

## Potential inversion with sub-barrier fusion data reexamined

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We invert experimental data for heavy-ion fusion reactions at energies well below the Coulomb barrier in order to directly determine the internucleus potential between the colliding nuclei. In contrast to the previous applications of the inversion formula, we explicitly take into account the effect of channel couplings on fusion reactions, by assuming that fusion cross sections at deep sub-barrier energies are governed by the lowest barrier in the barrier distribution. We apply this procedure to the  $^{16}\text{O} + ^{144}\text{Sm}$  and  $^{16}\text{O} + ^{208}\text{Pb}$  reactions, and find that the inverted internucleus potential are much thicker than phenomenological potentials. A relation to the steep fall-off phenomenon of fusion cross sections recently found at deep sub-barrier energies is also discussed.

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Nuclear reactions are primarily governed by the nucleus-nucleus potential. In particular, the Coulomb barrier, which appears due to a strong cancellation between the attractive nuclear force and the long-range repulsive Coulomb interaction, plays a decisive role in heavy-ion collisions. Several methods have been proposed to compute the real part of the internuclear potential. Among them, the double folding model has been often employed and has enjoyed a success in describing elastic and inelastic scattering for many systems [1–3]. The Woods-Saxon form has also often been used to parametrize the internuclear potential [4]. The surface region of the double folding potential can in fact be well parametrized by the Woods-Saxon form with the diffuseness parameter of around 0.63 fm, and such phenomenological potential has been as successful as the double folding potential.

In recent years, many experimental evidences have accumulated that show that the double folding potential fails to account for the *fusion* cross sections at energies close to the Coulomb barrier. That is, the double folding potential (and also the Woods-Saxon potential which fits elastic scattering) overestimates fusion cross sections at energies both above and below the Coulomb barrier, having an inconsistent energy dependence to the experimental fusion excitation function [5–11]. This trend is in accordance with the more recent measurements of fusion cross sections at extreme sub-barrier energies, that show a much steeper fusion excitation functions as compared with theoretical predictions [12].

Notice that the scattering process is sensitive mainly to the surface region of the nuclear potential, while the fusion reaction is also relatively sensitive to the inner part. The double folding potential and the Woods-Saxon potential are reasonable in the surface region [13]. However, it is not obvious whether they provide reasonable parametrizations inside the Coulomb barrier, where the colliding nuclei significantly overlap with each other [6,14,15]. This is so, particularly because the double folding potential takes into account only the so called knock-on exchange effect, ignoring all the other exchange effects originating from the antisymmetrization of the total wave function of the colliding system [16].

The purpose of this paper is to investigate the radial shape of the internucleus potential inside the Coulomb barrier and discuss its deviation from the conventional parametrizations.

To this end, we apply the inversion formula based on the WKB approximation [17] and determine the internuclear potential directly from the experimental data without assuming any parametrization. This method was used many years ago by Balantekin *et al.* [18]. They assumed a one-dimensional energy independent local potential, and found that the inversion procedure leads to an unphysical multivalued potential for heavy systems. This analysis has provided a clear evidence for inadequacy of the one-dimensional barrier passing model for heavy-ion fusion reactions, and has triggered to develop the coupled-channels approach.

Although the analysis of Balantekin *et al.* is important as it has clarified the dynamics of sub-barrier fusion reactions, it is not satisfactory from the point of view of determination of the internucleus potential. Since the experimental evidences for inadequacy of the double folding potential inside the Coulomb barrier is increasingly accumulating, it is intriguing to revisit this problem by taking into account the current understanding of sub-barrier fusion. In this connection, we mention that the main reason why Balantekin *et al.* obtained the unphysical internucleus potentials is that they did not take into account the channel coupling effect, which has by now been well understood in terms of barrier distribution [5,20–23]. Our idea here is to apply the inversion procedure only to the lowest barrier in the barrier distribution assuming that the fusion cross sections are determined only by it at deep sub-barrier energies. We will demonstrate below that the internucleus potentials thus obtained are well behaved and show a significant deviation from the conventional Woods-Saxon shape.

For a single channel system with a potential  $V(r)$ , the inversion formula relates the thickness of the potential, i.e., the distance between the two classical turning points at a given energy  $E$ , with the classical action  $S$  as [17,18]

$$t(E) \equiv r_2(E) - r_1(E) \quad (1)$$

$$= -\frac{2}{\pi} \sqrt{\frac{\hbar^2}{2\mu}} \int_E^{V_b} dE' \frac{\left(\frac{dS}{dE'}\right)}{\sqrt{E' - E}}, \quad (2)$$

where  $\mu$  is the reduced mass between the colliding nuclei and  $V_b$  is the height of the potential. The classical action  $S(E)$  is

given by

$$S(E) = \int_{r_1(E)}^{r_2(E)} dr \sqrt{\frac{2\mu}{\hbar^2}(V(r) - E)}, \quad (3)$$

and can be obtained once the penetrability  $P(E)$  is found in some way using the WKB relation  $P(E) = 1/[1 + e^{2S(E)}]$ .

