Properties of possible new unflavored mesons below 2.4 GeV

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The global features of spectrum of highly excited light nonstrange mesons can be well understood within both chiral symmetry restoration scenario combined with the relation $M^2 \sim J + n$ and within a nonrelativistic description based on the relation $M^2 \sim L + n$. The predictions of these two alternative classifications for missing states are different and only future experiments can distinguish between the two. We elaborate and compare systematically the predictions of both schemes, which may serve as a suggestion for future experiments devoted to the search for missing states.

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I. INTRODUCTION

In recent years many new data on unflavored mesons have appeared in the section "Further States" of the Particle Data Group [1]. The main source for these data came from the Crystal Barrel experiment, where plenty of new states were observed in the proton-antiproton annihilation in the energy range 1.9–2.4 GeV [2,3]. The obtained spectrum remarkably confirmed the approximate linearity of both Regge trajectories and radial Regge trajectories (or, equivalently, the equidistance of daughter trajectories). An important feature of the spectrum is that the slopes of both types of trajectories are almost equal, i.e., the following relation can be written (see, e.g., [4,5] for discussions):

$$M_i^2 \sim J + n + c_i, \tag{1}$$

where i denotes a set of quantum numbers, J is the spin, n is the "radial" quantum number, and c_i is a constant. Theoretically such type of mass formulas appeared in dual [6], hadron string [7], and AdS/QCD [8] models. The experimental spectrum of unflavored mesons reveals a clear-cut clustering of states near certain equidistant values of masses square [5], which implies that the constants c_i should be equal or differ by an integer. If we fit the experimental data by means of Eq. (1), the constants c_i will not be universal and a relation between different c_i *a priori* is not clear.

However, instead of Eq. (1), one can consider its nonrelativistic analog [9–12],

$$M_i^2 \sim L + n + c, \tag{2}$$

with the angular momentum of quark-antiquark pair L being related to the total spin J as $J = L, L \pm 1$ depending on the mutual orientation of the quark/antiquark spin s. It turns out that the angular momentum assignment can be chosen such that the constant c will be approximately universal, as is written in Eq. (2). This means, in particular, that L and quark spins s can be added as in the usual quantum mechanics. Such a physical picture is quite unexpected because light mesons are ultrarelativistic systems, therefore L and s cannot be separated, a conserved quantum number is the total spin J, while L would be conserved with the spinless quarks only. The validity of Eq. (2) could be a nontrivial consequence of the asymptotic suppression of the spin-orbital correlations in excited hadrons [9,12–14].

Relation (2) implies a duplication of states in the channels where the resonances can be created by different angular momentum. For instance, the vector mesons can have either $L = 0$ or $L = 2$ (the so-called S- and D-wave mesons in the quantum-mechanical terminology), hence, they are duplicated. Experimentally such a duplication is well seen [2,3]. In practice, the separation of resonances into states with different angular momentum can be achieved by using the polarization data. Following this method, the experiment of the Crystal Barrel Collaboration obtained a good separation for the states with $(C, I) = (+1, 0), (-1, 1)$ [3]. The separation in other channels should be tentatively guessed. As long as one accepts a nonrelativistic framework, the parity of the quark-antiquark pair is defined as $P = (-1)^{L+1}$. The states with maximal L at given mass are then parity singlets, associating them with the resonances on the leading Regge trajectories, we obtain a correct qualitative picture of the known experimental spectrum.

Another pattern of parity doubling is predicted by the chiral symmetry restoration (CSR) scenario (see [12] for a review). If effective CSR occurs high in the spectrum, the chiral multiplets become complete. In particular, this implies the absence of parity singlet states among highly excited hadrons. Within the CSR picture, the duplication of some trajectories appears due to an assignment of states on these trajectories to different chiral multiplets.

The classifications of states based on CSR and the ones based on Eq. (2) cannot coexist because the relativistic chiral basis and the nonrelativistic $n^{2s+1}L_J$ basis are incompatible [15], the chiral basis, however, can meet Eq. (1).

Thus, an intriguing problem emerges—which alternative (if any) is realized in nature? The answer can be provided by examining the phenomenological implications of the possibilities above, such as spectroscopic predictions. A phenomenological analysis of these predictions is still absent in the literature and the present paper is intended to fill in this gap, providing thereby a stimulus for the search of new states that distinguish between the two alternatives.

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We will show by an explicit assignment of mesons according to the quantum numbers (L, n) that relation (2) describes the spectrum of practically all confirmed and unconfirmed unflavored mesons except the masses of Goldstone bosons. There are only eight missing states below 2.4 GeV, which allow us to justify or falsify the classification in future. The CSR scenario predicts these eight states as well, but it predicts also many missing states beyond them.

