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Resonant relativistic corrections and the A_y problem

G. A. Miller[1,*] and A. Schwenk[1,2,†]
[1]*Department of Physics, University of Washington, Seattle, Washington 98195, USA*
[2]*TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada*
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We study relativistic corrections to nuclear interactions caused by boosting the two-nucleon interaction to a frame in which their total momentum does not vanish. These corrections induce a change in the computed value of the neutron-deuteron analyzing power A_y that is estimated using the plane-wave impulse approximation. This allows a transparent analytical calculation that demonstrates the significance of relativistic corrections. Faddeev calculations are, however, needed to conclude on the A_y puzzle.

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I. INTRODUCTION

One of the major unsolved problems in nuclear physics is the so-called A_y puzzle in nucleon-deuteron (Nd) scattering. The nucleon analyzing power A_y is the difference in differential cross sections for scattering of polarized nucleons [1]:

$$A_y = \frac{\frac{d\sigma}{d\Omega}|_\uparrow - \frac{d\sigma}{d\Omega}|_\downarrow}{\frac{d\sigma}{d\Omega}|_\uparrow + \frac{d\sigma}{d\Omega}|_\downarrow}, \tag{1}$$

where $\uparrow$ denotes the polarization normal to the reaction plane (spanned by the center-of-mass momentum of the incident and scattered nucleon). All modern nucleon-nucleon (NN) interactions lead to practically the same results: They under predict A_y by 30% for laboratory energies $E_{\rm N} \lesssim 30$ MeV (for a review, see Ref. [2]), whereas the predicted A_y is in very good agreement for higher energies. The contributions of the existing three-nucleon (3N) interactions to A_y are small at low energies [2–4]. A similar discrepancy is found for the deuteron vector analyzing power iT_{11} [2].

The NN contribution to A_y is directly related to the ${}^3P_j$ phase shifts [5], but it is very unlikely that uncertainties in these phases can resolve the puzzle [6]. For few MeV energies, A_y is maximal around a center-of-mass scattering angle $\theta \approx 100^\circ$. This is the location of the minimum of the differential cross section so that small effects are amplified in A_y. As a result, a number of small contributions to A_y have been investigated. For instance, magnetic moment interactions lead to a very small contribution to A_y near the maximum in pd scattering, but are only sizable at forward angles for nd [7,8]. Moreover, ad hoc solutions have been proposed that range from introducing a phenomenological 3N spin-orbit force [9] to including the effects of exchanging one pion in the presence of a two-nucleon correlation [10].

Recently, Fisher *et al.* [11] have shown that the A_y problem increases from a 30% discrepancy in Nd to a 100% puzzle in p^3He. Therefore, one can expect that the problem becomes even more pronounced for understanding heavier systems. In addition, the A_y discrepancy increases with the inclusion of the Coulomb interaction in the pd system [12–14].

In the three-body (or higher-body) system, not all pairs of particles are simultaneously in the *two-body* center-of-mass (c.m.) system, and therefore relativistic corrections [15–17] have to be taken into account:

$$\delta V \sim \frac{Q^2}{m^2} V_{\rm NN}, \tag{2}$$

where Q^2 includes at least one power of the two-body c.m. momentum $\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2$, m is the nucleon mass, and $V_{\rm NN}$ denotes the NN interaction in the c.m. frame (for $P = 0$). The modern understanding is to consider these corrections as 3N interactions, but using the formalism developed in Refs. [15–17] it is straightforward to include these effects to order $(Q/m)^2$ without any new parameters. The naive expectation is that relativistic corrections are small at low energies. This was confirmed for selected nd observables and for energies $E_n \geqslant 28$ MeV [18] (where there is no A_y problem).

In this work, we show that, in contrast to the naive expectation, relativistic boost effects may be important at low energies. This is due to spin-violating relativistic corrections, which couple relative NN S waves with the ${}^3P_j$ waves (combined with a change of the two-body c.m. angular momentum). We find that the interference with the large S-wave scattering lengths can lead to resonant enhancements of A_y at low energies. This effect would explain why predictions for A_y at $E_n \gtrsim 30$ MeV agree well with experiment. We present a transparent analytical calculation, based on using the plane-wave impulse approximation, that explores the effect of relativistic corrections on spin observables. The effects are small but significant and should be combined with a complete solution of the Faddeev equations.

This article is organized as follows. We begin in Sec. II with a brief discussion of the relevant notation and scattering formalism. In Sec. III, we classify all relativistic corrections to order $(Q/m)^2$ and their impact on the differential cross section and A_y. We calculate analytically their effect on A_y neglecting distortions. The central findings of this article are given in Eq. (30) and in Fig. 2. Our results and Eq. (30) are general and in a form that should be implemented in future Faddeev calculations. The reader familiar with the standard notation and 3N scattering can skip Sec. II. In Sec. IV, the contribution to A_y is estimated using benchmarked nd [3,19] phase shifts and pionless effective field theory (EFT) contact

*miller@phys.washington.edu
†schwenk@triumf.ca

interactions for the relativistic corrections δV. We conclude in Sec. V that relativistic boost effects may be important for a precise understanding of three-body spin observables.