In heavy-ion fusion reactions, it is well known that the  $S$ -wave penetrability for the Coulomb barrier can be approximately obtained from the fusion cross section  $\sigma_{\text{fus}}$  as [18,20–22]

$$P(E) = \frac{d}{dE} \left( \frac{E\sigma_{\text{fus}}}{\pi R^2} \right), \quad (4)$$

where the effective moment of inertia  $R$  may depend on energy [18,19]. This formula assumes that the number of classical turning point is two for all partial waves, and thus implicitly assumes a deep internuclear potential. In the previous application of the inversion formula by Balantekin *et al.*, they assumed that the penetrability so obtained was resulted from the penetration of a one-dimensional energy independent potential [18]. Instead, here we assume that the penetrability  $P$  is given as a weighted sum of contribution from many distributed barriers, where the distribution arises due to a coupling of the relative motion between the colliding nuclei to nuclear intrinsic degrees of freedoms such as collective vibrational or rotational excitations. In this eigenchannel picture, the penetrability is given by

$$P(E) = \sum_n w_n P_n(E), \quad (5)$$

where  $P_n$  is the penetrability for the  $n$ -th eigenbarrier and  $w_n$  is the corresponding weight factor. This concept has been well established by now from the experimental measurements for the barrier distribution [5,21] as well as from numerical calculations of coupled-channels equations [23,24]. In principle, the weight factors  $w_n$  depend on energy if the excitation energy for the intrinsic motion is not zero. However, the energy dependence is shown to be weak [24], and we assume in this paper that the weight factors are energy independent.

At energies below the lowest eigenbarrier (i.e., the adiabatic barrier) in the barrier distribution, one expects that only the lowest barrier contributes to the total penetrability,

$$P(E) \approx w_0 P_0(E). \quad (6)$$

This indicates that one can apply the inversion formula to the lowest eigenbarrier using fusion cross sections at deep sub-barrier energies, after correcting the weight factor. In order to demonstrate how this works, Figs. 1(a) and 1(b) show the second and the first derivatives of the measured  $E\sigma_{\text{fus}}$  [5] for the  $^{16}\text{O} + ^{144}\text{Sm}$  reaction, respectively. The former quantity is usually referred to as the fusion barrier distribution [5,21]. We use the point difference formula with  $\Delta E_{\text{c.m.}} = 1.8$  MeV to carry out the derivatives. For this system, one can clearly recognize that the barrier distribution has a double peaked structure. Correspondingly, the first derivative  $d(E\sigma_{\text{fus}})/dE$  appears to have two steps as a function of energy. If one neglects weak couplings to the double phonon state in the  $^{144}\text{Sm}$  nucleus [25], the double peaked structure of the barrier distribution can be interpreted to originate mainly from the coupling to the first 3-

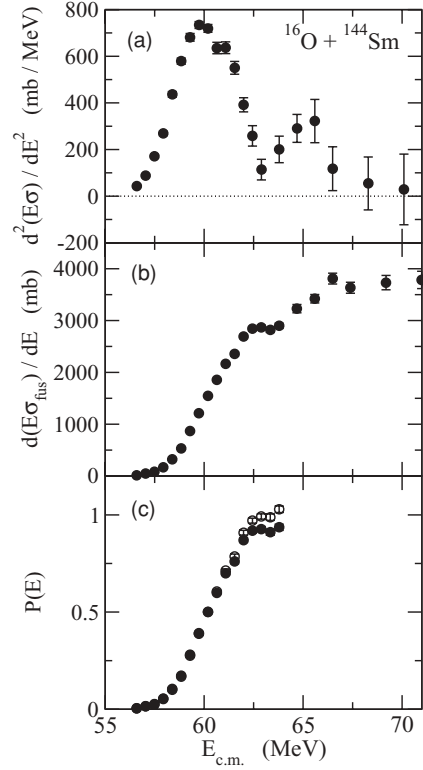


FIG. 1. (a) The experimental fusion barrier distribution for the  $^{16}\text{O} + ^{144}\text{Sm}$  reaction defined as  $d^2(E\sigma_{\text{fus}})/dE^2$ . The experimental data are taken from Ref. [5]. (b) The first derivative of  $E\sigma_{\text{fus}}$  for the  $^{16}\text{O} + ^{144}\text{Sm}$  reaction. (c) The same as (b), but normalized so that it is 0.5 at the energy of the lower peak in the barrier distribution shown in (a) (the filled circle). The open circles are obtained in a similar way, but by taking into account the energy dependence of the effective moment of inertia as given by Eq. (9).

