Extraction of the longitudinal and transverse response functions in $(e, e'p)$ reactions

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In this Brief Report we extract the longitudinal and transverse response functions from exclusive $(e, e'p)$ cross sections at fixed squared four-momentum transfer Q^2 in the quasielastic region. They are extracted in parallel kinematics by applying the Rosenbluth separation to the $(e, e'p)$ reactions. The distorted Coulomb effects are also taken into account on each extracted response functions. The Coulomb effects in the response functions are different from those of the corresponding cross sections.

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Medium and high energy electron scattering has been long acknowledged as a useful tool to investigate nuclear structure and properties in the quasielastic region. In particular, the exclusive $(e, e'p)$ reactions have given many fruitful results for the interference response functions from the target nuclei, which could not be obtained from the inclusive (e, e') reactions in the quasielastic region. In the plane wave Born approximation (PWBA), where the electrons are described by Dirac plane waves, the cross section for the exclusive $(\vec{e}, e'p)$ reactions with incoming polarized electron beam is given by

$$\frac{d^3\sigma}{dE_f d\Omega_f d\Omega_p} = K[v_L R_L + v_T R_T + v_{TT}\cos 2\phi_p R_{TT} + v_{LT}\cos\phi_p R_{LT} + h v_{LT'}\sin\phi_p R_{LT'}], \quad (1)$$

where R_L, R_T, R_{TT}, R_{LT}, and $R_{LT'}$ represent the longitudinal, transverse, transverse-transverse, longitudinal-transverse, and polarized longitudinal-transverse response functions, respectively. If the incident electron beam is unpolarized the fifth term (polarized longitudinal-transverse response function) disappears. In particular, the fifth term is known to vanish in the absence of the final state interaction of the knocked-out proton [1]. The incoming and outgoing electrons define the scattering plane (x-z plane) with $p_i^\mu = (E_i, \mathbf{p}_i)$ and $p_f^\mu = (E_f, \mathbf{p}_f)$, respectively. The three-momentum transfer $\mathbf{q}$ is along $\hat{\mathbf{z}}$ direction, ϕ_p is the azimuthal angle of the knocked-out proton measured with the electron plane, and h is the helicity of the polarized incident electron. K denotes the electron kinematics factor given by $pE_p\sigma_M/(2\pi)^3$ with the Mott cross section σ_M and the knocked-out proton momentum $\mathbf{p}$ and energy E_p. The functions v_L, v_T, etc., depend only on the electron kinematics given by

$$v_L = \frac{Q^4}{q^4}, \quad v_T = \tan^2\frac{\theta}{2} + \frac{Q^2}{2q^2}, \quad v_{TT} = \frac{Q^2}{2q^2},$$
$$v_{LT} = \frac{Q^2}{q^2}\left(\tan^2\frac{\theta}{2} + \frac{Q^2}{2q^2}\right)^{1/2}, \quad v_{LT'} = \frac{Q^2}{q^2}\tan\frac{\theta}{2}, \quad (2)$$

where θ represents the electron scattering angle. The four-momentum transfer is defined by $Q^2 = q^2 - \omega^2$ with the energy transfer ω.

By expanding the partial waves for the electron wave function, Ohio group [5] exactly treated the Coulomb distortion from the static Coulomb field of the target nucleus, referred to the full distorted wave Born approximation (DWBA). The full DWBA calculation can compare various nuclear models and furnish an invaluable check for several approximations of the Coulomb distortion. But it is numerically challenging and computational time increases rapidly with the higher incident electron energies. Moreover, it is not possible to express the cross section as a sum of bilinear products of the response functions such as Eq. (1), so that there is no way to investigate each response functions separately.

There are two approaches to treat the electron Coulomb distortion approximately. One is an analytic form for the electron wave functions with the Coulomb distortion based on the work of Lenz and Rosenfelder [6]. The approximation is that the electron momentum is replaced by a value shifted by the Coulomb potential at the origin [7,8], called the effective momentum approximation (EMA). While this approximation is well described for light nuclei and high incident electron energies it is not good for heavy nuclei and for intermediate electron energies [9]. Recently, Kim and Wright [10] improved the EMA using the Coulomb potential at $\frac{2}{3}R$ (R is a radius of target nuclei) instead of the value at the origin for the inclusive (e, e') reaction at high electron energies greater than 1 GeV.