The paper is organized as follows. In Sec. II we remind the reader of some phenomenological ideas concerning the origin of linear spectrum and estimate qualitatively an expected value for the constant c in Eq. (2). Section III contains our phenomenological analysis and predictions. We conclude in Sec. IV.

II. THEORETICAL DISCUSSIONS

Let us present some known heuristic arguments in favor of linear spectrum. For high radial or orbital excitation, a meson state can be considered quasiclassically as a pair of relativistic quarks interacting via a linear potential. Consequently, neglecting the quark spin, the meson mass can be written as

$$M = 2p + \sigma r, \tag{3}$$

where p is the relativistic quark momentum and σ is the string tension. The maximal length of the chromoelectric flux tube between the quarks is $l = M/\sigma$. Applying the quasiclassical (WKB) quantization condition,

$$\int_0^l p\, dr = \pi n, \tag{4}$$

with the momentum p taken from Eq. (3), one obtains

$$M^2 \sim n. \tag{5}$$

A "next-to-leading" correction to the presented picture can be considered. It comes from the Bohr-Sommerfeld quantization condition (4): n must be replaced by $n + \gamma$, where γ is a constant of order of unity characterizing the nature of turning points. In Eq. (3) one deals with a centrosymmetrical potential. It is well known (see, e.g., [16]) that in this case $\gamma = \frac{1}{2}$. Hence, the corrected linear spectrum is

$$M^2 \sim n + \tfrac{1}{2}. \tag{6}$$

Exactly this type of spectrum is predicted by the Lovelace-Shapiro dual amplitude [17], where $\gamma = \frac{1}{2}$ comes from the Adler self-consistency condition (at $p^2 = m_\pi^2$, the $\pi\pi$ scattering amplitude is zero). In some channels this spectrum appeared naturally within the QCD sum rules [18], where $\gamma = \frac{1}{2}$ stems from the absence of dimension-two gauge-invariant condensate. Recently the intercept $\frac{1}{2}$ has been reported within a holographic dual of QCD (the second reference in [8]).

Specific boundary conditions can lead to another value for γ. We mention the following possibilities: identified ends (closed string) correspond to $\gamma = 0$, S-wave states correspond to $\gamma = \frac{3}{4}$, infinite potential walls at the ends correspond to $\gamma = 1$. The first possibility is unrealistic for mesons, thus in a general case we expect γ to lie in the interval $\frac{1}{2} \leqslant \gamma \leqslant 1$.

According to Regge theory and simple hadron string considerations, M^2 is also linear in the angular momentum L (Chew-Frautschi formula). This suggests that n in Eq. (5) might be substituted by $n + L$, thus resulting in Eq. (2). Unfortunately, we are not aware of solid arguments for such a replacement.

The linear spectrum (5) is an exact result within a kind of dimension-two QCD, the 't Hooft model [19]. The next-to-leading correction to Eq. (5) within this model, however, is $O(\ln n)$ rather than a constant. In this respect we should remind the reader that the 't Hooft model is defined in a specific sequence of $N_c \to \infty$ limits, $m_q \to 0$ while $m_q \gg g \sim 1/\sqrt{N_c}$, where m_q denotes current quark mass and g is coupling constant. In contrast to QCD, we cannot set $m_q = 0$ from the very beginning. On the other hand, if one takes into account the masses of current quarks in the derivation above, the logarithmic corrections emerge naturally (see, e.g., [20]).

A delicate point in such kind of reasoning is the relative value of slope between radial and orbital trajectories. The matter is that $M^2 = 4\pi\sigma$ in the derivation above, but $M^2 = 2\pi\sigma$ according to the Chew-Frautschi formula. Naively, this leads to $M^2 \sim L + 2n$ rather than to Eq. (2). A possible reason is that the parity is not properly incorporated: It is related to the orbital motion (defined through L) in three space dimensions, but in one space dimension it is related to the reflections of wave functions. Considering the radial excitations of a one-dimensional object, one deals with the latter case, where the states alternate in parity, like in the 't Hooft model. The extraction of states with the same parity is then tantamount to enlarging of the slope by two times.

The note above is a particular manifestation of a general problem: A linear potential plus a semiclassical analysis produces a necessarily different angular and radial slopes, for this reason it may be suggestive only and by no means may serve for justification of Eq. (2). A derivation of Eq. (2) or Eq. (1) is a challenge for future quark models [21], presently these empirical relations do not have solid theoretical support. In particular, Eq. (2) implies the existence of a single "principal" quantum number, $N = L + n$, like in a hydrogen atom [11], a development of this analogy could be far reaching.