state in  $^{144}\text{Sm}$ . Notice that in general a barrier distribution has a peak at the energy equal to the height of a potential barrier. Assuming that the main peak of the barrier distribution around  $E_{\text{c.m.}} \sim 60$  MeV consists only of the contribution from the lowest eigenbarrier, we scale the first derivative  $d(E\sigma_{\text{fus}})/dE$  so that it has a value of 0.5 at the peak energy, which we assume to be identical to the position of the lowest barrier,  $V_b$ . The scaling factor corresponds to the product of the geometrical factor  $\pi R^2$  and the weight factor  $w_0$  [see Eqs. (4) and (6)]. The function thus obtained is shown by the filled circles in Fig. 1(c). This function can be interpreted as the penetrability for the lowest barrier, to which one can apply the inversion formula to determine the radial shape.

The inversion formula yields only the barrier thickness,  $t(E)$ , and one has to supplement either the outer or the inner turning points to determine the radial shape of the potential [18]. We estimate the *outer* turning point  $r_2(E)$  using the Coulomb interaction of point charge and the Woods-Saxon nuclear potential,

$$V_N(r) = -\frac{V_0}{1 + \exp[(r - R_0)/a]}, \quad (7)$$

with the range parameter of

$$R_0 = \sum_{i=P,T} (1.233A_i^{1/3} - 0.98A_i^{-1/3}) + 0.29 \text{ (fm)}, \quad (8)$$

and the diffuseness parameter of  $a = 0.63$  fm. We adjust the depth  $V_0$  in order to reproduce the barrier height  $V_b$  determined from the peak position of the barrier distribution. Since the Coulomb term dominates at the outer turning point, except for the region near the barrier top, the inverted potential is insensitive to the actual shape of nuclear potential employed to estimate the outer turning point. The Woods-Saxon potential (7) determines not only the outer turning point but also the position of the potential barrier,  $R_b$ . Following Ref. [18], we use

$$R(E) = \frac{1}{2} (R_b + Z_P Z_T e^2 / E), \quad (9)$$

for the effective moment of inertia in Eq. (4). The penetrability obtained by taking into account the energy dependence of the effective moment of inertia is denoted by the open circles in Fig. 1(c). Although the difference between the filled and open circles is not large, especially at energies below the barrier, the penetrability behaves slightly better if one considers the energy dependence of moment of inertia, since it is saturated at unity at high energies. In the actual application of the inversion formula shown below, we smooth the data points with a fifth-order polynomial fit to the function  $\ln[E\sigma_{\text{fus}}/\pi R(E)^2]$  [18]. We have confirmed that the results do not significantly change even if we use a higher order polynomial fit. We also fit the lowest peak of the barrier distribution using the Wong formula [26] in order to accurately estimate the barrier height  $V_b$ .

We have confirmed the accuracy of the inversion procedure using the theoretical fusion cross sections obtained by the computer code CCFULL [27]. For this purpose, we consider the  $^{16}\text{O} + ^{144}\text{Sm}$  system, and generate the fusion cross sections by taking into account the excitation to the first  $3^-$  state in  $^{144}\text{Sm}$ . We use the same parameters as in Ref. [28]. We find that the resultant inverted potential closely follows the adiabatic potential obtained by diagonalizing the coupling Hamiltonian at each position  $r$  [23,24]. This evidently justifies our procedure for the potential inversion discussed above.

We now invert the experimental data for the  $^{16}\text{O} + ^{144}\text{Sm}$  system shown by the open circles in Fig. 1(c) in order to obtain the radial dependence of the adiabatic barrier. The result of the inversion method is shown in Fig. 2. The uncertainty

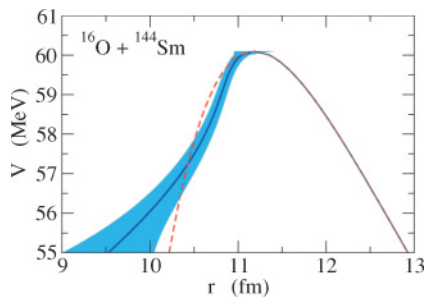


FIG. 2. (Color online) The adiabatic potential for the  $^{16}\text{O} + ^{144}\text{Sm}$  reaction obtained with the inversion method. The dashed line is a barrier due to a phenomenological Woods-Saxon potential.

of the inverted potential is estimated in the same way as in Ref. [18]. The dashed line shows the barrier due to the Woods-Saxon potential (7) used to estimate the outer turning points. One clearly sees that the inverted potential is much thicker than the phenomenological potential at low energies, although it is close to the phenomenological potential at energies close to the potential barrier. This trend is opposite to what Balantekin *et al.* found in the previous analysis. If there was an unresolved peak in the barrier distribution below the main peak, one would obtain a much thinner barrier than the phenomenological potential, as in the previous analysis. We have actually obtained such unphysical thin barriers for the  $^{17}\text{O} + ^{144}\text{Sm}$  and  $^{16}\text{O} + ^{148}\text{Sm}$  systems, where the main peak of the barrier distribution is not expected to correspond to the lowest eigenbarrier [5]. Having a thick barrier, rather than a thin barrier, we are convinced that the main peak of the barrier distribution for the  $^{16}\text{O} + ^{144}\text{Sm}$  reaction indeed consists of the lowest eigenbarrier. In this way, the potential inversion method could also be used to judge whether there is an unresolved barrier in the barrier distribution.