II. NOTATION AND SCATTERING FORMALISM

We follow the notation and conventions of Glöckle *et al.* [2] and define the nd scattering amplitude M by

$$M_{m_j',m_n';m_j,m_n}(\boldsymbol{q}',\boldsymbol{q}) = -\frac{2m}{3}(2\pi)^2\langle\phi_d, m_j'; \boldsymbol{q}', m_n'|U|\phi_d, m_j; \boldsymbol{q}, m_n\rangle, \qquad (3)$$

where m_j, m_n are the deuteron total angular momentum and nucleon spin magnetic quantum numbers, respectively, $\boldsymbol{q}, \boldsymbol{q}'$ are initial and final relative momenta of the nucleon in the nd c.m. system, and U denotes the transition amplitude. The relative momenta are on-shell related to the neutron laboratory energy E_n by $q = |\boldsymbol{q}| = |\boldsymbol{q}'| = \sqrt{\frac{8}{9}mE_n}$, and the c.m. scattering angle is $\cos\theta \equiv \widehat{\boldsymbol{q}}\cdot\widehat{\boldsymbol{q}'}$. Finally, the plane-wave states are normalized as $\langle \boldsymbol{p}'|plm\rangle \equiv i^{-l}Y_{lm}(\widehat{\boldsymbol{p}'})\delta(p'-p)/(p'p)$ with spherical harmonics $Y_{lm}(\widehat{\boldsymbol{p}})$, and thus $\langle\boldsymbol{p}|\boldsymbol{p}'\rangle = \delta^{(3)}(\boldsymbol{p}-\boldsymbol{p}')$ and $\langle\boldsymbol{p}|\boldsymbol{r}\rangle = e^{-i\boldsymbol{p}\cdot\boldsymbol{r}}/(2\pi)^{3/2}$.

In terms of the scattering amplitude, the spin-averaged differential cross section $d\sigma/d\Omega$ is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2j+1)(2s_{\rm N}+1)}\mathrm{Tr}(MM^\dagger) = \frac{1}{6}\sum_{m_j',m_n',m_j,m_n}\left|M_{m_j',m_n';m_j,m_n}(\boldsymbol{q}',\boldsymbol{q})\right|^2, \qquad (4)$$

where $j=1$ and $s_{\rm N}=1/2$ are the spin of the deuteron and nucleon, respectively. The nucleon analyzing power is defined by

$$A_i = \frac{\mathrm{Tr}(M\boldsymbol{\sigma}^i M^\dagger)}{\mathrm{Tr}(MM^\dagger)}, \qquad (5)$$

with Pauli matrices $\boldsymbol{\sigma}^i$ and standard conventions for the coordinate system: $\widehat{\mathbf{z}} = \widehat{\boldsymbol{q}}, \widehat{\mathbf{y}} = \widehat{\boldsymbol{q}}\times\widehat{\boldsymbol{q}'}/|\widehat{\boldsymbol{q}}\times\widehat{\boldsymbol{q}'}|$, and $\widehat{\mathbf{x}} = \widehat{\mathbf{y}}\times\widehat{\mathbf{z}}$. This directly leads to Eq. (1), if one chooses $\widehat{\mathbf{y}}$ as the spin quantization axis. Finally, due to parity conservation, $A_x = A_z = 0$ [1]. Using the Fourier transform of an operator representation for the deuteron wave function [20] ,

$$\widehat{\phi}_d(\boldsymbol{p}) \equiv \widetilde{\phi}_d^0(p) + \widetilde{\phi}_d^2(p)\frac{S_{12}(\widehat{\boldsymbol{p}})}{\sqrt{8}}, \qquad (6)$$

$$\langle\boldsymbol{p}, m_j'|\phi_d, m_j\rangle = \langle m_j'|\widehat{\phi}_d(\boldsymbol{p})|m_j\rangle, \qquad (7)$$

with tensor operator $S_{12}(\widehat{\boldsymbol{p}})$, the scattering amplitude can be expressed in a convenient operator form

$$M_{m_j',m_n';m_j,m_n}(\boldsymbol{q}',\boldsymbol{q}) = -\frac{2m}{3}(2\pi)^2\int d\boldsymbol{p}'\int d\boldsymbol{p}\langle\boldsymbol{p}', m_j'; \boldsymbol{q}', m_n'|\widehat{\phi}_d(\boldsymbol{p}')U\widehat{\phi}_d(\boldsymbol{p})|\boldsymbol{p}, m_j; \boldsymbol{q}, m_n\rangle. \qquad (8)$$

Finally, computing the analyzing power is simplified by coupling the deuteron total angular momentum $\boldsymbol{j}$ with the nucleon spin $\boldsymbol{s}_{\rm N}$ to a total spin $\boldsymbol{\Sigma} = \boldsymbol{j}+\boldsymbol{s}_{\rm N}$. In this basis, the spin matrix elements of the scattering amplitude are given by

$$M_{\Sigma',m_\Sigma';\Sigma,m_\Sigma}(\boldsymbol{q}',\boldsymbol{q}) = \sum_{m_j',m_j}(1\,m_j'\,1/2\,m_\Sigma'-m_j'|\Sigma' m_\Sigma')(1\,m_j\,1/2\,m_\Sigma-m_j|\Sigma m_\Sigma) \times M_{m_j',m_\Sigma'-m_j';m_j,m_\Sigma-m_j}(\boldsymbol{q}',\boldsymbol{q}). \qquad (9)$$