III. FITS AND PREDICTIONS

Using experimental masses from the Particle Data Group [1] one can perform a global fit of the data by the linear spectrum. Such an analysis was performed in [5]. The result is that on average the masses of well-known light nonstrange mesons behave as (in GeV^2)

$$M_{\text{exp}}^2 \approx 1.14(N + 0.54), \qquad N = 0, 1, 2. \tag{7}$$

One can consider the states observed by the Crystal Barrel experiment [3], which allow us to extend Eq. (7) to $N = 3, 4$. It turns out that both slope and intercept are then changed negligibly [5]. Comparing Eqs. (6) and (7) we see that our guess on the "next-to-leading" correction is compatible with the experimental data.

Partly following [2,3], we classify the light nonstrange mesons according to the values of (L, n), see Table I. As

TABLE I. Classification of light non-strange mesons according to the values of (L, n). The states with the lowest star rating (according to [3]) are marked by the question mark, the states, which presumably have a large admixture of strange quark, are marked by the double question mark.

$L \backslash n$	0	1	2	3	4
0	$\pi(140)$	$\pi(1300)$	$\pi(1800)$	$\pi(2070)$	$\pi(2360)$
	$\eta(548)(??)$	$\eta(1295)(??)$	$\eta(1760)$	$\eta(2010)$	$\eta(2285)$
	$\rho(770)$	$\rho(1450)$	$\rho(?)$	$\rho(1900)$	$\rho(2150)$
	$\omega(782)$	$\omega(1420)$	$\omega(?)$	$\omega(?)$	$\omega(2205)(?)$
1	$f_0(1370)$	$f_0(1770)$	$f_0(2020)$	$f_0(2337)$	
	$a_0(1450)(??)$	$a_0(?)$	$a_0(2025)$	$a_0(?)$	
	$a_1(1260)$	$a_1(1640)$	$a_1(1930)(?)$	$a_1(2270)(?)$	
	$f_1(1285)$	$f_1(?)$	$f_1(1971)$	$f_1(2310)$	
	$b_1(1230)$	$b_1(1620)(?)$	$b_1(1960)$	$b_1(2240)$	
	$h_1(1170)$	$h_1(1595)(?)$	$h_1(1965)$	$h_1(2215)$	
	$a_2(1320)$	$a_2(1680)$	$a_2(1950)(?)$	$a_2(2175)(?)$	
	$f_2(1275)$	$f_2(1640)$	$f_2(1934)$	$f_2(2240)$	
2	$\rho(1700)$	$\rho(2000)$	$\rho(2265)$		
	$\omega(1650)$	$\omega(1960)$	$\omega(2295)(?)$		
	$\pi_2(1670)$	$\pi_2(2005)$	$\pi_2(2245)$		
	$\eta_2(1645)$	$\eta_2(2030)$	$\eta_2(2267)$		
	$\rho_2(?)$	$\rho_2(1940)$	$\rho_2(2225)$		
	$\omega_2(?)$	$\omega_2(1975)$	$\omega_2(2195)$		
	$\rho_3(1690)$	$\rho_3(1982)$	$\rho_3(2300)(?)$		
	$\omega_3(1670)$	$\omega_3(1945)$	$\omega_3(2285)$		

seen from Table I, the states with equal $N = L + n$ are indeed approximately degenerate (one should read the data in a diagonal way, the frames are introduced for convenience). We will regard the averaged values of masses and widths at given N from [5] as predictions for unknown states in the mass region under consideration. Thus, for $M(N)$ we have (in MeV): $M(0) \approx 785$, $M(1) \approx 1325 \pm 90$, $M(2) \approx 1700 \pm 60$ $M(3) \approx 2000 \pm 40$, $M(4) \approx 2270 \pm 40$. Looking at Table I, we make the following predictions for the nonstrange mesons which still have not been observed.

(i) In the energy range 1700 ± 60 MeV there exists $a_0, f_1, \rho_2, \omega_2$, as well as the second ρ and ω mesons. Their widths are approximately $\Gamma = 200 \pm 70$ MeV. The state $X(1650)$ with $I^G(J^{PC}) = 0^-(?^{?-})$ cited in [1] might be a possible candidate for the predicted ω or ω_2 mesons. The state $X(1750)$ with $I^G(J^{PC}) = ?^?(1^{--})$ cited in [1] might be a possible candidate for the predicted ω or ρ mesons.

