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How much entropy is produced in strongly coupled quark-gluon plasma (sQGP) by dissipative effects?

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We argue that estimates of dissipative effects based on first-order hydrodynamics with shear viscosity are potentially misleading because higher order terms in the gradient expansion of the dissipative part of the stress tensor tend to reduce them. Using recently obtained sound dispersion relations in thermal $\mathcal{N}=4$ supersymmetric plasma, we calculate the *resummed* effect of these high-order terms for Bjorken expansion appropriate to heavy ion collisions such as those performed at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC). A reduction of entropy production is found to be substantial, up to an order of magnitude.

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The hydrodynamic description of matter created in high-energy collisions was proposed by Landau [1] more than 50 years ago, motivated by large coupling at small distance, as followed from the β functions of QED and scalar theories known at the time. Hadronic matter is of course described by QCD, in which the coupling runs in the opposite way. And yet, recent experiments performed at the BNL Relativistic Heavy Ion Collider (RHIC) have shown spectacular collective flows, well described by relativistic hydrodynamics. More specifically, one observed three types of flow: (i) outward expansion in the transverse plane or radial flow, (ii) azimuthal asymmetry or elliptic flow [2,3], and (iii) the recently proposed conical flow from quenched jets [4]. These observations lead to the conclusion that the quark-gluon plasma (QGP) in RHIC collisions is a near-perfect liquid in a strongly coupled regime [5]. The issue we discuss below is the following: At what initial time τ_0 is one able to start the hydrodynamic (hydro) description of heavy ion collisions without phenomenological or theoretical contradictions?

Phenomenologically, it was argued in Refs. [2,3] that elliptic flow is especially sensitive to τ_0. Indeed, ballistic motion of partons may quickly erase the initial spatial anisotropy on which this effect is based. In practice, hydrodynamics at RHIC is usually used starting from time $\tau_0 \sim 1/2$ fm, otherwise the observed ellipticity is not reproduced.

Can one actually use hydrodynamics reliably at such a short time? In gaslike systems (small Knudsen number, Kn = (mean free path)/(size) $\ll$1), one can compare viscous hydro predictions to Boltzmann kinetics. For low-energy heavy ion collisions, that issue was addressed in the past (see, e.g, Kapusta's paper [6], which focused on the amount of entropy produced during the hydrodynamic stage). A long history of systematic expansion beyond Navier-Stokes, such as various versions of Burnett second-order theory accurate to $O(\mathrm{Kn}^2)$, and some applications can be found, e.g., in Ref. [7].

Early stages of the collisions or conical flows from quenched jets do correspond to Kn $= O(1)$. Yet the strongly coupled QGP (sQGP) is believed to be very different from a Boltzmann gas, so a kinetic approach is inadequate. At the classical level, one should rely instead on the molecular dynamics developed in Ref. [8]. In this paper, however, we will use the anti-de-Sitter space/conformal field theory (AdS/CFT) correspondence as our main guide.

Specifically, we will ask the question: How small should the initial hydro time τ_0 be compared to a relevant "microscopic scale"? As a relevant observable, we will monitor dissipation via entropy production. We will vary τ_0 and see how much entropy is produced after it, comparing it to the "primordial" entropy at τ_0, $\Delta S/S_0$.

To set up the problem, let us start with a very crude dimensional estimate. If we think that the QCD effective coupling is large, i.e., $\alpha_s \sim 1$, and the only reasonable microscopic length is given by temperature,[1] then the relevant micro-to-macro ratio of scales is simply $T_0\tau_0$. With $T_0 \sim$ 400 MeV at RHIC, one finds this ratio to be close to unity. We are then led to a pessimistic conclusion: at such time, the application of any macroscopic theory, thermodynamic or hydrodynamic, seems to be impossible, since order one corrections are expected.