We use benchmarked nd partial waves for the scattering amplitude without relativistic corrections, so we briefly discuss the partial wave expansion. The states with good total spin Σ read $|p(ls)j; \boldsymbol{q}(j1/2)\Sigma m_\Sigma\rangle$, where $s=1$ is the spin of the deuteron and the nucleon motion can also be expanded in angular momenta $|q\lambda m_\lambda\rangle$. In these states the nd scattering amplitude is given by

$$M_{\Sigma',m_\Sigma';\Sigma,m_\Sigma}(\boldsymbol{q}',\boldsymbol{q}) = -\frac{2m}{3}(2\pi)^2\sum_{\lambda',m_\lambda',\lambda,m_\lambda} i^{\lambda-\lambda'}Y_{\lambda',m_\lambda'}(\widehat{\boldsymbol{q}'})Y^*_{\lambda,m_\lambda}(\widehat{\boldsymbol{q}}) \times\sum_{l,l'}\int p'^2dp'\widetilde{\phi}_d^{l'}(p')\int p^2dp\widetilde{\phi}_d^{l}(p)\langle p'(l'1)1; q'\lambda' m_\lambda'(1,1/2)\Sigma' m_\Sigma'|U|p(l1)1; q\lambda m_\lambda(1,1/2)\Sigma m_\Sigma\rangle. \qquad (10)$$

Next one couples the nucleon angular momentum with the total spin to the total angular momentum $\boldsymbol{J} = \boldsymbol{\lambda}+\boldsymbol{\Sigma}$, for which U is diagonal in J and independent of m_J, thus $m_\lambda' + m_\Sigma' = m_\lambda + m_\Sigma$. With $\widehat{\boldsymbol{q}} = \widehat{\mathbf{z}}$, we have $Y^*_{\lambda,m_\lambda}(\widehat{\boldsymbol{q}}) = \delta_{m_\lambda,0}\sqrt{\frac{2\lambda+1}{4\pi}}$, and consequently $m_J = m_\Sigma$ and $m_\lambda' = m_\Sigma - m_\Sigma'$. The second line in Eq. (10) in the coupled

$(\lambda\Sigma')Jm_J$ basis is independent of m_J and can be decomposed as $(\delta_{\lambda',\lambda}\delta_{\Sigma',\Sigma} - S^J_{\lambda',\Sigma';\lambda,\Sigma})/(4\pi i m q/3)$. With this at hand, the partial wave decomposition reads

$$M_{\Sigma',m'_\Sigma;\Sigma,m_\Sigma}(\boldsymbol{q}',\boldsymbol{q}) = \frac{i\sqrt{\pi}}{q}\sum_{\lambda',\lambda,J} i^{\lambda-\lambda'}\sqrt{2\lambda+1}\,Y_{\lambda',m_\Sigma-m'_\Sigma}(\widehat{\boldsymbol{q}'}) \times(\lambda' m_\Sigma - m'_\Sigma \Sigma' m'_\Sigma|Jm_\Sigma)(\lambda 0 \Sigma m_\Sigma|Jm_\Sigma)(\delta_{\lambda',\lambda}\delta_{\Sigma',\Sigma} - S^J_{\lambda',\Sigma';\lambda,\Sigma}), \tag{11}$$

where $S^J_{\lambda',\Sigma';\lambda,\Sigma}$ is given in terms of the nd phase shifts and mixing parameters [1] (see also Eqs. (209)–(214) in Ref. [2]).

III. RELATIVISTIC CORRECTIONS

Boost corrections to the two-nucleon interaction depend on the total momentum $\boldsymbol{P}$ of the pair and are obtained by satisfying the commutation relations of the Poincaré group [15]. To leading order in $(Q/m)^2$, the relativistic boost corrections are given in momentum space by (for the corresponding coordinate space expression, see Eq. (1.7) in Ref. [17])

$$\begin{aligned}\delta v_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k},\boldsymbol{P}) &= -\frac{P^2}{4m^2}v_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k}) + \frac{i}{8m^2}[(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2), v_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k})]\\ &\quad\times\boldsymbol{P}\cdot\boldsymbol{k} - \frac{i}{8m^2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\times\boldsymbol{P}\cdot(\boldsymbol{k}-\boldsymbol{k}')v_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k})\\ &\quad-\frac{1}{8m^2}(\boldsymbol{P}\cdot(\boldsymbol{k}-\boldsymbol{k}'))\boldsymbol{P}\cdot\nabla_{\boldsymbol{k}-\boldsymbol{k}'}v_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k}),\end{aligned} \tag{12}$$

where $v_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k},\boldsymbol{P})$ is the direct NN interaction in the c.m. system, with initial and final relative momenta $\boldsymbol{k} = (\boldsymbol{p}_1-\boldsymbol{p}_2)/2$ and $\boldsymbol{k}' = (\boldsymbol{p}'_1-\boldsymbol{p}'_2)/2$, and Eq. (12) accounts only for the direct term of the boost correction. As explained in Ref. [17] the Poincaré group commutation relations do not have a unique solution. The operator δv can have an additional term of the form

$$\delta v' = -i[\chi, H_0 + v], \tag{13}$$

where χ is a translationally invariant function and H_0 is the noninteracting Hamiltonian. One must pay attention to this term when studying scattering processes.

Some of the operators in Eq. (12) can be obtained from purely classical considerations [17]. The first term arises from treating the potential as a contribution to the nucleon mass and then expanding the relativistic energy operator. The final term of Eq. (12) is due to the effects of Lorentz contraction. The third term results from Thomas precession in which objects with spin precess when they accelerate, because rotations do not commute with boosts. The commutator term of Eq. (12) does not have an analog in classical mechanics.

