (E_i + E_f)/2} j_0(q r_{mn}),$$
(7)

where $R_A = 1.2A^{1/3}$ fm is the nuclear radius and j_0 is the spherical Bessel function. The term $h_K(q^2)$ in Eq. (7) includes the contributions arising from the induced currents and the finite nucleon size [20,28].

Next we write the nuclear matrix elements explicitly in the pnQRPA framework. They are given by

$$M_{K}^{(0v)} = \sum_{J^{\pi}, k_{1}, k_{2}, J'} \sum_{pp'nn'} (-1)^{j_{n}+j_{p'}+J+J'} \sqrt{2J'+1} \\ \times \left\{ \begin{array}{l} j_{p} & j_{n} & J \\ j_{n'} & j_{p'} & J' \end{array} \right\} (pp': J'||\mathcal{O}_{K}||nn': J') \\ \times \left(0_{f}^{+} \left\| [c_{p'}^{\dagger} \tilde{c}_{n'}]_{J} \right\| J_{k_{1}}^{\pi} \right) \langle J_{k_{1}}^{\pi} \big| J_{k_{2}}^{\pi} \rangle \left(J_{k_{2}}^{\pi} \left\| [c_{p}^{\dagger} \tilde{c}_{n}]_{J} \right\| 0_{i}^{\dagger} \right),$$

$$(8)$$

where k_1 and k_2 label the different pnQRPA solutions for a given multipole J^{π} . The operators \mathcal{O}_K inside the two-particle matrix element derive from Eqs. (5) and (6), and they can be written as

$$\mathcal{O}_F = h_F(r, E_k), \quad \mathcal{O}_{\text{GT}} = h_{\text{GT}}(r, E_k)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \quad r = |\mathbf{r}_1 - \mathbf{r}_2|.$$
(9)

The expression for the pnQRPA transition densities $(0_f^+||[c_{p'}^{\dagger}\tilde{c}_{n'}]_J||J_{k_1}^{\pi})$ and $(J_{k_2}^{\pi}||[c_p^{\dagger}\tilde{c}_n]_J||0_i^+)$, and for the overlap factor $\langle J_{k_1}^{\pi}|J_{k_2}^{\pi}\rangle$ can be found, e.g., in Refs. [11,17].

The traditional way to include the short-range correlations in the $0\nu\beta\beta$ decay calculations is by introducing the Jastrow correlation function $f_J(r)$, which depends on the relative distance $r = |\mathbf{r}_1 - \mathbf{r}_2|$ of two nucleons. In the Jastrow scheme, the uncorrelated operator \mathcal{O} is replaced by the correlated operator \mathcal{O}_J by a procedure

$$(0_f^+||\mathcal{O}||0_i^+) \to (0_f^+||\mathcal{O}_J||0_i^+) = (0_f^+||f_J\mathcal{O}f_J||0_i^+).$$
(10)

A typical choice for the function f_J in $0\nu\beta\beta$ calculations is [30]

$$f_J(r) = 1 - e^{-ar}(1 - br^2),$$
 (11)

with $a = 1.1 \text{ fm}^2$ and $b = 0.68 \text{ fm}^2$. Such application of the Jastrow correlation function is very rudimentary; consequently, the Jastrow correlation cuts out the $r \leq 1$ fm part from the relative two-particle wave function. In Ref. [26] it was demonstrated that this leads to overestimation of the effects of short-range correlations on the many-body wave function.

