Testing approximations beyond the proton-neutron quasiparticle random phase approximation

O. Civitarese,[1,*] J. G. Hirsch,[2,†] A. Mariano,[1,‡] and M. Reboiro[1,3,§]
[1]*Department of Physics. University of La Plata, C. C. 67 1900 La Plata, Argentina*
[2]*Instituto de Ciencias Nucleares, UNAM, A. P. 70-543, 04510 Mexico City, D. F., Mexico*
[3]*Faculty of Engineering, University of Lomas de Zamora, Km 2 (1836) Lavallol, Argentina*
(Received 14 February 2007; published 2 August 2007)

In this work we analyze the validity of recently proposed extensions of the Quasiparticle Random Phase Approximation (QRPA). Particularly, we focus our attention on the Fully Renormalized QRPA (FRQRPA). We found that the results of this approximation do not differ from the results of the QRPA. This finding is supported by a detailed comparison between both formalisms, their assumptions and approximations, in the context of realistic calculations.

DOI: 10.1103/PhysRevC.76.024303 PACS number(s): 21.60.Fw, 21.60.Jz

I. INTRODUCTION

The validity of the Quasiparticle Random Phase Approximation (QRPA) has been tested exhaustively since it was first proposed by Baranger [1]. Various extensions of the method and its drawbacks can be found in textbooks [2,3]. For the sake of concreteness we shall focus on the proton-neutron version of the QRPA (pn-QRPA), where the elementary building blocks are two quasiparticle configurations of one quasiproton and one quasineutron [4–6]. The literature on the subject is extensive, and for reasons of brevity we shall refer the reader to the articles quoted in Ref. [7], where the essentials of the method are described. During the last decades the interest in the use of the QRPA and pn-QRPA was prompted by the need to calculate energy spectra and transitions relevant for the study of exotic nuclear electroweak processes [7]. Particularly, the question about the comparison between weak-coupling shell model results [8] and pn-QRPA [5,6] has attracted considerable attention. The answer to this question, that is the need to treat in the pn-QRPA not only the dominant particle-hole channel of the residual interactions but also the particle-particle and hole-hole channels, was the starting point for a blooming theoretical activity. It was shown that a renormalization of the coupling constant of the particle-particle channel of the residual proton-neutron interaction, g_{pp}, was needed to explain the strong suppression of the observed matrix elements of the two-neutrino double-β-decay [5,6]. However, this renormalization also gives rise to instabilities of the pn-QRPA approach. This was coined the g_{pp} problem. In the years elapsed since the first articles on the subject appeared, the theory has been advanced in a sort of chaotic way. The use of renormalizations of the pn-QRPA, the renormalized pn-QRPA (RQRPA) of Ref. [9], revisited and questioned in Ref. [10], was followed by higher order corrections, perturbative approaches, second renormalizations, self-consistent attempts, etc., to name some of the more representative articles on the subject [11–14]. The uncertainties introduced by the use of renormalized couplings may be summarized as follows: the pn-QRPA is a theory of small amplitude vibrations around a given minimum, the chosen mean field [15]. The effect of the particle-particle interactions, when renormalized beyond certain values, is to produce the breakdown of the approximation; that is, the vibrations may acquire infinite amplitudes as a consequence of the deformation added to the mean field [16]. In other words, if the strength of the attractive channels of the interactions, i.e., the coupling constant g_{pp}, is unrealistically renormalized, the ground state and the first excited state of the pn-QRPA approximation will invert their roles. The mechanism is similar to the renormalization of attractive particle-hole interactions between like-particles, if one is trying to force the spherical QRPA to produce large $BE(2)$ values. In that case the first excited 2^+ state will become the ground state and the system will be permanently deformed. If one insists on (a) keeping the values of g_{pp} needed to produce a strong suppression of the double Gamow-Teller matrix element and (b) including extra terms of the pn-QRPA expansion, the equation of motion will be altered to the point of losing stability. The violation of the sum rule is thus a result of this kind of added nonconsistency, which is alien to the pn-QRPA approach. In terms of a perturbative expansion, to which the pn-QRPA is naturally amenable as a theory of small vibrations around the mean field point [15], one is mixing orders in a nonconsistent manner.

The effects of particle-particle correlations upon the pn-QRPA have been discussed in the context of a phase transition and modeled in the framework of exactly solvable models. The reader is kindly referred to Ref. [17] for details about this point.

As a general view, the use of schematic, solvable models greatly helps at the time of evaluating the validity of a certain approximation. If the proposed approximation, like any of the various extensions of the pn-QRPA, works satisfactory in the framework of a solvable model, then one has definite room for improvement in realistic cases. However, if the approximations do not work in the simple cases, one may doubt about the use of them in realistic cases.

