Revised ^{45}V(p, γ)^{46}Cr astrophysical reaction rate

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The astrophysical reaction rate of the ^{45}V(p, γ)^{46}Cr reaction, which is relevant to ^{44}Ti production in core-collapse supernovae, has been revised through a consistent application of the Thomas-Ehrman level displacement formalism. The new rate agrees well with that predicted by the NON-SMOKER statistical calculation with an ETFSI mass model.

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The isotope ^{44}Ti is of interest in astrophysics because of evidence for its production in core-collapse supernovae [1,2]. A major aim of satellite-based γ-ray observatories is to observe nucleosynthesis of ^{44}Ti through the 1.157 MeV γ-ray emission of its β-decay daughter, ^{44}Sc. In the case of the Cassiopeia A [3] and possibly the Vela [4] supernova remnants, such a signal has been seen. It is thought that production of ^{44}Ti in this environment takes place in the shock-heated Si layer just outside the collapsed core, as part of an α-rich freeze-out [5]. This is in the vicinity of the mass cut, that is, the region between the point at which material is successfully ejected and that at which it falls back onto the protoneutron star. Thus, there is the possibility that one could determine the location of the mass cut by comparing the amount of ^{44}Ti detected (ejected) against that which was expected to be produced overall. In this way one would have a powerful tool for testing the hydrodynamic aspects of a particular model. However, one of the ingredients needed to allow this comparison to be made is detailed knowledge of the rates of nuclear reactions involved in the production of ^{44}Ti. Work by The *et al.* [6] explored which nuclear reactions had the most impact on the ^{44}Ti abundances produced in core-collapse supernovae, and they found that uncertainties in the ^{45}V(p, γ)^{46}Cr reaction rate made a large contribution to the overall uncertainty. The rate of this reaction is unknown, but it is expected to be dominated by resonant contributions proceeding through states in ^{46}Cr.

In cases such as this, where a reaction proceeds via a proton resonant state in a proton-rich nucleus, the Thomas-Ehrman level displacement (TELD) formalism [7,8] is often found to be particularly useful. This usefulnessl largely derives from the fact that the states involved are above the particle decay threshold, resulting in proton partial widths that are frequently too narrow to be measured experimentally. Thus, it is worthwhile to turn to the charge symmetry of the nuclear force, making use of relatively abundant spectroscopic data of analog states in the mirror nucleus, to determine the properties of the astrophysically important states. Based on this, Horoi *et al.* [9] estimated the reaction rate through application of the TELD formalism to known spectroscopic information in the mirror nucleus of ^{46}Cr, ^{46}Ti. Calculating the rate as the sum of contributions from individual resonances, they found a rate about one order of magnitude smaller at $T_9 = 3$ than that predicted by the NON-SMOKER [10] statistical model calculation. However, a survey of the TELD literature [11] revealed inconsistency in the definition of critical parameters, leading to errors in calculations. We have used the consistent formalism presented in Ref. [11] to reevaluate the ^{45}V(p, γ)^{46}Cr reaction rate because of its astrophysical importance.

The level displacement of analog states in the ^{46}Ti-^{46}Cr pair can be represented as

$$\Delta^*_\lambda = E^*(^{46}\mathrm{Ti}) - E^*(^{46}\mathrm{Cr}), \tag{1}$$

where $E^*(^{46}$Ti) is the excitation energy of a particular level in ^{46}Ti, and $E^*(^{46}$Cr) is the excitation of its analog in ^{46}Cr. The TELD formalism allows evaluation of this level displacement as

$$\Delta^*_\lambda = \frac{3\hbar^2}{2M_c a_c^2}\theta_c^2 \left\{ [P_c(FF'_x + GG'_x)]_{|E=E_r} - \left(x\frac{W'_x}{W} \right)_{|E=E_b} \right\}, \tag{2}$$

where M_c is the reduced mass, a_c is the channel radius, θ_c^2 is the dimensionless reduced width, F, G and W are the regular and irregular Coulomb wave functions and the Whittaker function, and F', G' and W' are their derivatives, respectively. P_c is the Coulomb penetrability, and $x = kr$ where k is the wave number and r is the radial coordinate. The evaluations are made at $E_r = E^*(p) - S_p^p$ and $E_b = |E^*(n) - S_n^n|$, the energies relative to the respective nucleon thresholds.