For the  $^{16}\text{O} + ^{144}\text{Sm}$  reaction, the experimental data exist only from  $E_{\text{c.m.}} = 56.6$  MeV (that is, about 3.6 MeV below the lowest barrier height), and the inverted potential has a large uncertainty below this energy. In order to discuss the behavior of the potential far below the barrier, we next consider the  $^{16}\text{O} + ^{208}\text{Pb}$  reaction, for which the fusion cross sections were recently measured at deep sub-barrier energies [29]. Figure 3 shows the potential barrier for this system obtained with the inversion method. One sees that the behavior of the potential is qualitatively similar to that for the  $^{16}\text{O} + ^{144}\text{Sm}$  reaction shown in Fig. 2. Namely, the inverted potential is close to a phenomenological potential in the region near the barrier top, but it deviates largely at deep sub-barrier energies, where the thickness is much larger than the phenomenological potential. The thicker the potential is, the smaller the penetrability is, and also the stronger the energy dependence of the penetrability is. The thick potential barrier obtained for the  $^{16}\text{O} + ^{144}\text{Sm}$  and  $^{16}\text{O} + ^{208}\text{Pb}$  systems is thus consistent with the recent experimental observations [12,29] that the fusion excitation function is much steeper than theoretical predictions at deep subbarrier energies. Although the present analysis does not exclude a possibility of a shallow potential [15], the present study suggests that the origin of the steep fall-off phenomenon of fusion cross section can be at least partly attributed to the departure of internuclear potential from the Woods-Saxon shape.

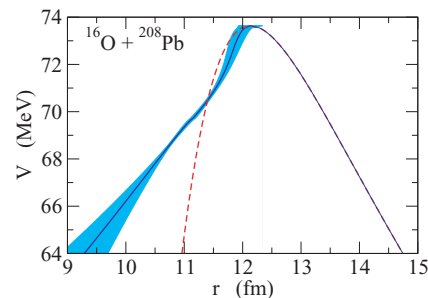


FIG. 3. (Color online) Same as Fig. 2, but for the  $^{16}\text{O} + ^{208}\text{Pb}$  reaction.

For the  $^{16}\text{O} + ^{208}\text{Pb}$  system shown in Fig. 3, the deviation of the inverted potential from the phenomenological potential starts to occur at around  $E = 70.4$  MeV. It is interesting to notice that this energy is very close to the potential energy at the contact configuration estimated with the Krappe-Nix-Sierk potential [30,31]. Inside the touching configuration, the potential represents the fission-like adiabatic potential energy surface. The effect of such one-body potential has been considered recently and is shown to account well for the steep fall-off phenomena of fusion cross sections [31]. The inverted potentials which we obtain are thus intimately related to the one-body dynamics for deep sub-barrier fusion reactions.

In summary, we applied the potential inversion method, which relates the potential penetrability to the thickness of the potential barrier, in order to investigate the radial dependence of the internucleus potential for heavy-ion fusion reactions. To this end, we assumed that the tunneling is well described by the lowest adiabatic barrier at deep sub-barrier energies, and extracted the penetrability by combining the experimental barrier distribution and fusion cross sections. We found that the resultant potential for the  $^{16}\text{O} + ^{144}\text{Sm}$  and  $^{16}\text{O} + ^{208}\text{Pb}$  systems is much thicker than a barrier obtained with a phenomenological Woods-Saxon potential. This indicates that the steep fall-off phenomenon of fusion cross sections

recently observed in several systems can be partly accounted for in terms of a deviation of internuclear potential from the Woods-Saxon shape.

The lowest peak in a barrier distribution is relatively well resolved in general for systems involved with a vibrational target. It would be an interesting future work to apply the inversion method systematically to such systems and discuss a global potential for heavy-ion collisions. Another problem is the dynamical effects after the touching configuration. In this paper, we exploited a barrier distribution picture for sub-barrier fusion, but assumed an energy independent potential for each distributed potential barrier. It would be an interesting work to discuss how the thick potential which we obtained in this paper is related to dynamical effects such as the coordinate dependence of reduced mass, and energy and angular momentum dissipations, that are other promising origins for the steep fall-off phenomenon of fusion cross sections [7,11,29].

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