Our procedure is to calculate the leading relativistic corrections δM in the basis $|\boldsymbol{p}, m_j; \boldsymbol{q}, m_n\rangle$ of Eq. (8) by accounting for the change in U, δU caused by δv of Eq. (12). In this exploratory study we use the plane-wave impulse approximation, which treats one of the nucleons as a spectator. Taking the matrix element of δU within plane-wave neutron-deuteron states yields δM. The use of the plane-wave impulse approximation enables us to analytically study the effect of relativistic boost corrections and make a first assessment of their importance. A full Faddeev calculation including distortions will eventually be needed to make a complete assessment. Our present use of the plane-wave impulse approximation has an additional advantage: The matrix element of the term $\delta v'$ of Eq. (13), taken between on-shell elastic scattering states, vanishes.

There are three contributions to δU arising from the three pairs in the nd system:

$$\delta U = \delta V_{12} + \delta V_{13} + \delta V_{23}. \tag{14}$$

Since relativistic corrections are of order $(Q/m)^2$, we keep only the central parts in $v_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k})$. Noncentral interactions start at $\mathcal{O}((Q/m_\pi)^2)$ in pionless effective field theory, which is relevant for the energies of interest. Therefore, the spin structure is limited to

$$v_{\sigma_1,\sigma_2} = v_{\mathbb{1}} + v_{\text{spin}}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2. \tag{15}$$

Furthermore we can neglect the last term in Eq. (12) of $\mathcal{O}((Q/m)^2(Q/m_\pi)^2)$.

The next step is to include the exchange term. We need to compute

$$\begin{aligned}\delta V_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k},\boldsymbol{P}) &= \delta v_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k},\boldsymbol{P})\\ &\quad - P_\sigma P_\tau \delta v_{\sigma_1,\sigma_2}(-\boldsymbol{k}',\boldsymbol{k},\boldsymbol{P}),\end{aligned} \tag{16}$$

where the spin (isospin) exchange operator is P_σ (P_τ) and $V_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k}) = v_{\sigma_1,\sigma_2}(\boldsymbol{k}',\boldsymbol{k}) - P_\sigma P_\tau v_{\sigma_1,\sigma_2}(-\boldsymbol{k}',\boldsymbol{k})$ denotes the antisymmetrized interaction. Writing the commutator term of Eq. (12) explicitly and using the property that

$\{(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2), P_\sigma\} = 0$ leads directly to the result

$$\delta V_{\sigma_1,\sigma_2}(\boldsymbol{k}', \boldsymbol{k}, \boldsymbol{P}) = -\frac{P^2}{4m^2} V_{\sigma_1,\sigma_2}(\boldsymbol{k}', \boldsymbol{k}) - \frac{i}{8m^2} V_{\sigma_1,\sigma_2}(\boldsymbol{k}', \boldsymbol{k})(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\boldsymbol{P}\cdot\boldsymbol{k} + \frac{i}{8m^2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\times\boldsymbol{P}\cdot\boldsymbol{k}' V_{\sigma_1,\sigma_2}(\boldsymbol{k}', \boldsymbol{k}). \quad (17)$$

In antisymmetrized states, relativistic corrections thus have the form of V (boost corrections in)−(boost corrections out) V.

It is evident that the first (P^2) term in Eq. (17) will lead to a relativistic correction to the nd scattering amplitude that is a scalar in spin and of general structure

$$\delta M_{P^2} \sim \mathbb{1}\frac{Q^2}{m^2} \quad \text{and} \quad \boldsymbol{S}\cdot\boldsymbol{\sigma}_3\frac{Q^2}{m^2}, \quad (18)$$

where $\boldsymbol{S} = (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)/2$ is the deuteron spin and the nucleon spin operator is given by $\boldsymbol{s}_{\rm N} = \boldsymbol{\sigma}_3/2$. In the following we will show that the spin-violating (sv) relativistic corrections [the last two terms in Eq. (17)] lead to terms of the form

$$\delta M_{\rm sv} \sim \boldsymbol{S}^y\frac{Q^2}{m^2} \quad \text{and} \quad \boldsymbol{\sigma}_3^y\frac{Q^2}{m^2}. \quad (19)$$

The leading contributions to the differential cross section and to A_y are from the interference of δM of $\mathcal{O}((Q/m)^2)$ with the leading nd scattering amplitude at low energies. Similar to the above considerations for the two-nucleon interaction, the leading operators in M are given by the central part

$$M_{S,\sigma_3} = M_{\mathbb{1}} + M_{\rm spin}\boldsymbol{S}\cdot\boldsymbol{\sigma}_3 + \mathcal{O}((Q/m_\pi)^2). \quad (20)$$

We can now evaluate the relativistic corrections to the nucleon analyzing power δA_y and to the differential cross section $\delta(d\sigma/d\Omega)$:

$$\delta A_y = \frac{\mathrm{Tr}(\delta M\boldsymbol{\sigma}^y M^\dagger + M\boldsymbol{\sigma}^y\delta M^\dagger)}{\mathrm{Tr}(MM^\dagger)} - A_y\frac{\mathrm{Tr}(\delta MM^\dagger + M\delta M^\dagger)}{\mathrm{Tr}(MM^\dagger)}, \quad (21)$$

$$\delta\frac{d\sigma}{d\Omega} = \frac{1}{6}\mathrm{Tr}(\delta MM^\dagger + M\delta M^\dagger). \quad (22)$$