As an other approach, since the middle of 1990's, Kim and Wright [9,11,12] developed the approximation of the Coulomb distorted electron wave functions to solve the above difficult problems related to the full DWBA calculation. The essence of the approximation is that the electron wave functions contain r-dependent momentum and the parametrization of the elastic scattering phase shifts in terms of the angular momentum. This electron wave function has a "plane-wave-like" form which *directly* allows the extraction of the various response functions such as the PWBA calculation. This approximation showed a good agreement of about 1–2% with the full DWBA calculations near the peaks of the cross sections for heavy target nucleus [11]. At the high incident electron energies

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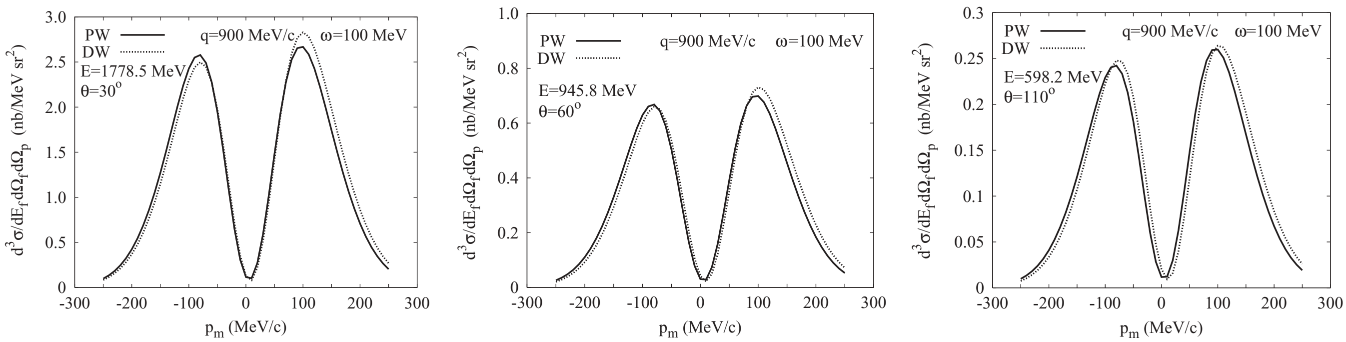

FIG. 1. The cross sections in terms of the missing momentum at $q = 900$ MeV/c and $\omega = 100$ MeV. The incident energies and the scattering angles are 1778.5 MeV and 30°, 945.8 MeV and 60°, and 598.2 MeV and 110°, respectively. The solid curves denote the results of the PWBA and the dotted lines are for the approximate DWBA. The outgoing proton is knocked-out from the $p_{1/2}$ shell of ^{16}O.

such as JLab type [13], the approximation described the experimental data very well [12]. This approximation of the electron wave function in the presence of the static Coulomb potential is called the approximate DWBA.

There have appeared some theoretical works for investigating the response functions. For example, Picklesimer and Van Orden [2] developed theoretical descriptions of the response functions for the $(\vec{e}, e'\vec{N})$ reactions, which coincidentally polarize the incident electron beam and the knocked-out nucleons from nuclei in a relativistic frame. References [3,4] studied the electron scattering for both polarized incident electron and polarized targets, $\vec{A}(\vec{e}, e'N)B$, in the framework of an impulse approximation. Especially, the Pavia group [4] directly calculated the longitudinal and transverse response functions from the hadron current in parallel kinematics in terms of three-momentum transfer.

In this Brief Report, we extract the longitudinal and transverse response functions from the approximate DWBA calculations for the exclusive $(e, e'p)$ reactions by using the Rosenbluth separation. We also investigate the effects of the Coulomb distortion from extracted response functions. In all calculations, we use a relativistic single particle model based on σ-ω model [14] for the bound state nucleon and a relativistic optical model [15] for an outgoing proton combined with the free relativistic nucleon current operator. This model contains the final state interaction of the outgoing proton.