The excitation energies in ^{46}Cr have been recalculated and are presented in Table I. The assumption $\theta_c^2 (= \theta_p^2 = \theta_n^2) = 0.1$ made in Ref. [9] has been used in the present work. It can be seen that these revised energies are quite different from those of Ref. [9], in which mistakes were made in calculating the Whittaker functions and their derivatives and inconsistent parameter definitions were used.

To estimate an astrophysical reaction rate, one also needs to estimate the widths of these states. In Ref. [9], the proton partial decay width defined by Blatt and Weisskopf [12] was written as

$$\Gamma_p = \frac{2\hbar P_\ell \theta_p^2}{a_c}\left(\frac{2E}{M_c}\right)^{\frac{1}{2}}. \tag{3}$$

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TABLE I. Selected positive parity levels in ^{46}Ti, their possible spins, and the proton partial decay widths of their ^{46}Cr analogs, comparing present and previous results. Energy and width are in units of MeV.

			Present		Previous [9]	
$E_x(^{46}\text{Ti})$	J^π	ℓ_{tr}	$E_x(^{46}\text{Cr})$	Γ_p	$E_x(^{46}\text{Cr})$	Γ_p
5.080	2^+	1	4.927	1.38×10^{-46}	5.152	1.0×10^{-14}
	2^+	3	4.960	4.82×10^{-35}	5.644	9.0×10^{-9}
5.321	2^+	1	5.165	1.77×10^{-14}	5.388	2.2×10^{-9}
5.363	2^+	1	5.206	3.38×10^{-13}	5.430	8.0×10^{-9}
	2^+	3	5.240	1.56×10^{-14}	5.918	4.9×10^{-7}
5.515	2^+	1	5.356	3.74×10^{-10}	5.578	2.8×10^{-7}
	2^+	3	5.391	8.81×10^{-12}	6.066	2.4×10^{-6}
6.025	$2^+, 4^+$	1	5.856	7.25×10^{-6}	6.076	1.9×10^{-4}
6.118	2^+	1	5.947	1.91×10^{-5}	6.166	4.1×10^{-4}
	2^+	3	5.989	3.60×10^{-7}	6.650	1.5×10^{-4}
6.424	$2^+, 4^+$	1	6.246	2.18×10^{-4}	6.464	2.9×10^{-3}
6.550	$2^+, 4^+$	1	6.369	4.70×10^{-4}	6.586	5.4×10^{-3}
	$2^+, 4^+$	3	6.417	1.08×10^{-5}	7.066	1.1×10^{-3}

The factor P_ℓ, however, was misinterpreted as P_c, the Coulomb penetrability, in contrast to $P_\ell = 1/(F^2+G^2)$ in Refs. [12,13]. Here, following French [14], we have evaluated the proton partial decay widths using the relation

$$\Gamma_p = \frac{3\hbar^2 P_c \theta_p^2}{M_c a_c^2}, \tag{4}$$

and we present the resulting widths in Table I. It is apparent that the new widths are considerably smaller, with the impact that direct experimental measurement via resonant proton elastic scattering is now clearly precluded.

The γ-ray decay widths for these states must also be calculated. Recently, the low-lying states in ^{46}Cr were investigated by Garrett *et al.* [15]; these precise excitation energies in ^{46}Cr were used in the present calculation rather than the analog energies in ^{46}Ti [9]. The Weisskopf estimates of the widths [16] are given as $\Gamma_W(E1) = 6.8 \times 10^{-8} A^{2/3} E_\gamma^3$ and $\Gamma_W(E2) = 4.9 \times 10^{-14} A^{4/3} E_\gamma^5$, where the widths are in units of MeV. It appears that the factor $A^{2/3}$ was missed in Ref. [9], their calculated $\Gamma_W(E1)$ value being approximately a factor of 2 larger than it should be. For states with $J^\pi = 2^+$ the dominant contributions for $E1$ and $E2$ are $\Gamma_W(E1)$ arising from the γ-ray decay to the 3^- (3.197 MeV) state, and $\Gamma_W(E2)$ arising from γ-ray decay to the 0^+ ground state, respectively. For states with $J^\pi = 4^+$, the dominant contributions are $\Gamma_W(E1)$ from the γ-ray decay to the 3^- (3.197 MeV) state as well as to the 4^- (3.594 MeV) state, and $\Gamma_W(E2)$ from the γ-ray decay to 2^+ (0.892 MeV) state, respectively. The calculated Γ_W values are listed in Table II. However, known systematics in the region around $A = 46$ [17] suggest a hindrance factor for the dipole transitions ($E1$) of about $10^{-3} \sim 10^{-6}$, with an average factor of 10^{-4}, and an enhancement factor for $E2$ transitions of about between 1 and 60, with an average factor of 15. We find therefore in Table II that typically $\Gamma_W(E1) \geqslant 200\Gamma_W(E2)$;