Since A_y is small, we can neglect the second term in Eq. (21). For the leading contributions, it then follows that only $\delta M_{\rm sv}$ contributes to δA_y,

$$\delta A_y = \frac{\mathrm{Tr}(\delta M_{\rm sv}\boldsymbol{\sigma}^y M^\dagger + M\boldsymbol{\sigma}^y\delta M_{\rm sv}^\dagger)}{\mathrm{Tr}(MM^\dagger)} + \mathcal{O}\left(\frac{Q^4}{m^2 m_\pi^2}\right), \quad (23)$$

and only δM_{P^2} contributes to $\delta(d\sigma/d\Omega)$. A straightforward calculation of the spin-violating relativistic corrections arising from $V_{\mathbb{1}}(\boldsymbol{k}', \boldsymbol{k}) + V_{\rm spin}(\boldsymbol{k}', \boldsymbol{k})\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \equiv (1 - P_\sigma P_\tau P_k)(v_{\mathbb{1}} + v_{\rm spin}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$ yields

$$\delta V^{\rm sv}_{\sigma_1,\sigma_2}(\boldsymbol{k}', \boldsymbol{k}, \boldsymbol{P}) = -\frac{i}{8m^2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\times\boldsymbol{P}\cdot(\boldsymbol{k} - \boldsymbol{k}')(V_{\mathbb{1}}(\boldsymbol{k}', \boldsymbol{k}) - V_{\rm spin}(\boldsymbol{k}', \boldsymbol{k})) + \frac{1}{4m^2}(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\times\boldsymbol{P}\cdot(\boldsymbol{k} + \boldsymbol{k}')V_{\rm spin}(\boldsymbol{k}', \boldsymbol{k}). \quad (24)$$

The resulting spin-violating interactions connect the two-body ${}^3P_j$ waves (which are crucial for A_y) with the two-body S waves. The S waves are resonant at low energies with large scattering lengths, $a_0 \equiv a_{{}^1\mathrm{S}_0} = -23.768 \pm 0.006$ fm and $a_1 \equiv a_{{}^3\mathrm{S}_1} = 5.420 \pm 0.001$ fm [21], and therefore the interference with the ${}^3P_j$ waves can lead to a resonant enhancement of these relativistic corrections at low energies. For higher energies, the S-wave phase shifts decrease, so the effect of the spin-violating interactions decreases.

Including isospin and restricting two-nucleon interactions to S waves, the central part of the antisymmetrized two-body interaction can be written as

$$V_{i,3} = \tfrac{1}{8}[V_0(1 - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_3)(3 + \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_3) + V_1(3 + \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_3)(1 - \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_3)], \quad (25)$$

where $i = 1, 2$ and $\boldsymbol{\tau}_{i,3}$ denote Pauli matrices that operate in isospin space and $V_{0,1}$ are projections on $s = 0, 1$ states. The operator $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_3$ vanishes in nd states, and thus we have

$$V_{\mathbb{1}} = \tfrac{3}{8}(V_0 + V_1) \quad \text{and} \quad V_{\rm spin} = \tfrac{1}{8}(V_1 - 3V_0). \quad (26)$$

We use leading-order $(Q/m_\pi)^0$ pionless EFT contact interactions [22] where the operators V_0 and V_1 are momentum independent:

$$V_i = \frac{C_i}{2\pi^2 m} \quad \text{with} \quad C_i = \frac{1}{\frac{1}{a_i} - \mu}, \quad (27)$$

for $i = 0, 1$. Here, μ is the renormalization scale in dimensional regularization with a power-divergence subtraction scheme [22]. A similar expression is obtained for a momentum-cutoff regularization. The operator of Eq. (24) therefore has the form of a spin-violating operator $(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)$ dotted into a momentum vector that induces transitions between spin triplet (singlet)/relative S-wave and spin singlet (triplet)/relative P-wave states. The momentum vector $\boldsymbol{k}$ ($\boldsymbol{k}'$) in Eq. (24) explicitly projects on incoming (outgoing)

P-wave states. The change in the orbital angular momentum is compensated with a corresponding change of the two-body c.m. angular momentum, so that the total angular momentum is preserved.

At low energies we can take the P-wave states to be plane waves, but we include iterated S-wave interactions V_i in the initial state (for $\boldsymbol{k}'$) and final state (for $\boldsymbol{k}$). This leads to replacing C_i by

$$C_i \to \frac{C_i}{1 + C_i(\mu + i\sqrt{mE_{\rm rel}})} = \frac{1}{\frac{1}{a_i} + i\sqrt{mE_{\rm rel}}}, \qquad (28)$$

where $E_{\rm rel}$ is the relative energy (in the two-body c.m. of system). As a result of these initial and final state interactions, we find that the operators V_i that enter in Eq. (24) are independent of the renormalization scale μ.