In this article we address the question of the validity of an approach, named the fully renormalized QRPA (FRQRPA), proposed in Refs. [18,19]. The authors of Refs. [18,19] argue

*civitare@fisica.unlp.edu.ar
†hirsch@nucleares.unam.mx
‡mariano@fisica.unlp.edu.ar
§reboiro@fisica.unlp.edu.ar

that they have found a remedy to the instabilities produced by the renormalization of the strength g_{pp} by replacing the two quasiparticle operators by their expression as proton-neutron (particle-hole) operators and by adjusting the BCS parameters accordingly. If true, it would be a great improvement but, unfortunately, for reasons that we discuss in this article, it seems that the proposed method suffers from inconsistencies that invalidate the claims about its rightness.

In a previous effort [20] we showed that the approximations of Refs. [18,19] do indeed suffer from some inconsistencies. The proof was based on the results of calculations performed in a schematic model. In this article we discuss in details the derivation of all the equations, since not all of the needed elements have been presented before. To test the approximations of Refs. [18,19], we apply the formalism to the case of a multilevel single particle basis and we use a realistic interaction that is the same as that used in Refs. [18,19]. The theory is presented in Sec. II. In Sec. III we present and discuss the results of a realistic calculation for the case of ^{76}Ge. Conclusions are drawn in Sec. IV.

II. FORMALISM

In the following, we briefly review the currently available procedure for dealing with the quasiparticle random phase approximation treatment of a Hamiltonian that includes single particle, pairing, and proton-neutron two-body interactions. Although the procedure is rather well known, we review again here the main results of the formalism to guide the discussion.

A. The BCS sector

The quasiparticle transformation is defined by

$$\alpha_q^\dagger = u_q a_q^\dagger - v_q a_{\bar{q}}, \tag{1}$$

where the subindex q stands for the complete set of quantum numbers needed to specify a single particle state. The transformation of the single particle plus pairing sector of the Hamiltonian to the quasiparticle basis yields the single quasiparticle Hamiltonian

$$H_{\text{s.p.}} + H_{\text{pairing}} = H_0 + \sum_q 2\Omega_q \epsilon_q \hat{\mathcal{N}}_q, \tag{2}$$

where H_0 is the correlated ground state energy, ϵ_q is the quasiparticle energy, and $\hat{\mathcal{N}}_q$ is the quasiparticle number operator $\hat{\mathcal{N}}_{p,n} = \frac{[\alpha_q^\dagger \alpha_q]^0}{\sqrt{2\Omega_q}}$ for protons (p) and neutrons (n), with $2\Omega_q = 2j_q + 1$. Additional two and four quasiparticle terms, which appear in the transformation of the single particle plus pairing Hamiltonian to the quasiparticle basis, are neglected because their vacuum expectation values vanish when taken with respect to the BCS vacuum.

For a general (nonseparable) two-body interaction, the use of the BCS formalism leads to the expressions

$$\begin{aligned} \epsilon_q &= \bar{e}_q A_q + \Delta_q B_q, \\ 0 &= -\Delta_q A_q + \bar{e}_q B_q, \end{aligned} \tag{3}$$

where

$$\bar{e}_q = e_q - \lambda_{p(n)} + \sum_{q'} \sqrt{\frac{\Omega_{q'}}{\Omega_q}} v_{q'}^2 F(qq, q'q', 0), \tag{4}$$

is the single particle energy corrected by self-energy terms, $\lambda_{p(n)}$ is the chemical potential for protons (neutrons). The quantity

$$\Delta_q = -\frac{1}{2} \sum_{q'} \sqrt{\frac{\Omega'_q}{\Omega_q}} u_{q'} v_{q'} G(qq, q'q', 0). \tag{5}$$

is the state dependent gap equation. In these equations $F(qq, q'q', 0)$ and $G(qq, q'q', 0)$ are the matrix elements of the monopole interaction in the particle-particle and particle-hole channels, respectively. The factors A_q and B_q are defined as

$$\begin{aligned} A_q &= \left(u_q^2 - v_q^2\right) \\ B_q &= 2u_q v_q. \end{aligned} \tag{6}$$

The occupation numbers u_q and v_q are constrained by the conditions

$$\begin{aligned} Z &\equiv \langle 0|\hat{\mathcal{Z}}|0\rangle \\ &= \sum_p 2\Omega_p v_p^2 + \sum_p \left(u_p^2 - v_p^2\right) \langle 0|\hat{\mathcal{N}}_p|0\rangle, \end{aligned} \tag{7}$$

$$\begin{aligned} N &\equiv \langle 0|\hat{\mathcal{N}}|0\rangle \\ &= \sum_n 2\Omega_n v_n^2 + \sum_n \left(u_n^2 - v_n^2\right) \langle 0|\hat{\mathcal{N}}_n|0\rangle. \end{aligned} \tag{8}$$

In the above equations, $\hat{\mathcal{N}}$ and $\hat{\mathcal{Z}}$ are the neutron and proton particle-number operators while $\hat{\mathcal{N}}_n$ and $\hat{\mathcal{N}}_p$ are the neutron and proton quasiparticle number operators, respectively. The second term, in both equations, vanishes when $|0\rangle$ is the BCS correlated ground state. If $|0\rangle$ includes correlations that go beyond the BCS level, the contribution of the second term is smaller than the first one by a factor $1/\Omega_q$.