(ii) In the energy range 2000 ± 40 MeV there exists the second ω meson. Its width is approximately $\Gamma = 220 \pm 70$ MeV. The state $X(1975)$ with $I^G(J^{PC}) = ?^?(?^{??})$ cited

TABLE I. *(Continued.)*

$L \backslash n$	0	1	2	3	4
3	$f_2(2001)$	$f_2(2293)$			
	$a_2(2030)$	$a_2(2255)$			
	$f_3(2048)$	$f_3(2303)$			
	$a_3(2031)$	$a_3(2275)$			
	$b_3(2032)$	$b_3(2245)$			
	$h_3(2025)$	$h_3(2275)$			
	$f_4(2018)$	$f_4(2283)$			
	$a_4(2005)$	$a_4(2255)$			
	$\rho_3(2260)$				
	$\omega_3(2255)$				
4	$\rho_4(2230)$				
	$\omega_4(2250)(?)$				
	$\pi_4(2250)$				
	$\eta_4(2328)$				
	$\rho_5(2300)$				
	$\omega_5(2250)$				

in [1] might be a possible candidate for the predicted ω meson.

(iii) In the energy range 2270 ± 40 MeV there exists a_0 meson. Its width is approximately $\Gamma = 270 \pm 60$ MeV. The states $X(2210)$ and $X(2340)$ with $I^G(J^{PC}) = ?^?(?^{??})$ cited in [1] might be possible candidates for the predicted a_0 meson.

Thus, the nonrelativistic $n^{2s+1}L_J$ assignment based on Eq. (2) predicts eight nonstrange mesons in the energy range 1.6–2.3 GeV which have never been observed and are awaiting their discovery.

Consider predictions of the CSR scenario based on Eq. (1). Evidently, all eight missing states above should also follow from this scenario if effective CSR takes place above 1.7 GeV. We will enumerate the predictions which go beyond these eight new mesons.

(i) 1700 ± 60 MeV. The indications on CSR are not solid in this mass region. Nevertheless, if CSR happens we may expect in the minimal scenario the appearance of parity partners for ρ_3 and ω_3 mesons—new a_3 and f_3 mesons, respectively. If CSR leads to parity-chiral multiplets described in [12] the $(1,0) \oplus (0,1)$ and $(\frac{1}{2},\frac{1}{2})$ representations of $SU(2)_L \times SU(2)_R$] then we should expect also the second ρ_3 and ω_3 mesons and their $(\frac{1}{2},\frac{1}{2})$ chiral partners, the h_3 and b_3 mesons.

(ii) 2000 ± 40 MeV. We should expect at least the parity partners for a_4 and f_4 mesons—the states ρ_4 and ω_4. If CSR results in parity-chiral multiplets described in [12] then we should expect also the second a_4 and f_4 states, their chiral partners η_4 and π_4, and the second ρ_3 and ω_3 mesons [all carry the representation $(\frac{1}{2},\frac{1}{2})$].

(iii) 2270 ± 40 MeV. We should expect at least the parity partners for ρ_5 and ω_5 mesons—the states a_5 and f_5. If CSR leads to parity-chiral multiplets described in [12] then we should expect also the second ρ_5 and ω_5 states, their chiral partners h_5 and b_5, and the second a_4 and f_4 mesons [all carry the representation $(\frac{1}{2},\frac{1}{2})$].

Thus, the CSR scenario combined with a clustering of states expressed by Eq. (1) leads to a richer spectrum of high excitations.

IV. CONCLUSIONS

We have provided in a concise form the concrete spectroscopic predictions which follow from recent discussions on global features of a light nonstrange meson spectrum.

The assumption that relation (2) does not depend on quantum numbers of unflavored nonexotic mesons allows us to provide the whole spectrum with two input parameters only, the universal slope and intercept. The quasiclassical and some other arguments indicate that these inputs could be related. Fixing the physical values for the slope and intercept, universal relation (2) gives 100 nonstrange mesons below 2.4 GeV, see Table I. Except in some rare cases, e.g., the Goldstone bosons, the agreement with the masses of known confirmed resonances from the Particle Data Group [1] and unconfirmed states observed by Crystal Barrel [3] is impressive. There exist only eight missing states which have never been observed. The predictions for their masses and widths are given and possible candidates are indicated. We do not see any theoretical reasons why those states should be absent in nature, most likely they still have been not detected experimentally. The seemingly random (factor isospin) distribution of missing states on the spectrum supports our belief.

Relation (2) is in odds with the Lorentz group (angular momentum L is not conserved quantum number in relativistic quark-antiquark pair) and chiral symmetry restoration. Both obstacles can be overcome if one accepts relation (1), the number of predicted states below 2.4 GeV is then substantially larger.

The discovery of the indicated missing resonances in future experiments will constitute a crucial test for the two alternatives discussed in the paper, providing thereby an important step forward toward establishing final order in the spectroscopy of light mesons.

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