Next we calculate the contributions of the spin-violating relativistic corrections to δA_y for the nd system (where the Coulomb interaction does not operate). We neglect distortions that would involve the nucleon treated as a spectator. We work in the three-body c.m. system $\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3 = 0$, where nucleons $1, 2$ constitute the deuteron and nucleon 3 is the free neutron. Because the matrix element of $\delta V_{12}^{\rm sv}$ vanishes when evaluated in the deuteron eigenstate, we need only to evaluate two contributions to the spin-violating collision operator $\delta U_{\rm sv} = \delta V_{13}^{\rm sv} + \delta V_{23}^{\rm sv}$, shown diagrammatically in Fig. 1. We employ Jacobi momenta and the incoming nucleon momenta are expressed as $\boldsymbol{p}_1 = \boldsymbol{p} - \boldsymbol{q}/2$, $\boldsymbol{p}_2 = -\boldsymbol{p} - \boldsymbol{q}/2$, and $\boldsymbol{p}_3 = \boldsymbol{q}$ (with primed momentum labels for the outgoing nucleons). The second contribution $\delta V_{23}^{\rm sv}$ can be obtained from $\delta V_{13}^{\rm sv}$ by replacing $\boldsymbol{\sigma}_1 \to \boldsymbol{\sigma}_2$, $\boldsymbol{p} \to -\boldsymbol{p}$, and $\boldsymbol{p}' \to -\boldsymbol{p}'$. Because the deuteron is even in momentum ($l = 0, 2$), we can change variables in Eq. (8) back to $-\boldsymbol{p} \to \boldsymbol{p}$ and $-\boldsymbol{p}' \to \boldsymbol{p}'$. Consequently, the contribution of $\delta V_{23}^{\rm sv}$ to $\delta M_{\rm sv}$ is identical to the contribution of $\delta V_{13}^{\rm sv}$ after replacing $\boldsymbol{\sigma}_1 \to \boldsymbol{\sigma}_2$ in the latter.

Inserting this into Eq. (24) (with $1, 2 \to 1, 3$ and $1, 2 \to 2, 3$), we obtain for the total leading $(Q/m)^2$ relativistic corrections relevant for δA_y

$$\delta U_{S,\boldsymbol{\sigma}_3}^{\rm sv}(\boldsymbol{p}, \boldsymbol{q}', \boldsymbol{q}) = \delta^{(3)}(\boldsymbol{p}' - (\boldsymbol{p} + \Delta))\delta \widetilde{U}_{S,\boldsymbol{\sigma}_3}^{\rm sv}(\boldsymbol{p}, \boldsymbol{q}', \boldsymbol{q}), \qquad (29)$$

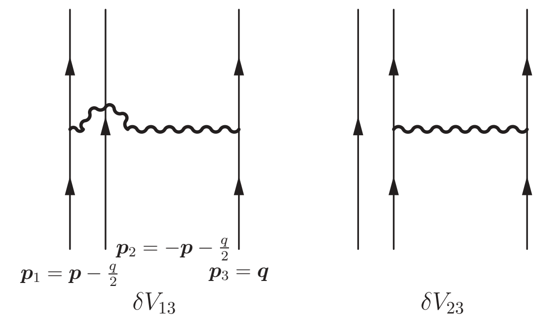

FIG. 1. Contributions of the spin-violating interactions to the relativistic corrections $\delta U_{\rm sv}$ and our conventions for the Jacobi momenta. In Born approximation, the low-energy coefficients are given by the C_i of Eq. (27), and in the plane-wave impulse approximation we use the C_i of Eq. (28).

where the δ function accounts for the conservation of the two-body c.m. momentum, the momentum transfer is $\Delta \equiv (\boldsymbol{q} - \boldsymbol{q}')/2$, and we have

$$\delta \widetilde{U}_{S,\boldsymbol{\sigma}_3}^{\rm sv}(\boldsymbol{p}, \boldsymbol{q}', \boldsymbol{q}) = \left[-\frac{i}{4m^2}(\boldsymbol{S} - \boldsymbol{\sigma}_3) \times \left(\boldsymbol{p} + \frac{\boldsymbol{q}}{2}\right) \cdot (\boldsymbol{q}' - \boldsymbol{q}) \frac{3V_0 + V_1}{4} - \frac{1}{2m^2}(\boldsymbol{S} \times \boldsymbol{\sigma}_3) \times \left(\boldsymbol{p} + \frac{\boldsymbol{q}}{2}\right) \cdot (\boldsymbol{q} + \boldsymbol{q}') \frac{V_1 - 3V_0}{8} \right]. \qquad (30)$$

Here the relative momentum arguments of $V_{0,1}$ are $\boldsymbol{k} = \frac{\boldsymbol{p}}{2} - \frac{3\boldsymbol{q}}{4}$ and $\boldsymbol{k}' = \frac{\boldsymbol{p}}{2} + \frac{\boldsymbol{q}}{4} - \boldsymbol{q}'$. The result of Eq. (30) is general and useful as input to Faddeev calculations, in which the term is dressed by the effects of initial and final state strong interactions.

Using Eq. (30) in Eq. (8), we obtain our final expression for the relevant change in the scattering amplitude:

$$\delta M_{m_j',m_n';m_j,m_n}^{\rm sv}(\boldsymbol{q}', \boldsymbol{q}) = -\frac{2m}{3}(2\pi)^2 \int d\boldsymbol{p} \langle m_j', m_n' | \widehat{\phi}_d(\boldsymbol{p} + \Delta) \delta \widetilde{U}_{S,\boldsymbol{\sigma}_3}^{\rm sv}(\boldsymbol{p}, \boldsymbol{q}', \boldsymbol{q}) \widehat{\phi}_d(\boldsymbol{p}) | m_j, m_n \rangle. \qquad (31)$$

Next, we estimate the impact of these spin-violating boost corrections on A_y based on pionless EFT contact interactions for $V_{0,1}$ and using benchmarked nd phase shifts from Kievsky *et al.* [3,19] for M. This has the advantage that δA_y can be evaluated analytically and the physics is transparent.