There are two kinematics commonly used in the analysis of the exclusive $(e, e'p)$ experiments. One is the perpendicular kinematics where the magnitude of the knocked-out proton $\mathbf{p}$ is equal to the magnitude of the momentum transfer $\mathbf{q}$ ($|\mathbf{p}| = |\mathbf{q}|$) and the polar angle of the $\mathbf{p}$ is detected in terms of the $\mathbf{q}$, so-called ω-q constant kinematics. In this kinematics, all terms in Eq. (1) do not disappear and it is possible to extract the transverse, longitudinal-transverse, and polarized longitudinal-transverse response functions by subtracting the cross section with the left and right sides [16] from the experimental data and/or the full DWBA calculation. But there is no way to extract the longitudinal and transverse-transverse response functions because the relevant electron kinematics factors have a function of the four-momentum transfer Q^2 only, i.e., no dependence of the scattering angle θ.

Hence we have to choose the other kinematics, namely, the parallel kinematics to extract the longitudinal and transverse response functions. In the parallel kinematics, where the knocked-out proton momentum $\mathbf{p}$ is along the momentum transfer $\mathbf{q}$, the interference response functions in Eq. (1) disappear, so that the longitudinal and transverse terms only remain. In this Brief Report, we separate the longitudinal and

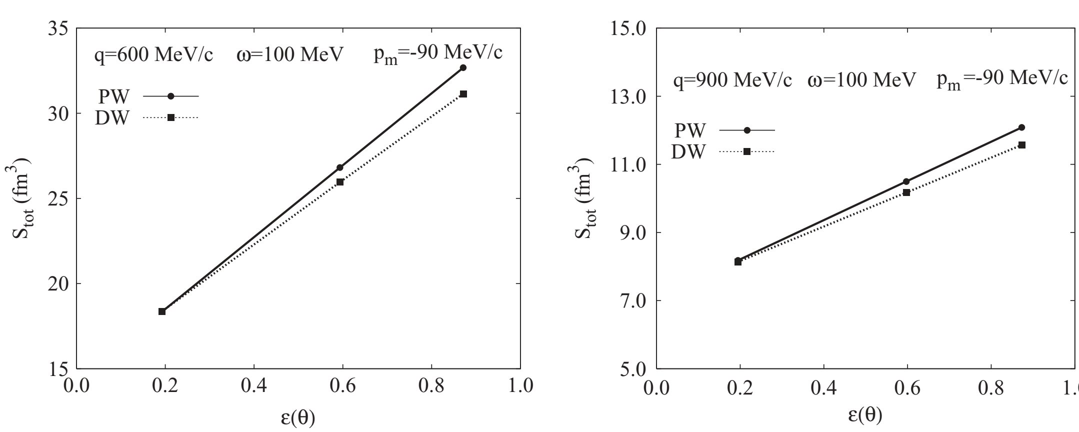

FIG. 2. The Rosenbluth plots for a constant three-momentum transfer 600 MeV/c and 900 MeV/c, and the energy transfer 100 MeV at the missing momentum $p_m = -90$ MeV/c.

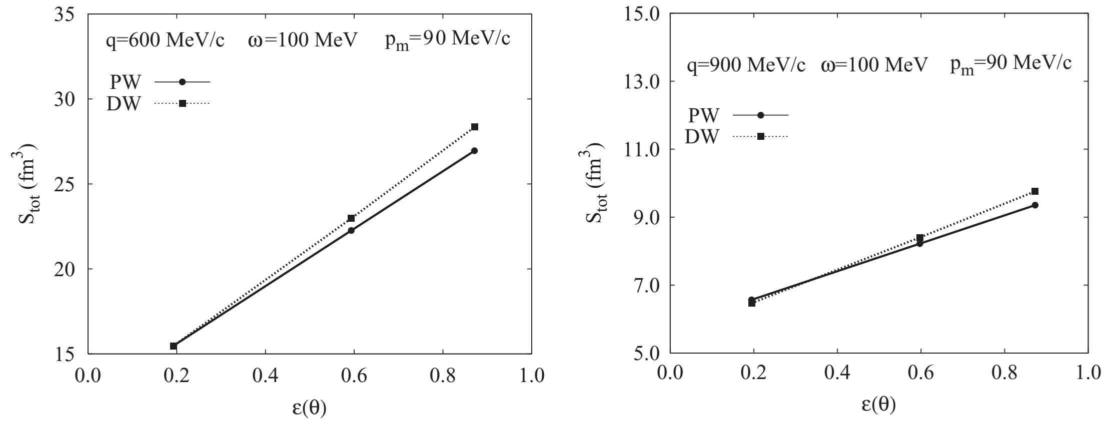

FIG. 3. The same as Fig. 2 except the missing momentum 90 MeV/c.

transverse response functions from the cross section in the parallel kinematics and investigate the Coulomb effect on the response functions extracted for the $(e, e'p)$ reactions.