Let us then perform the first approximation, including the explicit viscosity term to the first order. The zero-order (in the mean free path) stress tensor used in the ideal hydrodynamics has the form

$$T^{(0)}_{\mu\nu} = (\epsilon + p)\, u_\mu u_\nu + p\, g_{\mu\nu}, \tag{1}$$

while dissipative corrections are induced by gradients of the velocity field. The well-known first-order corrections are due to shear (η) and bulk (ξ) viscosities, that is,

$$\delta T^{(1)}_{\mu\nu} = \eta\left(\nabla_\mu u_\nu + \nabla_\nu u_\mu - \tfrac{2}{3}\Delta_{\mu\nu}\nabla_\rho u_\rho\right) + \xi(\Delta_{\mu\nu}\nabla_\rho u_\rho). \tag{2}$$

In this equation, we used the following projection operator onto the matter rest frame:

$$\nabla_\mu \equiv \Delta_{\mu\nu}\partial_\nu, \quad \Delta_{\mu\nu} \equiv g_{\mu\nu} - u_\mu u_\nu. \tag{3}$$

The energy-momentum conservation $\partial^\mu T_{\mu\nu}$ at this order corresponds to the Navier-Stokes equation.

Because colliding nuclei are Lorentz-compressed, the largest gradients at early time are longitudinal, along the beam direction. The expansion at this time can be approximated

[1] Note we have ignored, e.g., Λ_{QCD}.

by the well-known Bjorken rapidity-independent setup [9], in which hydrodynamic equations depend on only one coordinate: proper time $\tau = \sqrt{t^2 - x^2}$.

$$\frac{1}{\epsilon + p}\frac{d\epsilon}{d\tau} = \frac{1}{s}\frac{ds}{d\tau} = -\frac{1}{\tau}\left(1 - \frac{(4/3)\eta + \xi}{(\epsilon + p)\tau}\right), \tag{4}$$

where we have introduced the entropy density $s = (\epsilon + p)/T$. Note that for traceless $T_{\mu\nu}$ (conformally invariant plasma), the bulk viscosity $\xi = 0$.

For reasons which will soon become clear, let us compare this equation to another problem in which large longitudinal gradients appear as well, namely, a sound wave in the medium. The dispersion relation (the pole position) for a sound wave with frequency ω and wave vector q is, at small q,

$$\omega = c_s q - \frac{i}{2}q^2\Gamma_s, \quad \Gamma_s \equiv \frac{4}{3}\frac{\eta}{\epsilon + p}. \tag{5}$$

Notice that the right hand side of Eq. (4) contains precisely the same combination of viscosity and thermodynamic parameters as appears in the sound attenuation problem: the length Γ_s, which measures directly the magnitude of the dissipative corrections. At proper times $\tau \sim \Gamma_s$, one has to abandon the hydrodynamics altogether, as the dissipative corrections cannot be ignored.

For the entropy production in Eq. (4), the first correction to the ideal case is $(1 - \Gamma_s/\tau)$. Since the correction to one is negative, it reduces the rate of the entropy decrease with time. An equivalent statement is that the total positive sign shows that some amount of entropy is generated by the dissipative term. Danielewicz and Gyulassy [10] have analyzed Eq. (4) in great detail considering various values of η. Their results indicate that the entropy production can be substantial.

Our present study is motivated by the following argument. If the hydrodynamic description is forced to begin at early time τ_0 which is *not* large compared to the intrinsic microscale $1/T$, then limiting dissipative effects to the first gradient only $(\delta T^{(1)}_{\mu\nu})$ is parametrically not justified and higher order terms have to be accounted for. Ideally, those effects need to be *resummed*. As a first step, however, we may attempt to guess their sign and estimate the magnitude.

Formally, one can think of the dissipative part of the stress tensor $\delta T_{\mu\nu}$ as expended in a series containing all derivatives of the velocity field u, $\delta T^1_{\mu\nu}$ being the first term in the expansion. In the general $3+1$ dimensional case, there are many structures, each entering with a new and independent viscosity coefficient. We call them higher order viscosities, and the expansion is somewhat similar to a twist expansion. For the $1+1$ Bjorken problem, the appearance of the extra terms modifies Eq. (4), which can be written as a series in inverse proper time

$$\frac{\partial_\tau(s\tau)}{s\,(\tau\,T)} = 4\frac{\eta}{s}\left[\frac{1}{3}\,\frac{1}{(\tau\,T)^2} + \sum_{n=2}^{\infty}\frac{c_n}{(T\,\tau)^{2n}}\right]. \tag{6}$$