In the standard BCS approximation, the second term of these equations is neglected. In the standard approach, the self-energy terms of the quasiparticle energy are neglected and the equations for the occupation factors v_q and u_q and the quasiparticle energies are solved, variationally, obeying average conservation of particle number, for neutrons and protons, separately. Because the equations are ordered in powers of the factor $2\Omega_q$, the BCS solution is said to be valid at order 1 in this expansion parameter, in a manner that is fully analogous to classification of the terms of the pairing interaction in the one shell case [21]. This is a crucial aspect of the calculations, because corrections to the QRPA should not be larger than the terms that are neglected in the BCS treatment. This is just the case of the approximations claimed in Refs. [18,19], as we show later on.

B. The QRPA sector

Once the single particle and the pairing sector of the Hamiltonian are approximately diagonalized in the quasiparticle basis, the residual interaction may be transformed to the

harmonic one-phonon basis defined by the states

$$|kJM\rangle \equiv \Gamma^{\dagger}(kJM)|0\rangle, \tag{9}$$

for each angular momentum J, projection M, and eigenvalue index k. The one-phonon creation operator is constructed as a linear combination of creation and annihilation of proton-neutron quasiparticle pairs,

$$\Gamma^{\dagger}(kJM) = \sum_{pn}(X_{pn}(kJ)A^{\dagger}_{pn}(JM) - Y_{pn}(kJ)A_{pn}(\overline{JM})), \tag{10}$$

with

$$A^{\dagger}_{pn}(JM) = [\alpha^{\dagger}_{p}\alpha^{\dagger}_{n}]^{JM}. \tag{11}$$

In the quasiboson picture, the pair operators do obey boson commutation rules

$$\langle 0|[A_{pn}(JM), A^{\dagger}_{p'n'}(J'M')]|0\rangle = \delta_{pp'}\delta_{nn'}\delta_{JJ'}\delta_{MM'}, \tag{12}$$

and the ground state correlations are described, at the QRPA order of approximation, by the condition

$$\Gamma(kJM)|0\rangle = 0. \tag{13}$$

In matrix form, the QRPA equations have the form

$$\begin{pmatrix} A(J) & B(J) \\ B^{*}(J) & A^{*}(J) \end{pmatrix}\begin{pmatrix} X(kJ) \\ Y(kJ) \end{pmatrix} = \omega_{kJ}\begin{pmatrix} X(kJ) \\ -Y(kJ) \end{pmatrix}, \tag{14}$$

with

$$\begin{aligned} A_{pn,p'n'} &= \langle 0|[A_{pn}, [H, A^{\dagger}_{p'n'}]]|0\rangle \\ B_{pn,p'n'} &= -\langle 0|[A_{pn}, [H, A_{p'n'}]]|0\rangle. \end{aligned} \tag{15}$$

This eigenvalue problem yields the QRPA amplitudes X and Y and the eigenfrequency ω. It is fully consistent with the BCS treatment of pairing correlations and the one phonon energies belong to the same order of approximation concerning the classification in powers of Ω.[1] The QRPA matrix does not have exchange terms and the phonons do not include self-energy corrections coming from the quasiparticle mean field. Both corrections are of lower order and the couplings with the quasiparticles are also of lower order and they can be treated perturbatively (for the classification of terms see, for instance, Refs. [16,21]).

C. The fully renormalized QRPA

The FRQRPA [18,19] advocates the use of the pair operator

$$\overline{A}^{\dagger}_{pn} = \left(A^{\dagger}_{pn} + \frac{u_n v_n B^{\dagger}_{pn} - u_p v_p B_{pn}}{v_n^2 - v_p^2}\right)(\overline{D}_{pn})^{-1/2}, \tag{16}$$

with

$$\overline{D}_{pn} \equiv 1 - \langle 0|\hat{\mathcal{N}}_p + \hat{\mathcal{N}}_n|0\rangle - \left(1 - v_p^2 - v_n^2\right)\mathcal{R}_{np} \tag{17}$$

and

$$\mathcal{R}_{np} = \frac{\langle 0|\hat{\mathcal{N}}_p - \hat{\mathcal{N}}_n|0\rangle}{v_n^2 - v_p^2} \tag{18}$$

in the definition of the one-phonon operator

$$\overline{\Gamma}^{\dagger}(kJM) = \sum_{pn}(\overline{X}_{pn}(kJ)\overline{A}^{\dagger}_{pn}(JM) - \overline{Y}_{pn}(kJ)\overline{A}_{pn}(\overline{JM})). \tag{19}$$