To circumvent the difficulties associated with the Jastrow correlations, the more refined unitary correlation operator method (UCOM) [25] was used in the $0\nu\beta\beta$ decay calculations of Ref. [26]. In UCOM one obtains the correlated many-body state $|\tilde{\Psi}\rangle$ from the uncorrelated one as

$$|\tilde{\Psi}\rangle = C|\Psi\rangle,\tag{12}$$

where *C* is the unitary correlation operator. The operator *C* is a product of two unitary operators: $C = C_{\Omega}C_r$, where C_{Ω}

describes short-range tensor correlations and C_r the central correlations. Because of the unitary of the operator C the norm of the correlated state is conserved. Moreover, since the correlated matrix element of the operator \mathcal{O} can be written as

$$\langle \tilde{\Psi} | \mathcal{O} | \tilde{\Psi}' \rangle = \langle \Psi | C^{\dagger} \mathcal{O} C | \Psi' \rangle = \langle \Psi | \tilde{\mathcal{O}} | \Psi' \rangle, \qquad (13)$$

it is therefore equivalent to use either correlated states or correlated operators. For the Fermi and Gamow-Teller $0\nu\beta\beta$ nuclear matrix elements, the effect of the tensor correlation operator C_{Ω} vanishes and one is thus left only with the central correlation operator. The UCOM parameters used in our $0\nu\beta\beta$ calculations are the Bonn-A parameters taken from [31].

III. NUMERICAL APPLICATION

In Ref. [27] we applied the pnQRPA to compute the $0\nu\beta\beta$ nuclear matrix elements of ⁷⁶Ge and ⁸²Se in the model space containing the $1p-0f-2s-1d-0g-0h_{11/2}$ single-particle orbitals, both for protons and neutrons. Here we add to this model space the spin-orbit partner $0h_{9/2}$ of the $0h_{11/2}$ orbital, both for protons and neutrons, to describe the decays of ⁹⁶Zr and $^{100}\mbox{Mo}.$ For the rest of the decays, namely, for the $^{116}\mbox{Cd},$ ¹²⁸Te, ¹³⁰Te, and ¹³⁶Xe decays, we extended the proton and neutron model spaces to include the 1p-0f-2s-1d-0g-2p-1f-0h single-particle orbitals. The single-particle energies were obtained from a spherical Coulomb-corrected Woods-Saxon potential with a standard parametrization, optimized for nuclei near the line of β stability. Slight adjustments were made for some of the energies at the vicinity of the proton and neutron Fermi surfaces to reproduce better the low-energy spectra of the neighboring odd-A nuclei and those of the intermediate nuclei.

We have used the Bonn-A *G* matrix as the two-body interaction, and we have renormalized it in the standard way [32,33] by fitting the pairing parameters of the BCS by comparing with the phenomenological pairing gaps, extracted from the atomic mass tables. The particle-hole parameter g_{ph} of the pnQRPA affects the position of the giant Gamow-Teller resonance, and its value was fixed by the available data on the location of the giant state. Because of this phenomenological renormalization of the two-body interaction, we did not perform an additional UCOM renormalization [34].

After fixing all the Hamiltonian parameters, the only free parameter left is the proton-neutron particle-particle parameter g_{pp} of the pnQRPA. We obtained the physical values of g_{pp} by using the method of Refs. [20,27]. Consequently, we used the extracted experimental matrix elements of Ref. [20] that include the experimental error limits and the uncertainty in the value of the axial-vector coupling constant $1.0 \le g_A \le 1.25$. The resulting intervals of the experimental $2\nu\beta\beta$ matrix elements are shown in the second column of Table I, and they are scaled by the electron rest mass according to Eq. (2). By performing the pnQRPA calculations of the $2\nu\beta\beta$ matrix elements, the ranges of experimental matrix elements were subsequently converted to the intervals of g_{pp} values shown in column three of Table I. For some cases, there exists log ft data on β^- decay from the first 1⁺ state of the intermediate nucleus to the

TABLE I. Values of the g_{pp} parameter extracted from the data. First column shows the decay and the second column the matrix element values extracted from the $2\nu\beta\beta$ decay data by Ref. [20]. Third column gives the range of g_{pp} corresponding to the matrix elements of the second column. The g_{pp} ranges of the last column were extracted from the available β^- decay data.