TABLE II. Deduced resonant properties of ^{46}Cr states, assuming all states have $\ell = 1$ capture. See text for details.

$E_x(^{46}\text{Cr})$	J^π	Γ_p	$\Gamma_\gamma^{\text{max}}$	$\Gamma_\gamma^{\text{mean}}$	$\Gamma_W(E1)$	$\Gamma_W(E2)$
4.927	2^+	1.38×10^{-46}	1.4×10^{-6}	3.5×10^{-7}	4.5×10^{-6}	2.3×10^{-8}
5.165	2^+	1.77×10^{-14}	1.8×10^{-6}	4.5×10^{-7}	6.7×10^{-6}	3.0×10^{-8}
5.206	2^+	3.38×10^{-13}	1.9×10^{-6}	4.6×10^{-7}	7.1×10^{-6}	3.1×10^{-8}
5.356	2^+	3.74×10^{-10}	2.1×10^{-6}	5.3×10^{-7}	8.8×10^{-6}	3.6×10^{-8}
5.856	2^+	7.25×10^{-6}	3.4×10^{-6}	8.4×10^{-7}	1.6×10^{-5}	5.6×10^{-8}
	4^+	7.25×10^{-6}	1.5×10^{-6}	3.7×10^{-7}	2.7×10^{-5}	2.4×10^{-8}
5.947	2^+	1.91×10^{-5}	3.6×10^{-6}	9.0×10^{-7}	1.8×10^{-5}	6.0×10^{-8}
6.246	2^+	2.18×10^{-4}	4.6×10^{-6}	1.2×10^{-6}	2.5×10^{-5}	7.7×10^{-8}
	4^+	2.18×10^{-4}	2.2×10^{-6}	5.4×10^{-7}	4.1×10^{-5}	3.6×10^{-8}
6.369	2^+	4.70×10^{-4}	5.1×10^{-6}	1.3×10^{-6}	2.8×10^{-5}	8.5×10^{-8}
	4^+	4.70×10^{-4}	2.4×10^{-6}	6.0×10^{-7}	4.7×10^{-5}	4.0×10^{-8}

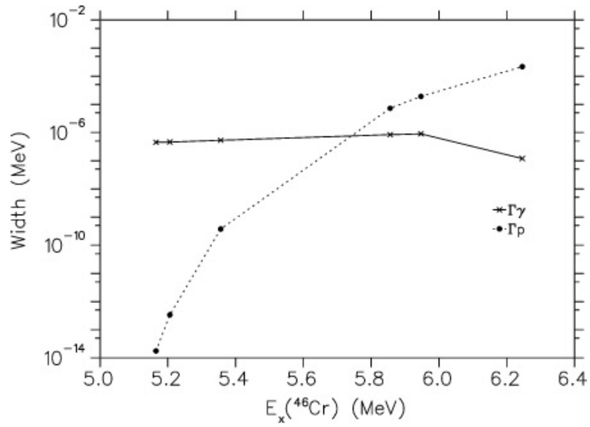

FIG. 1. Calculated proton and γ-ray partial decay widths, as a function of the excitation energy in ^{46}Cr. The transition between which width is most important in determining the astrophysical reaction rate occurs at approximately 5.75 MeV excitation. We show the mean γ-ray decay widths, but the steepness of the Γ_p curve means the conclusion is virtually unchanged within the limits of the maximum or minimum estimate for Γ_γ.