Note that the second term of $\overline{D}_{pn}$ is just the density-dependent term of the first renormalized QRPA, D_{pn}, of Ref. [9]. The operator $B^{\dagger}_{pn}(JM) = [\alpha^{\dagger}_{p}\alpha_{\overline{n}}]^{JM}$ is a one quasiparticle operator and it appears naturally in the quasiparticle transformation of the proton-neutron pair operator to the quasiparticle basis. However, because it annihilates a quasineutron and creates a quasiproton, its expectation value on the QRPA vacuum vanishes identically. Terms of the form $\alpha_q\alpha_q$, which is the commutator of $[B^{\dagger}_{pn}, A_{pn}]$, vanish. At this point it is worth mentioning that these terms do vanish already in the BCS approach, unless the constraints imposed on the BCS variation are removed. In other words, to admit the replacement of the operators A by $\overline{A}$ one should also admit that the BCS occupation factors u_q and v_q are not constrained either by the unitary nature of the BCS transformation or by the single quasiparticle structure of the BCS mean field. For example, v_q^2 would not be determined by the spectroscopy, the average number of particles would not coincide with the actual number of particles, and the vacuum would include quasiparticle excitations.

D. Separable interactions

In the following we analyze the structure of the equations of the previous subsection for the interaction [22,23]

$$\begin{aligned} H_{pn} = \frac{1}{2J+1}\sum_{pn,M}&\langle p||\mathcal{O}(J)||n\rangle\langle p'||\mathcal{O}(J)||n'\rangle^{*} \\ &\times\{\chi([a^{\dagger}_{p}a_{\overline{n}}]^{JM}[a^{\dagger}_{p'}a_{\overline{n'}}]^{\dagger\overline{JM}} + [a^{\dagger}_{p}a_{\overline{n}}]^{\dagger\overline{JM}}[a^{\dagger}_{p'}a_{\overline{n'}}]^{JM}) \\ &-\kappa([a^{\dagger}_{p}a^{\dagger}_{\overline{n}}]^{JM}[a^{\dagger}_{p'}a^{\dagger}_{\overline{n'}}]^{\dagger\overline{JM}} + [a^{\dagger}_{p}a^{\dagger}_{\overline{n}}]^{\dagger\overline{JM}}[a^{\dagger}_{p'}a^{\dagger}_{\overline{n'}}]^{JM})\}. \end{aligned} \tag{20}$$

In the standard pn-QRPA the pair operators that appear in the definition of the Hamiltonian are replaced by their expressions in terms of quasiparticles, namely,

$$\begin{aligned} [a^{\dagger}_{p}a_{\overline{n}}]^{JM} &= u_p v_n A^{\dagger}_{pn}(JM) + v_p u_n A_{pn}(\overline{JM}) \\ &\quad + u_p u_n B^{\dagger}_{pn}(JM) - v_p v_n B_{pn}(\overline{JM}), \\ [a^{\dagger}_{p}a^{\dagger}_{n}]^{JM} &= u_p u_n A^{\dagger}_{pn}(JM) - v_p v_n A_{pn}(\overline{JM}) \\ &\quad - u_p v_n B^{\dagger}_{pn}(JM) - v_p u_n B_{pn}(\overline{JM}). \end{aligned} \tag{21}$$

The matrix elements of the pn-QRPA equations are calculated by keeping only the contributions coming from the commutators of the operators $A^{\dagger}_{pn}(JM)$ and $A_{pn}(\overline{JM})$. In the FRQRPA, instead, one works with the operators $\overline{A}^{\dagger}_{pn}(JM)$ and $\overline{A}_{pn}(\overline{JM})$,

[1]For the case of a multilevel single particle basis, we refer to the effective shell degeneracy as Ω, instead of talking about each's level factor Ω_q. This is customarily done in the framework of nuclear field theory (for details see Refs. [16,21] and or in effective single shell models [3].

keeping all terms of the commutators, regardless of the order of the contributions, namely,

$$
\begin{aligned}
[a_p^\dagger a_{\overline{n}}]^{JM} &= (u_p v_n \overline{A}^\dagger_{pn}(JM) + v_p u_n \overline{A}_{pn}(\overline{JM}))\overline{D}^{1/2}_{pn}, \\
[a_p^\dagger a_n^\dagger]^{JM} &= (u_p u_n \overline{A}^\dagger_{pn}(JM) - v_p v_n \overline{A}_{pn}(\overline{JM}))\overline{D}^{1/2}_{pn} \\
&\quad - \frac{u_p v_n}{v_n^2 - v_p^2} B^\dagger_{pn}(JM) + \frac{v_p u_n}{v_n^2 - v_p^2} B_{pn}(\overline{JM}).
\end{aligned} \tag{22}
$$

The bar over the operators A and $A^\dagger$ follows from the definitions of the FRQRPA, and the commutators with the one quasiparticle operators B are given by

$$
\begin{aligned}
\left[\overline{D}^{1/2}_{p'n'}\overline{A}_{p'n'}, B_{pn}\right] &= [A_{p'n'}, B_{pn}] + \delta_{pp'}\delta_{nn'} u_p v_p \frac{\hat{\mathcal{N}}_p - \hat{\mathcal{N}}_n}{v_n^2 - v_p^2} \\
\left[\overline{D}^{1/2}_{p'n'}\overline{A}_{p'n'}, B^\dagger_{pn}\right] &= [A_{p'n'}, B^\dagger_{pn}] + \delta_{pp'}\delta_{nn'} u_n v_n \frac{\hat{\mathcal{N}}_p - \hat{\mathcal{N}}_n}{v_n^2 - v_p^2}.
\end{aligned} \tag{23}
$$