IV. RESULTS

We can transform variables in Eq. (31) from $\boldsymbol{p} \to \boldsymbol{p} - \Delta/2$. For momentum-independent interactions $V_{0,1}$, terms linear in $\boldsymbol{p}$ in $\widetilde{U}^{\rm sv}$ integrate to zero after this variable transformation. Therefore, we can replace $\boldsymbol{p} + \boldsymbol{q}/2$ by $-\Delta/2 + \boldsymbol{q}/2 = (\boldsymbol{q} + \boldsymbol{q}')/4$ in Eq. (30), and as a result the $\boldsymbol{S} \times \boldsymbol{\sigma}_3$ term vanishes. Note that a relatively small quantity $(\boldsymbol{q} + \boldsymbol{q}')/4$ determines the change in the computed A_y. Furthermore, we simplify the integral by approximating $E_{\rm rel}$ of Eq. (28) by zero. The relative energy is very low, $E_{\rm rel} = E_d + 2E_n/3 - 3(\boldsymbol{p} + \frac{\boldsymbol{q}}{2})^2/(4m)$ (with deuteron binding energy $E_d = -2.22$ MeV), and we have $E_{\rm rel} < 0$ for the energy of interest ($E_n = 3$ MeV). So the imaginary part vanishes. Because we do not include effective range corrections, we further neglect the energy dependence of the real part of Eq. (28). It is necessary to reexamine this treatment within the framework of a Faddeev calculation that we advocate below. We thus find

$$\delta M^{\rm sv}_{m_j',m_n';m_j,m_n}(\boldsymbol{q}',\boldsymbol{q}) = \frac{i}{24m^2}(3C_0+C_1)\int d\boldsymbol{p}\langle m_j',m_n'|\widehat{\phi}_d(\boldsymbol{p}+\Delta/2)[(\boldsymbol{S}-\boldsymbol{\sigma}_3)\cdot\boldsymbol{q}\times\boldsymbol{q}']\widehat{\phi}_d(\boldsymbol{p}-\Delta/2)|m_j,m_n\rangle. \quad (32)$$

Because $\boldsymbol{S}$ commutes with the tensor operator $S_{12}(\widehat{\boldsymbol{p}})$, we can move the operator $[(\boldsymbol{S}-\boldsymbol{\sigma}_3)\cdot\boldsymbol{q}\times\boldsymbol{q}']$ to the right of $\widehat{\phi}_d(\boldsymbol{p}-\Delta/2)$ and insert a one operator in deuteron spin space $\mathbb{1}=\sum_{m_j''}|m_j''\rangle\langle m_j''|$. Using

$$\int d\boldsymbol{p}\langle m_j'|\widehat{\phi}_d(\boldsymbol{p}+\Delta/2)\widehat{\phi}_d(\boldsymbol{p}-\Delta/2)|m_j''\rangle = \delta_{m_j',m_j''}+\mathcal{O}(\Delta^2), \quad (33)$$

we can neglect the momentum dependence of the charge form factor, as well as the magnetic and quadrupole form factors of the deuteron. With $\boldsymbol{q}\times\boldsymbol{q}'=q^2\sin\theta\,\widehat{\mathbf{y}}$, we have for the spin-violating boost corrections in operator form

$$\delta M_{\rm sv} = iR(\boldsymbol{S}-\boldsymbol{\sigma}_3)^y = i\frac{q^2\sin\theta}{24m^2}(3C_0+C_1)(\boldsymbol{S}-\boldsymbol{\sigma}_3)^y. \quad (34)$$

Here we have for convenience combined all factors into the coefficient R. Combining our results with Eq. (21) leads to

$$\delta A_y = iR\frac{\mathrm{Tr}\big(S^y\boldsymbol{\sigma}_3^y M^\dagger - MS^y\boldsymbol{\sigma}_3^y + M - M^\dagger\big)}{\mathrm{Tr}(MM^\dagger)}. \quad (35)$$

The necessary spin matrix element follows from the Wigner-Eckert theorem:

$$\langle m_j'|\boldsymbol{S}^y|m_j\rangle = i[(1m_j'11|1m_j)+(1m_j'1-1|1m_j)]. \quad (36)$$

We are now in the position to study the impact on A_y. For the nd scattering amplitude M in Eq. (35) we use the phase shifts from Kievsky *et al.* [3,19]. These are based on the Argonne v_{18} NN and the Urbana 3N interaction for J^P up to $7/2^+$ (from Table 2 in Ref. [3]) and on the Argonne v_{14} NN interaction for $7/2^-$ and $9/2 \leqslant J \leqslant 13/2$ (from Tables I and II in Ref. [19]). No parameters are adjusted. As a check, we reproduce the differential cross section of Ref. [3], which is in very good agreement with experiment.

The effect of the spin-violating boost corrections on A_y is shown in Fig. 2 for $E_n = 3$ MeV and in comparison to the data from McAninch *et al.* [23]. We see that the influence of the spin-violating relativistic corrections is to increase the computed value of $A_y(\theta)$ by about 10% at the peak. This contribution is significant. It shows that relativistic effects may be relevant even at very low energies because of resonant enhancements. However, this effect alone is too small to solve the A_y puzzle. We therefore explore the effects of initial and final state interactions with the nucleon that has been treated as a spectator so far, and, finally, we discuss the energy dependence of these boost effects.