Decay	m.e. $(2\nu\beta\beta)$	$g_{\rm pp}(2\nu\beta\beta)$	$g_{\rm pp}(\beta^-)$
96 Zr \rightarrow 96 Mo	0.026-0.112	1.06-1.11	_
$^{100}Mo \rightarrow ^{100}Ru$	0.107-0.181	1.07-1.09	1.07-1.08
$^{116}Cd \rightarrow ^{116}Sn$	0.058-0.102	0.97-1.01	0.82-0.84
$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$	0.011-0.037	0.89-0.92	0.86-0.88
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	0.014-0.054	0.84-0.90	_
136 Xe \rightarrow 136 Ba	≤0.023	≥0.74	-

ground state of the double β daughter nucleus. Applying the above procedure to these data leads to the experimental β^- matrix elements and the corresponding ranges of g_{pp} , listed in the last column of Table I.

As can be seen from Table I, the values of g_{pp} extracted from the β^- and $2\nu\beta\beta$ data are (roughly) compatible for the decays of ¹⁰⁰Mo and ¹²⁸Te, whereas for the ¹¹⁶Cd decay this is not the case. This discrepancy was already pointed out in Ref. [21]. The reason for this discrepancy is not clear, but an interesting observation is that the ¹¹⁶Cd decay obeys very closely the single-state dominance hypothesis, whereas the ¹²⁸Te decay obeys it slightly less and the ¹⁰⁰Mo decay the least, as clearly shown in Table 1 of Ref. [35].

IV. DISCUSSION OF RESULTS

In Table II we show the evolution of the values of the $0\nu\beta\beta$ nuclear matrix elements as we add more corrections to the bare matrix element. First we list the g_{pp} value used, which was taken to be in the middle of the g_{pp} interval of Table I. The following columns list the bare matrix element (b.m.e.), the matrix element including the higher-order terms of the nucleonic weak current (b.m.e.+A), and the matrix element with finite nucleon size effects added (b.m.e.+A+B). In the last two columns we have added either the Jastrow (C) or UCOM (D) short-range correlations. The value $g_A = 1.25$ was used in these calculations.

Table II shows that the Jastrow method produces a much larger reduction in the magnitude of $M^{(0\nu)}$ than the UCOM. The UCOM (D) with higher-order term (A) and finite nucleon size (B) corrections included seems to reduce the magnitude of $M^{(0\nu)}$ by a rough factor of 2/3 from its bare value. A similar scaling factor was also present for the $0\nu\beta\beta$ results of ⁷⁶Ge and ⁸²Se in Ref. [27]. At the same time one obtains a reduction factor of 1/2 for the corresponding Jastrow (C) results.

We visualize the g_{pp} dependence of $M^{(0\nu)}$ in Fig. 1. The calculations included the higher-order term (A), finite-size (B), and UCOM (D) corrections. The calculations are shown for the interval $g_{pp} \leq 1.1$, the upper limit lying near the breaking point of the pnQRPA for all shown nuclear systems.

TABLE II. Total matrix element $M^{(0\nu)}$ of Eq. (3) computed by correcting the bare matrix element (b.m.e) for the higher-order terms of the nucleonic weak current (A), for the finite nucleon size (B), and for either the Jastrow (C) or UCOM (D) correlations. The value of g_{pp} used is indicated in the second column.

Nucleus	$g_{ m pp}$	b.m.e.	+A	+A+B	+A+B+C	+A+B+D
⁹⁶ Zr	1.085	-5.308	-4.814	-3.736	-2.454	-3.521
¹⁰⁰ Mo	1.08	-6.126	-5.571	-4.358	-2.914	-4.113
¹¹⁶ Cd	0.99	-5.726	-5.172	-4.263	-3.169	-4.076
¹²⁸ Te	0.905	-7.349	-6.673	-5.260	-3.563	-4.979
¹³⁰ Te	0.87	-6.626	-6.021	-4.777	-3.285	-4.530
¹³⁶ Xe	0.74	-4.715	-4.269	-3.478	-2.537	-3.317