In consequence, the matrix elements of the FRQRPA matrix, for a given multipolarity J, are written

$$
\begin{aligned}
&A_{pn,p'n'}(J) \\
&= \left[(\epsilon_p + \epsilon_n) - 2\left(\epsilon_p u_p^2 v_p^2 - \epsilon_n u_n^2 v_n^2\right)\frac{R_{np}}{\left(v_n^2 - v_p^2\right)\overline{D}_{pn}}\right] \\
&\quad \times \delta_{pp'}\delta_{nn'} + 2\chi \frac{\langle p||\mathcal{O}(J)||n\rangle\langle p'||\mathcal{O}(J)||n'\rangle^*}{2J+1} \\
&\quad \times (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'})\overline{D}^{1/2}_{pn}\overline{D}^{1/2}_{p'n'} \\
&\quad - 2\kappa \frac{\langle p||\mathcal{O}(J)||n\rangle\langle p'||\mathcal{O}(J)||n'\rangle^*}{2J+1} \\
&\quad \times \left[u_p u_n u_{p'} u_{n'} \frac{\widetilde{\widetilde{D}}_{pn}\widetilde{\widetilde{D}}_{p'n'}}{\overline{D}^{1/2}_{pn}\overline{D}^{1/2}_{p'n'}} + \frac{v_p v_n v_{p'} v_{n'}\widetilde{D}_{pn}\widetilde{D}_{p'n'}}{\overline{D}^{1/2}_{pn}\overline{D}^{1/2}_{p'n'}}\right], \\
&B_{pn,p'n'}(J) \\
&= -2(\epsilon_p - \epsilon_n) u_p v_p u_n v_n \frac{R_{np}}{\left(v_n^2 - v_p^2\right)}\delta_{pp'}\delta_{nn'} \\
&\quad + 2\chi \frac{\langle p||\mathcal{O}(J)||n\rangle\langle p'||\mathcal{O}(J)||n'\rangle^*}{2J+1} \\
&\quad \times (v_p u_n u_{p'} v_{n'} + u_p v_n v_{p'} u_{n'})\overline{D}^{1/2}_{pn}\overline{D}^{1/2}_{p'n'} \\
&\quad + 2\kappa \frac{\langle p||\mathcal{O}(J)||n\rangle\langle p'||\mathcal{O}(J)||n'\rangle^*}{2J+1} \\
&\quad \times \left[u_p u_n v_{p'} v_{n'} \frac{\widetilde{\widetilde{D}}_{pn}\widetilde{D}_{p'n'}}{\overline{D}^{1/2}_{pn}\overline{D}^{1/2}_{p'n'}} + \frac{v_p v_n u_{p'} u_{n'}\widetilde{D}_{pn}\widetilde{\widetilde{D}}_{p'n'}}{\overline{D}^{1/2}_{pn}\overline{D}^{1/2}_{p'n'}}\right],
\end{aligned} \tag{24}
$$

where the following notation has been introduced

$$
\begin{aligned}
\widetilde{D}_{pn} &= 1 - \langle 0|\hat{\mathcal{N}}_p + \hat{\mathcal{N}}_n|0\rangle - \left(u_p^2 + u_n^2\right)\mathcal{R}_{np}, \\
\widetilde{\widetilde{D}}_{pn} &= 1 - \langle 0|\hat{\mathcal{N}}_p + \hat{\mathcal{N}}_n|0\rangle + \left(v_p^2 + v_n^2\right)\mathcal{R}_{np}.
\end{aligned} \tag{25}
$$

In deriving the above equations we have requested, for consistency, that the expectation value of the commutators of the bare operators on the BCS vacuum vanishes identically, that is, $\langle [A_{p'n'}, B_{pn}]\rangle_0 = \langle [A_{p'n'}, B^\dagger_{pn}]\rangle_0 = 0$.[2]

To conclude, the above FRQRPA expressions do indeed coincide trivially with the pn-QRPA ones if $\mathcal{R}_{np} = 0$ and $\langle 0|\hat{\mathcal{N}}_p + \hat{\mathcal{N}}_n|0\rangle = 0$, while the pn-RQRPA is retrieved if $\mathcal{R}_{np} = 0$, as it can be verified straightforwardly.