We have put T here simply for dimensional reasons: clearly $T\tau$ is a micro-to-macro scale ratio which determines convergence of these series and the total amount of produced entropy. Similarly, the sound wave dispersion relation becomes nonlinear as we go beyond the lowest order:

$$\begin{aligned} \omega &= \Re[\omega(q)] + i\,\Im[\omega(q)], \\ \frac{\Re[\omega]}{2\pi\,T} &= c_s\,\frac{q}{2\pi\,T} + \sum_{n=1}^{\infty} r_n\left(\frac{q}{2\pi\,T}\right)^{2n+1}, \\ \frac{\Im[\omega]}{2\pi T} &= -\frac{4\pi\eta}{s}\left[\frac{1}{3}\left(\frac{q}{2\pi T}\right)^2 + \sum_{n=2}^{\infty}\eta_n\left(\frac{q}{2\pi T}\right)^{2n}\right]. \end{aligned} \tag{7}$$

Based on T-parity arguments, we keep only odd (even) powers of q for the real (imaginary) parts of ω. The coefficients c_n, r_n, and η_n are related since they originate from the very same gradient expansion of $T_{\mu\nu}$. Although both the entropy production series above and sound absorption should converge to a sign-definite answer, the coefficients of the series may well be of alternating sign (as we will see shortly).

Clearly, keeping these next-order terms can be useful only provided there is some microscopic theory which would make it possible to determine the values of the high-order viscosities. For strongly coupled QCD plasma, this information is at the moment beyond current theoretical reach, and we have to rely on models. A particularly useful and widely studied model of QCD plasma is the $\mathcal{N} = 4$ supersymmetric plasma, which is also conformal (CFT). The AdS/CFT correspondence [11] (see Ref. [12] for review) relates the strongly coupled gauge theory description to the weakly coupled gravity problem in the background of the AdS_5 black hole metric. Remarkably, certain information on higher order viscosities in the CFT plasma can be found in the literature, and we exploit this possibility below.

The viscosity-to-entropy ratio ($\eta/s = 1/4\pi$) deduced from AdS [13] turns out to be quite a reasonable approximation to the values appropriate for the RHIC data description. Thus one may hope that the information on the higher viscosities gained from the very same model can be trusted as a model for QCD. Admittedly having no convincing argument in favor, we simply assume that the viscosity expansion of the QCD plasma displays very similar behavior, both qualitative and quantitative, as its CFT sister.

Our estimates are based on the analysis of the quasinormal modes in the AdS black hole background by Kovtun and Starinets [14]. The dispersion relation for the sound mode, calculated in Ref. [14], is shown in Fig. 1. The real and imaginary parts of ω correspond to the expressions given in Eq. (7). At $q \to 0$, they agree with the leading-order hydrodynamic dispersion relation for sound [Eq. (5)] in which damping is quadratic in momentum.

The first important observation is that the next-order coefficient η_2 is *negative*, reducing the effect of the first (Navier-Stokes) term when gradients are large. The second is that $|\Im[\omega]|$ has a maximum at $q/2\pi T \sim 1$, and at large q the imaginary part starts to decrease. This means that the expansion (7) has a radius of convergence $q/2\pi T \sim 1$.

This behavior of sound is not common; usually the dissipation grows until the inverse momenta reach the interparticle distance and sound modes lose their meaning. But the CFT liquid is also not usual, that is, the scale T and inverse interparticle distance are infinitely separated; the latter goes

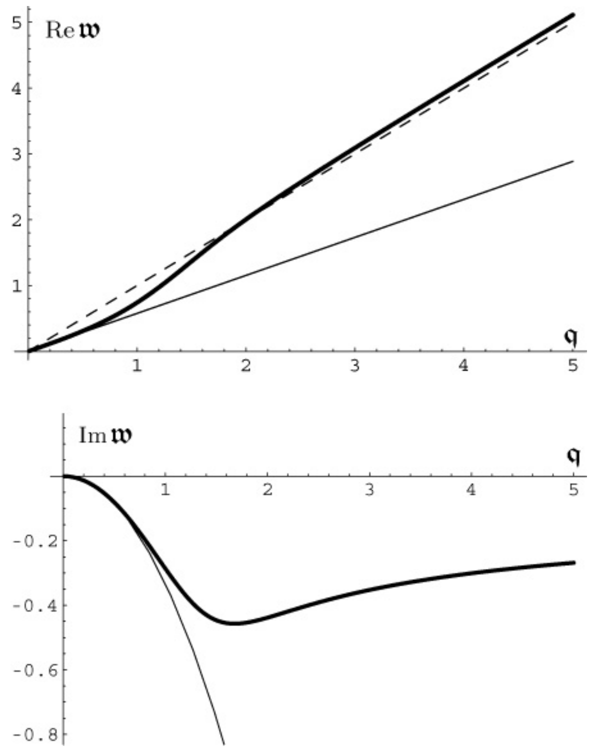

FIG. 1. Sound dispersion (real and imaginary parts) obtained from the analysis of quasinormal modes in the AdS black hole background. Both ω and q are plotted in units of $2\pi T$. The result and figure are taken from Ref. [14].

to infinity with the number of colors $N_c^{2/3} \to \infty$, and thus sound modes may exist at all momenta.