From Eq. (1), using the Rosenbluth separation for the total structure function of the inclusive (e, e') reaction [17], we redefine the total response function in the parallel kinematics

$$S_{\rm tot}(q, \theta, p_m) = \frac{1}{K}\frac{q^4}{Q^4}\epsilon(\theta)\frac{d^3\sigma}{dE_f d\Omega_f d\Omega_p} = \epsilon(\theta)R_L + \frac{q^2}{2Q^2}R_T, \tag{3}$$

where the missing momentum is determined by the kinematics $\mathbf{p}_m = \mathbf{p} - \mathbf{q}$. The virtual photon polarization is given by

$$\epsilon(\theta) = \left[1 + \frac{2q^2}{Q^2}\tan^2\frac{\theta}{2}\right]^{-1}. \tag{4}$$

This equation is a linear function of the independent variable $\epsilon(\theta)$ with slope R_L and intercept proportional to R_T.

In order to extract the longitudinal and transverse response functions, the three-momentum and energy transfers are kept constant, and the incident electron energies and the scattering angles are varied. We choose the three electron scattering angles, $\theta = 30^\circ$, 60°, and 110°, the three-momentum transfers $q = 600$, 900 MeV/c, and the energy transfer $\omega = 100$ MeV. In these kinematics, the kinetic energies of the knocked-out proton are varied in terms of the missing momentum. The detected proton is knocked-out from the $p_{1/2}$ orbit of ^{16}O in the all calculations. Unfortunately, although the experimental data were measured from NIKHEF [18], Saclay [19], and JLab [13], the number of these data are not enough to use the Rosenbluth separation in the parallel kinematics.

In Fig. 1, we show the cross sections for the cases of three kinematics at $q = 900$ MeV/c and $\omega = 100$ MeV. The electron kinematics are the incident electron energy $E_i = 1778.5$ MeV and the scattering angle $\theta = 30^\circ$, $E_i = 945.8$ MeV and $\theta = 60^\circ$, and $E_i = 598.2$ MeV and $\theta = 110^\circ$. The solid and the dotted curves represent the results of the PWBA and the approximate DWBA, labeled 'PW' and 'DW', respectively. The positions of the peaks for the dotted lines are shift to the right side with larger scattering angles. The electron Coulomb distortion is larger at the forward angle than at the backward angle since the longitudinal term contributes relatively large to the cross section comparing the transverse term.

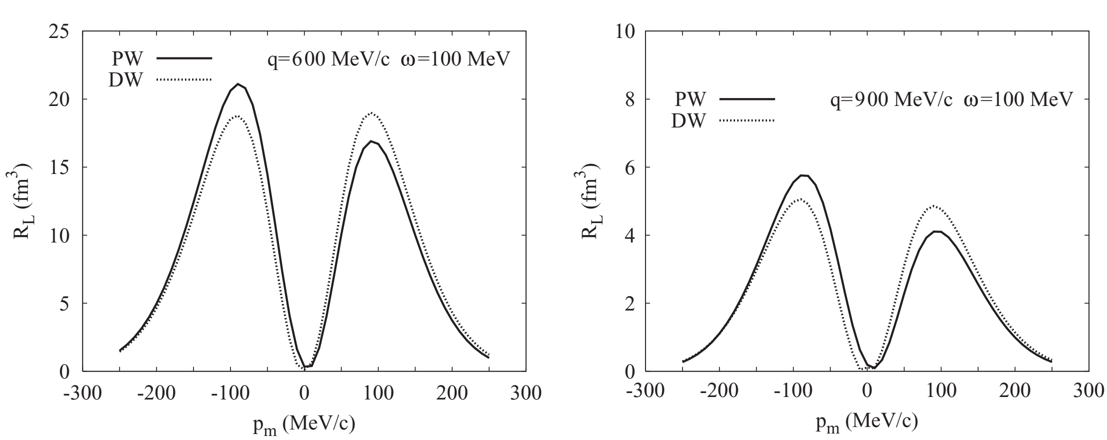

FIG. 4. The longitudinal response functions for the $p_{1/2}$ shell of ^{16}O at $q = 600$, 900 MeV/c and the energy transfer $\omega = 100$ MeV. The solid lines are the results for the PWBA calculations and the dotted curves are for the approximate DWBA calculation.