E. Quasiparticle occupations

If one thinks of the FRQRPA as an extension of the QRPA, based on a sort of variation around a correlated vacuum different from the BCS or QRPA ones, it should obey self-consistency. Therefore, one may define the correlated vacuum,

$$
|0\rangle = N_0 e^{\hat{S}_c}|BCS\rangle, \tag{26}
$$

in analogy with the definition of the QRPA vacuum, where the correlation operator $\hat{S}_c$ is

$$
\hat{S}_c = \frac{1}{2}\sum_{pnp'n'J}\sqrt{(2J+1)}C(J)_{pnp'n'}[\overline{A}^\dagger_{pn}(J)\overline{A}^\dagger_{p'n'}(J)]^0, \tag{27}
$$

with the coefficient

$$
C(J)_{pnp'n'} = \sum_k Y^*(J)_{pn,k} X^*(J)^{-1}_{p'n',k}.
$$

Thus, the quasiparticle occupations $\langle \hat{\mathcal{N}}_q\rangle$ would be given by the explicit form

$$
\begin{aligned}
\langle \hat{\mathcal{N}}_p\rangle &= \sum_{kJn'}\frac{(2J+1)}{2\Omega_p}\overline{D}_{pn'}|Y(J)_{pn',k}|^2, \\
\langle \hat{\mathcal{N}}_n\rangle &= \sum_{kJp'}\frac{(2J+1)}{2\Omega_n}\overline{D}_{p'n}|Y(J)_{p'n,k}|^2,
\end{aligned} \tag{28}
$$

and the actual values should be consistent with the values given by the original BCS condition, otherwise the consistency of the model would not be fulfilled. In other words, to find out a vacuum with correlations that go beyond the pn-QRPA correlations means that one should be able to find out new quasiparticle occupancies that should also obey the BCS conditions. If this is not achieved, then the starting BCS mean field values should be changed and the pn-QRPA procedure should be repeated until consistency is achieved. This is not the case of the FRQRPA of Refs. [18,19]. In fact, finite nontrivial values of $\mathcal{R}_{np}$, that is, $\mathcal{R}_{np} \neq .0$, of Eq. (17), cannot be determined in the numerical calculations, as we have shown already for the case of a single shell case [20] (see also Ref. [24]).

[2]Some of the terms that appear in the expressions for the matrix elements of the FRQRPA matrix, Eq. (23), differ in sign, as compared with the expressions reported in Refs. [18,19]. This may be due to typos in Refs. [18,19]. In our case, these expressions are consistent with the definitions given in Eq. (21).

III. RESULTS AND DISCUSSION

In this section we present the results of the calculations that we have performed using the formalisms described in the previous section. We have taken 0^+ and 1^+ excitations of ^{76}As, by performing BCS and pn-QRPA, RQRPA, and FRQRPA calculations on ^{76}Ge. The single particle basis that we have used in our calculations includes the states with principal oscillator quantum numbers $N = 3$ and 4 for protons and neutrons. The single particle energies are the solutions of the Wood-Saxon potential, with Coulomb corrections added for the proton states. The matrix elements of a δ-force interaction were used to solve the state-dependent gap equations and single quasiparticle occupancies. The residual proton-neutron interaction is the interaction introduced in Refs. [22,23], which is the same as that used in Refs. [18,19]. We have taken the same single particle space, interactions, and couplings of Refs. [18,19] to avoid further uncertainties in the comparison of results.

With all these elements we have solved the eigenvalue equations in the different approximations described in the previous section. Figure 1(a) shows the results corresponding to the first excited 1^+ state of ^{76}As. The energies are measured with respect to the ground state of ^{76}Ge. The coupling constant of the particle-particle proton-neutron interactions is taken as a variable. The correlation between these results and those of the Ikeda sum rule is also evident. Contrary to the claim of Refs. [18,19], the FRQRPA follows closely the results of the pn-QRPA in the region of the coupling where it is still possible to find out the solutions of both approximations. We have included the results of the RQRPA approximation to show that in this framework the collapse is shifted at the expense of violating the Ikeda sum rule [10], something that is again in contradiction with the results of Refs. [18,19].

Figure 2 shows the results of 0^+ excitations and those corresponding to the sum rule for Fermi transitions. Again for this case the features found for 1^+ excitations are repeated, signaling the contradiction between the claims of Refs. [18,19] and the actual results of the calculations. The results of Figs. 1 and 2 closely resemble the results already obtained for the schematic model situation that we have presented in our previous publication [20].