In Fig. 2 we have plotted the multipole decomposition of the total $0\nu\beta\beta$ matrix element $M^{(0\nu)}$ for all calculated decays. The upper (lower) end of each bar corresponds to the lower (upper) end of the g_{pp} interval of Table I. For the ¹³⁶Xe decay we can only give upper limits since only the lower limit of g_{pp} is known, as indicated in the last line of Table I. From Fig. 2 one can see that the widest spread appears in the bar corresponding to the 1^+ contribution. Furthermore, the g_{pp} interval extracted from the $2\nu\beta\beta$ data confines the 1⁺ contribution in a striking way: for ⁹⁶Zr and ¹⁰⁰Mo the 1⁺ contribution is of opposite sign to the other contributions. This interference with the rest of the contributions reduces the magnitude of $M^{(0\nu)}$ for these two decays. Another notable feature is that the 1^- contribution is always the leading one, the 2⁻ contribution being usually of comparable size. This pattern is different from the one of ⁷⁶Ge and 82 Se decays where the 2⁻ contribution was the dominant one [26]. All these observations are in qualitative agreement with the results of Ref. [20].

We have collected the obtained g_{pp} limits of Table I and their corresponding $0\nu\beta\beta$ matrix elements of Eq. (3) in Table III. It is worth pointing out that the magnitudes of the tensor matrix element $M_T^{(0\nu)}$ are quite small and not indicated in the table. As discussed earlier, the g_{pp} limits arise from both the $2\nu\beta\beta$ and β decay data. For ¹³⁶Xe, only the experimental lower limit of the $2\nu\beta\beta$ half-life is available, yielding only a lower limit for g_{pp} . For ¹²⁸Te the available β decay data give

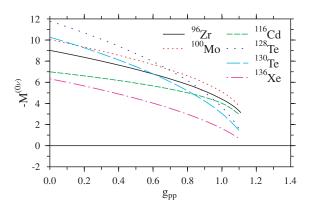


FIG. 1. (Color online) Calculated values of $M^{(0\nu)}$ for the indicated $0\nu\beta\beta$ decays as functions of g_{pp} . The UCOM (D) was used with $g_A = 1.25$ including all the other (+A+B) corrections.

 $g_{\rm pp} = 0.86$, and for ¹¹⁶Cd they give $g_{\rm pp} = 0.82$. In the last column of Table III, we also tabulate our predicted half-life limits in units of yr/($\langle m_{\nu} \rangle$ [eV])². The β decay data yield a different $g_{\rm pp}$ interval than the data on $2\nu\beta\beta$ decays for ¹¹⁶Cd and ¹²⁸Te. This discrepancy is especially striking for ¹¹⁶Cd. The implications of this discrepancy and its cure are still open questions [21].

Our results for $0\nu\beta\beta$ nuclear matrix elements disagree with those of Refs. [20] and [28]. In fact, just recently, the Tübingen-Caltec Collaboration [36] corrected their results for a coding error in their computer program. The results of the Erratum [36] agree nicely with our results for the Jastrow corrected nuclear matrix elements. This means that one can safely say that the Jastrow short-range correlations reduce the values of matrix elements some 25–40%. On the other hand, for the UCOM we obtain only a 4–16% reduction. For this reason, our present UCOM corrected matrix elements are larger than the Jastrow corrected ones. Such differences give rise to big differences in the predicted $0\nu\beta\beta$ half-lives for a given value of the effective neutrino mass $\langle m_{\nu} \rangle$. This invariably affects the sensitivity estimates for the presently running and planned double β experiments.

TABLE III. Calculated $0\nu\beta\beta$ nuclear matrix elements, the g_{pp} and g_A values used and the resulting half-lives. The UCOM and other corrections are included. The half-lives $t_{1/2}^{(0\nu)}$ are expressed in units of $yr/(\langle m_{\nu}\rangle [eV])^2$.