We have considered the effects of boosting the interaction between nucleons 13 and 23, while treating the nucleon 2 and nucleon 1 as a spectator. The total momentum of the boosted pair is effectively $(\boldsymbol{q}+\boldsymbol{q}')/4$. When for instance the projectile neutron interacts with nucleon 2 before interacting with nucleon 1, the total momentum of the 13 nucleon-pair will be increased due to the attractive interaction between nucleons 2 and 3, and we expect our boost effect to be enhanced. We explore the size of a 23 interaction with a schematic square-well $^1S_0$ potential of Ref. [20], which has a depth $V_0 = 13.4$ MeV and range $R = 2.65$ fm. Using conventional NN interactions, we estimate the probability to find two nucleons in a deuteron closer than R to be about 50%, so that a 23 interaction can be followed by a 13 interaction about half of the time. If this occurs, the relative momentum inside the well κ is given by

$$\kappa^2 = \frac{3q^2}{4}+mV_0 = m\left(\frac{2E_n}{3}+V_0\right), \quad (37)$$

so that $\kappa \approx 0.6\,\mathrm{fm}^{-1}$. These prescattering contributions occur about half of the time, and thus the relevant average momentum is $\approx 0.3\,\mathrm{fm}^{-1}$. This value is about three times larger than $|\boldsymbol{q}+\boldsymbol{q}'|/4 \approx 0.1\,\mathrm{fm}^{-1}$. Therefore, we expect that

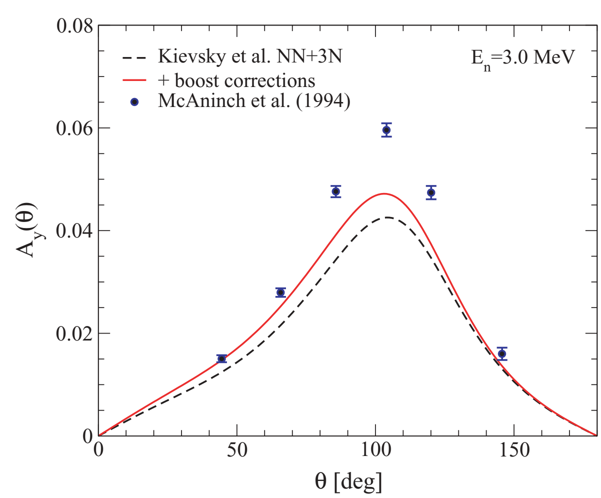

FIG. 2. (Color online) The nd analyzing power A_y for $E_n = 3$ MeV as a function of center-of-mass scattering angle θ. The dashed curve is based on nd phase shifts obtained from NN and 3N interactions [3,19] (see text for details). The solid curve includes our results for the boost corrections without distortion. The data are taken from McAninch *et al.* [23].

distortion effects will increase A_y further. This simple estimate should be taken only as an assessment that initial state interactions can make a large contribution to the boost effects.

With increasing energy, the resonant enhancement of the spin-violating relativistic corrections decreases. This is due to both the effective range r_i (and the decrease of the S-wave phase shifts with increasing energy),

$$\frac{1}{a_i} \to \frac{1}{a_i} - \frac{r_i m E_{\rm rel}}{2}, \tag{38}$$

and the impact of the imaginary part $i\sqrt{mE_{\rm rel}}$. A detailed study of the energy dependence of these boost effects is beyond the scope of this article and will be left to a future investigation [24].

V. SUMMARY AND FUTURE STEPS

In this article, we have presented the first estimate of the effects of relativistic boost corrections on the nd analyzing power A_y. We have focused on spin-violating relativistic corrections at order $(Q/m)^2$, which can be important at low energies because of a resonant enhancement from the large S-wave scattering lengths. Because boost corrections depend on the two-body c.m. momentum, the modern viewpoint is to consider their effects as 3N interactions. We have used the formalism of Refs. [15–17], where it is straightforward to include relativistic corrections to order $(Q/m)^2$ without any new parameters. The relevant spin-violating contribution to the nd transition amplitude is given in Eq. (30).

These corrections induce a 10% change in the computed value of the nd analyzing power A_y for laboratory energy $E_n = 3$ MeV. This is a small, but significant contribution of the sign necessary to resolve the A_y puzzle. Our result was estimated using the plane-wave impulse approximation, which leads to a transparent analytical calculation. The present study is clearly not complete. The effects of initial and final state interactions allow for additional contributions. For instance, the effects of δV_{12} would not vanish (as in the present calculation) if initial or final state interactions excited the deuteron. Faddeev calculations that include distortions are therefore needed to conclude on the A_y puzzle. The results presented here are mainly intended to stimulate the interest of the few-body community to include relativistic corrections in their complete solutions of the 3N problem.

For energies $E_n \gtrsim 30$ MeV, the predicted A_y based on microscopic NN and 3N interactions (without relativistic corrections) is in very good agreement with experiment. Our present results are not in contradiction to these findings, because the resonant enhancement of our spin-violating boost corrections decreases with energy. A detailed study of the energy dependence will be presented in a future article [24]. In addition, future work will estimate the scaling to larger systems and the impact on the A_y puzzle in n^3H scattering [24].

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