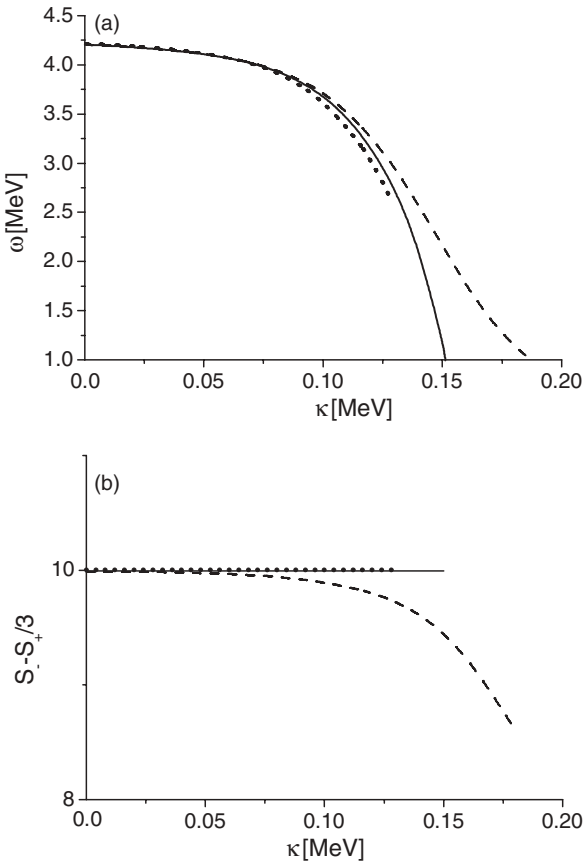

FIG. 1. (a) Energy of the first excited state $J^\pi = 1^+$ of ^{76}As, ω, and (b) the total strength $(S_- - S_+)/3$ of the Ikeda sum rule. The energy is given in units of MeV. The solid line is the pn-QRPA values, the dashed line represents the RQRPA values, and dotted line is the FRQRPA results. The values are shown as functions of the particle-particle proton-neutron coupling κ. The particle-hole proton-neutron interaction is fixed at the value $\chi = 0.21$ MeV.

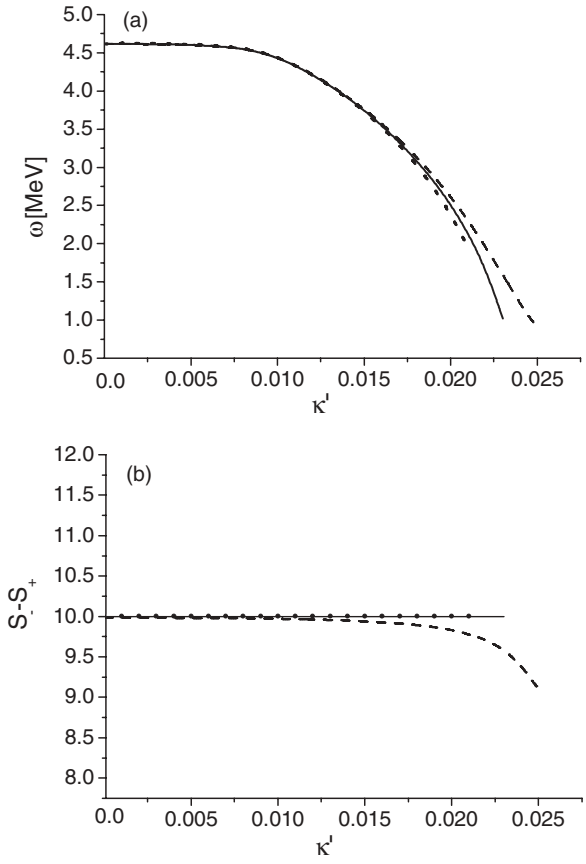

FIG. 2. (a) Energy of the first excited state $J^\pi = 0^+$ of ^{76}As, ω, and (b) the total strength $(S_- - S_+)$ of the Ikeda sum rule. The energy is given in units of MeV. The solid line is the pn-QRPA values, the dashed line represents the RQRPA values, and the dotted line is the FRQRPA results. The values are shown as functions of the particle-particle proton-neutron coupling $\kappa' = 4\kappa/V_{\text{pair}}$ [18,19]. The particle-hole proton-neutron interaction is fixed at the value $\chi = 0.025$ MeV.

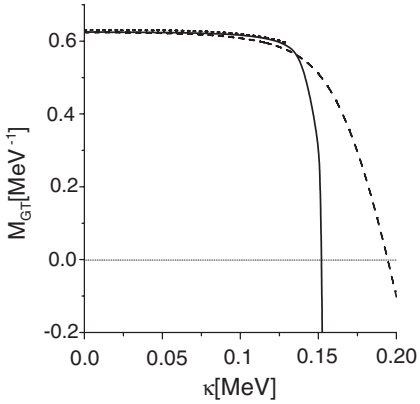

FIG. 3. Double Gamow-Teller matrix element, M_{GT}, for two-neutrino double-β-decay ground-state to ground-state transitions [7], as a function of the strength κ. The results obtained with pn-QRPA (solid line), RQRPA (dashed line), and FRQRPA (dotted line) are given in units of MeV^{-1}.