Why does the AdS/CFT construction predict a reduced damping at large momenta? Unfortunately, there are no analytic results for the AdS black hole quasinormal frequencies. We have reproduced the numerical results of Kovtun and Starinets [14]. While doing so, we reached a qualitative understanding of the phenomenon as it emerges from the gravity side: the momentum q entering the Schrodinger-type equation for the quasinormal modes in AdS plays a role of angular momentum, and the $O(q^2)$ term in this equation is essentially the centrifugal term. The centrifugal potential, as usual, acts toward reducing the wave functions near the AdS origin (or rather the black hole horizon where absorption takes place). Thus quasinormal frequencies generically emerge with the damping term decreasing with q. This observation is consistent with the results known for the quasinormal modes of the usual four-dimensional Schwartschield black hole, the problem set up 50 years ago by Regge and Wheeler [15]. Thanks to significant progress made in recent years, the modes are now known analytically [16,17]. Despite significant differences between this case and the AdS black hole in five dimensions, one indeed finds the orbital term reducing the damping, as argued above.

The lowest frequency in the sound channel is a special case. Its imaginary part cannot decrease with q since it starts from zero at $q = 0$; so it must grow first, before decreasing. Analytic results for the AdS modes would certainly help clarify the nature of the phenomenon, but the real challenge is to understand it from the gauge theory side.

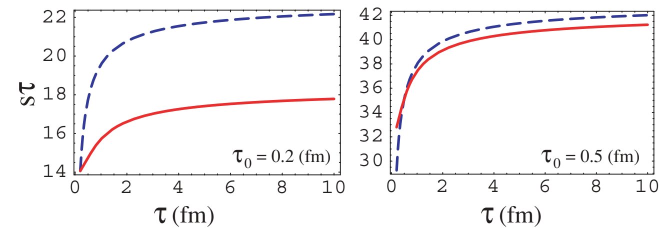

FIG. 2. (Color online) Entropy production as a function of proper time for initial time $\tau_0 = 0.2$ fm (left) and $\tau_0 = 0.5$ fm (right). The initial temperature $T_0 = 300$ MeV. The dashed (blue) curves correspond to the first-order (shear) viscosity approximation Eq. (4). The solid curve (red) is the all-order dissipative resummation Eq. (6).

To estimate the effect of higher viscosities on entropy production in the Bjorken setup, we first identify τ in Eq. (6) with $2\pi/q$ in Eq. (7). [This identification is naturally suggested by comparing the first term in the bracket in Eq. (6) with its partner in Eq. (7).] Indeed, while in proper time and spatial rapidity τ, η coordinates, the factors of $1/\tau$ come from its curved geometry, in the original coordinates t, x one may still think of those factors as coming from expansion in longitudinal space derivatives, with τ being simply an instantaneous longitudinal size. Thus we will identify the coefficients c_n with η_n. Both sound attenuation and entropy production in question are one-dimensional problems associated with similar longitudinal gradients and presumably similar physics. In practice, we use the curve for the imaginary part of ω (Fig. 1) as an input for the right hand side of Eq. (6).

The numerical results are shown in Figs. 2 and 3, in which we compare our estimates with the "conventional" shear viscosity results from Eq. (4). To be fully consistent with the model, we set $\eta/s = 1/4\pi$. We also set the initial temperature $T_0 = 300$ MeV, while the standard equation of state $s = 4k_{\mathrm{SB}}T^3$. For the coefficient k_{SB} we use the "QCD" value

$$k_{\mathrm{SB}} = \frac{\pi^2}{90}\left(2(N_c-1)^2 + \frac{7}{2}N_c n_f\right), \quad n_f = 3, \quad N_c = 3.$$