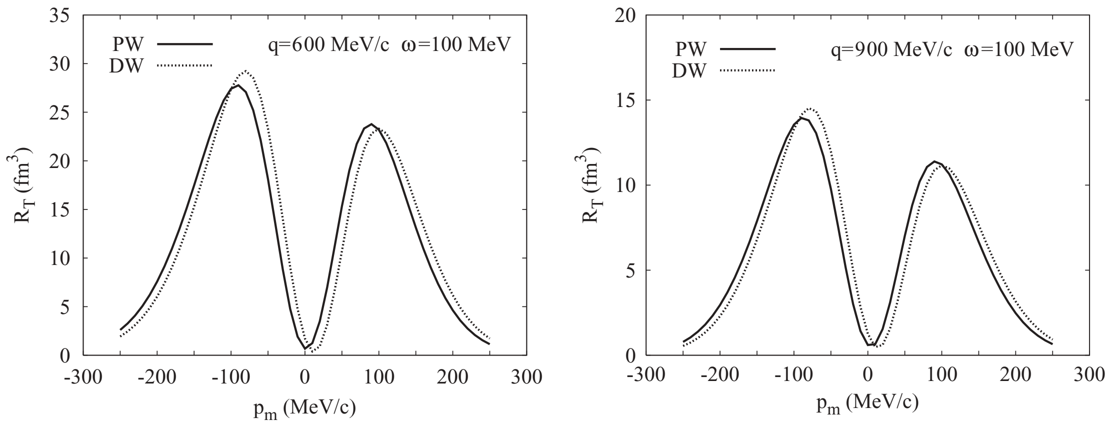

FIG. 5. The transverse response functions for the $p_{1/2}$ shell of ^{16}O at $q = 600$, 900 MeV/c and the energy transfer $\omega = 100$ MeV. The solid lines are the results for the PWBA calculations and the dotted curves are for the approximate DWBA calculation.

In Figs. 2 and 3, we show the Rosenbluth separation plots in terms of $\epsilon(\theta)$ in Eq. (3) for a constant three-momentum transfers 600 MeV/c and 900 MeV/c and the energy transfer 100 MeV at the missing momenta $p_m = -90$ and 90 MeV/c, which are the positions of the peak for the cross sections. The points represent the values of the total response functions at each angles. The dotted lines are the best fits for three points and become a straight line. The Rosenbluth plots of the PWBA become a straight line like the solid lines. The Coulomb distortions increase as the virtual photon polarization $\epsilon(\theta)$ increases, where the scattering angles decrease. These are the same results as Fig. 1.

In Figs. 4 and 5, the longitudinal and transverse response functions are extracted from the slopes and intercepts in Figs. 2 and 3 according to Eq. (3). The Coulomb distortion effects of the longitudinal response function are larger than those of the transverse response function. While the positions of the peaks and the shapes are similar to the cross sections, the effects of the electron Coulomb distortion appear to be larger on the longitudinal and transverse response functions than on the corresponding cross sections. The Rosenbluth separation is still a best tool for extracting the various response functions in the presence of the electron Coulomb distortion from the experimental data and/or the full DWBA calculation although the separation may not be valid.

In this work, we extract the longitudinal and transverse response functions in the parallel kinematics for the exclusive $(e, e'p)$ reaction. We follow the Rosenbluth separation to obtain the longitudinal and transverse response functions which is commonly used in the inclusive (e, e') reaction. The effects of the electron Coulomb distortion on the response functions appear to be different from those on the corresponding cross section while the shape and the positions of the peaks are similar to the cross sections. In conclusion, this method furnishes the information of the longitudinal and transverse response functions for experiments and/or the full DWBA calculation.

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