Nucleus	$g_{ m pp}$	g_A	$M_F^{(0\nu)}$	$M_{ m GT}^{(0 u)}$	$M^{(0\nu)}$	$t_{1/2}^{(0\nu)}$
⁹⁶ Zr	1.06	1.00	1.350	-2.969	-4.319	6.1×10^{23}
	1.11	1.25	1.261	-2.315	-3.117	4.7×10^{23}
¹⁰⁰ Mo	1.07	1.00	1.583	-3.266	-4.849	6.2×10^{23}
	1.09	1.25	1.543	-2.950	-3.931	3.8×10^{23}
¹¹⁶ Cd	$0.82 (\beta^{-} \text{ decay})$	1.25	1.427	-4.021	-4.928	2.3×10^{23}
	0.97	1.00	1.310	-3.372	-4.682	6.3×10^{23}
	1.01	1.25	1.275	-3.124	-3.935	3.6×10^{23}
¹²⁸ Te	$0.86 \ (\beta^{-} \text{ decay})$	1.25	1.939	-4.276	-5.509	5.2×10^{24}
	0.89	1.00	1.866	-3.975	-5.841	1.1×10^{25}
	0.92	1.25	1.792	-3.650	-4.790	6.9×10^{24}
¹³⁰ Te	0.84	1.00	1.699	-3.743	-5.442	5.3×10^{23}
	0.90	1.25	1.575	-3.219	-4.221	3.5×10^{23}
¹³⁶ Xe	0.74	1.00	1.104	-2.615	-3.719	1.1×10^{24}

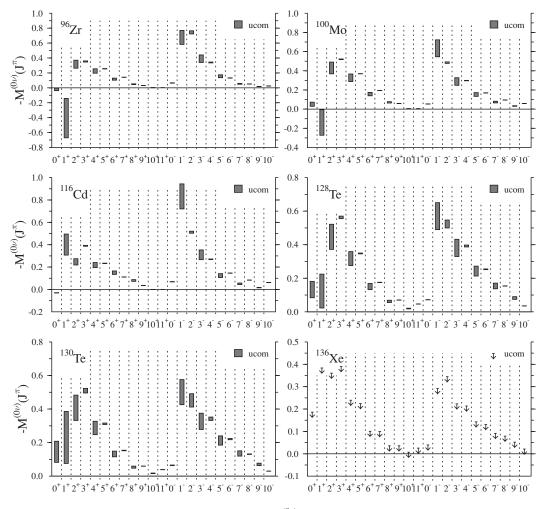


FIG. 2. Multipole decomposition of $M^{(0\nu)}$ for the calculated $0\nu\beta\beta$ decays.

V. SUMMARY AND CONCLUSIONS

We have calculated the $0\nu\beta\beta$ nuclear matrix elements for the decays of 96 Zr, 100 Mo, 116 Cd, 128 Te, 130 Te, and 136 Xe by using the proton-neutron quasiparticle random-phase approximation with realistic two-body interactions in realistic singleparticle spaces. We have corrected the bare matrix elements for higher-order terms of the nucleonic weak currents, for the nucleon's finite- ize, and for the nucleon-nucleon short-range correlations. The short-range correlations were included by using the unitary correlation operator formalism. This method is superior to the rudimentary Jastrow procedure traditionally adopted for the $0\nu\beta\beta$ calculations.

The UCOM reduces the magnitudes of the matrix elements less than the Jastrow procedure. This leads to larger

- S. Fukuda *et al.*, Phys. Rev. Lett. **86**, 5651 (2001); Q. R. Ahmad *et al.*, *ibid.* **89**, 011301 (2002); H. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002); K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003); M. H. Ahn *et al.*, *ibid.* **90**, 041801 (2003); S. N Ahmed *et al.*, *ibid.* **92**, 181301 (2004).
- [2] H. V. Klapdor-Kleingrothaus *et al.*, Mod. Phys. Lett. A 16, 2409 (2001).

matrix elements and shorter $0\nu\beta\beta$ half-lives as compared to some recent calculations quoted in the literature. This has a notable influence on the estimated neutrino-mass sensitivities of the presently running and future double β experiments.