Figure 2 presents the results for entropy production as a function of proper time for two initial times $\tau_0 = 0.2$ and $\tau_0 = 0.5$ fm. The dashed lines correspond to the first-order result in Eq. (4), while the solid curves include the higher order

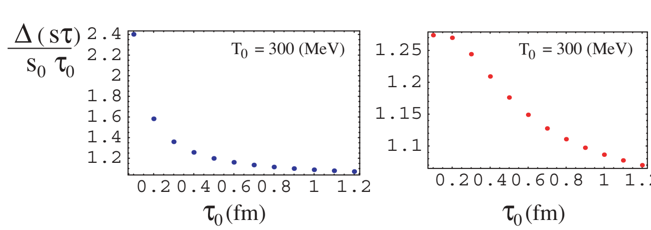

FIG. 3. (Color online) Fraction of entropy produced during the hydro phase as a function of initial proper time. The initial temperature $T_0 = 300$ MeV. The left (blue) points correspond to the first-order (shear) viscosity approximation. The right (red) points are for the all-order resummation.

viscosity corrections. Noticeably, there is a dramatic effect toward reduction of the entropy production as we start the hydro evolution at earlier times (the effect is almost invisible on the temperature profile). This is the central message of the present paper.

Figure 3 illustrates the relative amount of entropy produced during the hydro phase as a function of initial time. If the first-order hydrodynamics is launched at very early times, the hydro phase produces an excessive amount of entropy by up to 250%. (Such a large discrepancy is not seen in the RHIC data.) In sharp contrast, the result from the resummed viscous hydrodynamics is very stable and does not produce more than some 25% of initial entropy, even if pushed to start from extremely early times. The right-hand figure displays the absence of any pathological explosion at small τ_0.

It is worth commenting that we carried the analysis using the minimal value for the ratio $\eta/s = 1/4\pi$. We expect that if this ratio is taken larger, the discrepancy between the first-order dissipative hydro and all orders will be even stronger.

Before concluding this paper, we note that a practical implementation of relativistic viscous hydrodynamics has followed the Israel-Stewart second-order formalism (for recent publications see Ref. [18]) in which one introduces an additional parameter, the relaxation time for the system. Then the dissipative part of the stress tensor is found as a solution of an evolution equation, with the relaxation time being its parameter. For the Bjorken setup, the dissipative tensor thus obtained has all powers in $1/\tau$ and might resemble the expansion in Eqs. (6) and (7). The use of AdS/CFT may shed light on the interrelation between the two approaches: the first step in this direction has been made recently [19], resulting in a numerically very small relaxation time.

Finally, why can macroscopic approaches such as hydrodynamics be rather accurate at such a short time scale? In trying to answer this central question, one should keep in mind that $1/T$ is *not* the shortest microscopic scale. The interparton distance is much smaller, $\sim 1/(T N_{\rm dof}^{1/3})$, where the number of effective degrees of freedom $N_{\rm dof} \sim 40$ in QCD; while $N_{\rm dof} \sim N_c^2 \to \infty$ in the AdS/CFT approach.

In summary, we have argued that the higher order dissipative terms strongly reduce the effect of the usual viscosity. Therefore an "effective" viscosity-to-entropy ratio, found from a comparison of Navier-Stokes results and experimental data, can even be below the (proposed) lower bound of $1/4\pi$. We conclude that it is not impossible to use a hydrodynamic description of a RHIC collision starting from very early times. In particular, our study suggests that the final entropy observed and its "primordial" value obtained right after collision should indeed match, with an accuracy of 10–20%.

Note added: After this paper was submitted for archiving and publication, we learned of the results of three independent works [20] addressing viscous corrections to the elliptic flow. It emerges from Ref. [20] that only very small η/s, close or even smaller than its lower bound $1/4\pi$, is needed to fit the data. However, at early times, when the elliptic flow is formed, the next-order viscous effects pointed out in our paper are important. They may partially cancel the first-order viscous effect and thus explain this puzzling discrepancy.

We are thankful to Adrian Dumitru whose results (presented in his talk at Stony Brook, see also Ref. [21]) inspired us to think about the issue of entropy production during the hydrodynamic phase. He emphasized to us the important problem of matching the final entropy measured after the late hydro stage with the early-time partonic predictions, based on approaches such as the color glass condensate. This work is supported by the U.S. DOE through Grant Nos. DE-FG02-88ER40388 and DE-FG03-97ER4014.

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