Finally, we discuss the results corresponding to the matrix element for two-neutrino double-β-decay transitions, of the ground-state to ground-state type, as direct application of the approximations described in the previous section. Figure 3 shows the results of the calculations corresponding to the double Gamow-Teller matrix element [7] M_{GT}. We have taken the strength κ as a variable and performed, for each value of the particle-particle coupling constant, the calculation of the spectrum of 1^+ excitations, and the corresponding wave functions, and used them to calculate M_{GT} [7]. As can be seen from the results shown in Fig. 3, both the pn-QRPA and FRQRPA approximations yield similar values and the matrix element, within both approximations, vanishes at the same value of $\kappa \approx 0.15$ MeV. The RQRPA gives values that go further, to larger values of κ, and they do vanish at $\kappa \approx 0.2$ MeV. This trend, too, is similar to the one found in the schematic model space of Ref. [20]. The differences between the pn-QRPA and the RQRPA have been pointed out in details elsewhere [21], as well as the physical meaning of both approximations. For the sake of the present discussion, we continue with the comparison between the pn-QRPA and the FRQRPA. So far we have not found a significant difference between them and the explanation of this similarity is rather obvious, because we did not get solutions of the FRQRPA equations with $\mathcal{R}_{np} \neq 0$ that are, at the same time, consistent with the constraints fixed by the particle number and phonon-vacuum conditions.

IV. CONCLUSIONS

In this work we report comparative results for excited states and sum rules for charge-dependent Fermi transitions and Gamow-Teller transitions, obtained by applying the proton-neutron QRPA approximation and some recently proposed modified version of it, the fully renormalized pn-QRPA of Refs. [18,19]. The calculations were performed using a multilevel single particle basis and a realistic interaction. It was found that the results show the same trend of the results that we obtained previously for the case of a single shell model space and a separable, exactly solvable, interaction. For all purposes the results of the approximation [18,19], when carried out correctly, follow closely those of the pn-QRPA method. This trend is to be expected from the order of magnitude of the correction introduced in the approach of Refs. [18,19] to the occupation numbers and the vacuum, which coincide with those neglected in the standard pn-QRPA approach. Therefore, we are forced to conclude that, contrary to the claims of Refs. [18,19], the results of FRQRPA do not represent an improvement with respect to the pn-QRPA approach. From a more theoretical, inspired point of view, FRQRPA omits the consistency required of theories where the different terms of a given Hamiltonian should be ordered in a consistent manner before assessing the correctness of a given approximation [21]. The results discussed in the present work, for the case of a realistic model space and a realistic interaction, do indeed confirm our previous conclusions [20] extracted from a schematic, single shell situation.

ACKNOWLEDGMENTS

This work was partly supported by CONICET (Argentina), CONACYT, and the DGAPA-UNAM (Mexico).

[1] M. Baranger, Phys. Rev. **120**, 957 (1960).
[2] P. Ring and P. Shuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980).
[3] D. J. Rowe, Phys. Rev. **175**, 1283 (1968).
[4] D. Cha, Phys. Rev. C **27**, 2269 (1983).
[5] O. Civitarese, A. Faessler, and T. Tomoda, Phys. Lett. **B194**, 11 (1987).
[6] P. Vogel and M. R. Zirnbauer, Phys. Rev. Lett. **57**, 3148 (1986).
[7] J. Suhonen and O. Civitarese, Phys. Rep. **300**, 123 (1998).
[8] W. C. Haxton and G. J. Stephenson, Jr., Prog. Part. Nucl. Phys. **12**, 409 (1984).
[9] J. Toivanen and J. Suhonen, Phys. Rev. Lett. **75**, 410 (1995); Phys. Rev. C **55**, 2314 (1997).
[10] J. G. Hirsch, O. Civitarese, and M. Reboiro, Phys. Rev. C **60**, 024309 (1999).
[11] K. Muto, Phys. Lett. **B391**, 243 (1997).
[12] A. Bobyk, W. A. Kaminski, and F. Simkovic, Phys. Rev. C **63**, 051301(R) (2001).
[13] F. Simkovic, J. Schwieger, G. Pantis, and Amand Faessler, Found. Phys. **27**, 1275 (1997).
[14] F. Simkovic, A. A. Raduta, M. Veselsky, and A. Faessler, Phys. Rev. C **61**, 044319 (2000).
[15] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1975), Vol. 2.
[16] D. R. Bes and O. Civitarese, Nucl. Phys. **A705**, 297 (2002).
[17] J. Hirsch, P. Hess, and O. Civitarese, Phys. Lett. **B412**, 1 (1997).

[18] L. Pacearescu, V. Rodin, F. Simkovic, and A. Faessler, Phys. Rev. C **68**, 064310 (2003).
[19] V. Rodin and A. Faessler, Phys. Rev. C **66**, 051303(R) (2002).
[20] O. Civitarese, M. Reboiro, and J. G. Hirsch, Phys. Rev. C **71**, 014318 (2005).
[21] D. R. Bes, O. Civitarese, and N. N. Scoccola, Phys. Lett. **B446**, 93 (1999).
[22] V. A. Kuz'min and V. G. Soloviev, Nucl. Phys. **A486**, 118 (1988).
[23] O. Civitarese and J. Suhonen, Nucl. Phys. **A578**, 62 (1994).
[24] A. Mariano and J. G. Hirsch, Phys. Rev. C **57**, 3015 (1998); Phys. Rev. C **58**, 2736 (1998).