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- [3] H. V. Klapdor-Kleingrothaus *et al.*, Phys. Lett. **B586**, 198 (2004).
- [4] R. Arnold et al., Phys. Rev. Lett. 95, 182302 (2005).
- [5] C. Arnaboldi et al., Phys. Rev. Lett. 95, 142501 (2005).
- [6] A. S. Barabash, Phys. At. Nucl. 67, 458 (2004).
- [7] S. R. Elliott and P. Vogel, Annu. Rev. Nucl. Part. Sci. 52, 115 (2002).

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- [8] S. R. Elliott and J. Engel, J. Phys. G 30, R183 (2004).
- [9] S. M. Bilenky and S. T. Petcov, hep-ph/0405237.
- [10] V. M. Gehman and S. R. Elliott, J. Phys. G: Nucl. Part. Phys. 34, 667 (2007).
- [11] J. Suhonen and O. Civitarese, Phys. Rep. 300, 123 (1998).
- [12] W. C. Haxton and G. J. Stephenson Jr., Prog. Part. Nucl. Phys. 12, 409 (1984).
- [13] E. Caurier, F. Nowacki, A. Poves, and J. Retamosa, Nucl. Phys. A654, 973c (1999).
- [14] E. Caurier et al., Rev. Mod. Phys. 77, 427 (2005).
- [15] P. Vogel and M. R. Zirnbauer, Phys. Rev. Lett. 57, 3148 (1986).
- [16] O. Civitarese, A. Faessler, and T. Tomoda, Phys. Lett. B194, 11 (1987).
- [17] J. Suhonen, From Nucleons to Nucleus: Concepts of Microscopic Nuclear Theory (Springer, Berlin, 2007).
- [18] J. Toivanen and J. Suhonen, Phys. Rev. Lett. 75, 410 (1995).
- [19] J. Toivanen and J. Suhonen, Phys. Rev. C 55, 2314 (1997).
- [20] V. A. Rodin, A. Faessler, F. Šimkovic, and P. Vogel, Nucl. Phys. A766, 107 (2006).
- [21] J. Suhonen, Phys. Lett. B607, 87 (2005).
- [22] O. Civitarese and J. Suhonen, Phys. Lett. B626, 80 (2005); Nucl. Phys. A761, 313 (2005).
- [23] J. Engel, P. Vogel, and M. R. Zirnbauer, Phys. Rev. C 37, 731 (1988).

- [24] G. A. Miller and J. E. Spencer, Ann. Phys. (NY) 100, 562 (1976).
- [25] H. Feldmeier, T. Neff, R. Roth, and J. Schnack, Nucl. Phys. A632, 61 (1998).
- [26] M. Kortelainen, O. Civitarese, J. Suhonen, and J. Toivanen, Phys. Lett. B647, 128 (2007).
- [27] M. Kortelainen and J. Suhonen, Phys. Rev. C 75, 051303(R) (2007).
- [28] F. Šimkovic, G. Pantis, J. D. Vergados, and A. Faessler, Phys. Rev. C 60, 055502 (1999).
- [29] M. Doi, T. Kotani, and E. Takasugi, Prog. Theor. Phys. Suppl. 83, 1 (1985).
- [30] T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).
- [31] T. Neff and H. Feldmeier, Nucl. Phys. A713, 311 (2003).
- [32] J. Suhonen, T. Taigel, and A. Faessler, Nucl. Phys. A486, 91 (1988).
- [33] J. Suhonen, Nucl. Phys. A563, 205 (1993); A700, 649 (2002).
- [34] R. Roth, H. Hergert, P. Papakonstantinou, T. Neff, and H. Feldmeier, Phys. Rev. C 72, 034002 (2005).
- [35] O. Civitarese and J. Suhonen, Nucl. Phys. A653, 321 (1999).
- [36] V. A. Rodin, A. Faessler, F. Šimkovic, and P. Vogel, arXiv:0706.4304v1 [nucl-th], and to appear as Erratum in Nucl. Phys. A.