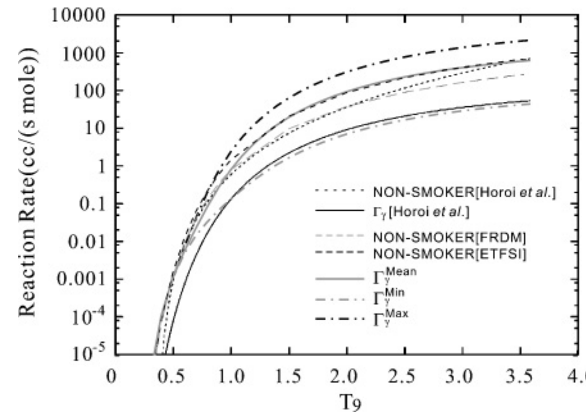

FIG. 2. Revised resonant reaction rates of ^{45}V$(p,\gamma)^{46}$Cr. For comparison, the previous results (with NON-SMOKER and Γ_γ [9]) are shown. The present NON-SMOKER rates with FRDM and ETFSI mass models are shown in dashed lines as well.

but by taking the above hindrance and enhancement factors into account, the actual $\Gamma_W(E2)$ is much larger than the actual $\Gamma_W(E1)$. The maximum and mean values of Γ_γ are estimated via $\Gamma_\gamma^{\max} = 10^{-3}\Gamma_W(E1) + 60\Gamma_W(E2)$ and $\Gamma_\gamma^{\text{mean}} = 10^{-4}\Gamma_W(E1) + 15\Gamma_W(E2)$, respectively. Obviously the actual $\Gamma_W(E2)$ widths dominate. The minimum Γ_γ ($\Gamma_\gamma^{\min}$) is found to be approximately equal to the calculated $\Gamma_W(E2)$. The relevant quantities are calculated and listed in Table II. It is worth noting that the shell-model calculation made in Ref. [9] indicated that the total Γ_γ width varies from 10^{-8} to 5×10^{-7} MeV, and thus the dominant width should be $\Gamma_W(E2)(+\to+$ transition). Therefore the present $\Gamma_\gamma^{\text{mean}}$ values broadly agree with those of the shell-model calculations.

For temperatures likely to occur in a supernova environment, only the proton and γ-ray decay channels are open. Additionally, the resonances are appropriately described as both narrow and well spaced. Thus, taking the total width as the sum of the proton and γ-ray decay widths, one can evaluate the astrophysical reaction rate using the Breit-Wigner single-level narrow-resonance formula [13]. Here, it is the smaller of the widths that determines the resonance strength, and thus, as can be seen from Fig. 1, our results suggest that for ^{46}Cr excitation energies of less than about 5.75 MeV, the astrophysical reaction rate is determined by the proton decay width; above this energy, it is the γ-ray decay width that determines the rate.

The new reaction rates are compared with those of Ref. [9] in Fig. 2. We notice that there was a factor E/k^2 missed in Eq. (6) of Horoi *et al.* [9], and therefore our results differ from theirs in two respects: (i) the revised energies and widths and (ii) a missing factor of E/k^2. In Fig. 2 we only show results using Γ_γ of $J^\pi = 2^+$ for states with 2^+ or 4^+ assignment, since the differences arising from alternative 2^+ and 4^+ assignments are quite small. The present lower-limit rate (indicated by a grey dot-dash line) is close to the previous calculation (with $\Gamma_\gamma = 10^{-3}\Gamma_W$ [9]) above $T_9 \geqslant 1.0$ since present $\Gamma_\gamma^{\min}$ values are of the same order as the previous values. The discrepancies below $T_9 \leqslant 1.0$ are caused by the usage of different state energies and proton widths; presently the excitation energies derived from the TELD method (listed in Table II) have been used in calculating the rates, while those derived from the shell-model calculations were used in the previous work [9]. The NON-SMOKER rates (in a stellar environment) with FRDM and ETFSI mass models [18] are shown in Fig. 2 as dashed lines. The rate estimated with the ETFSI mass model is about 2.6 times larger than that estimated with a FRDM mass model at higher temperature. The estimate based on the ETFSI mass model agrees well with that from the SMOKER code calculation [19]. For comparison, the NON-SMOKER rate calculated in Ref. [9] is also shown in Fig. 2, and it is quite different from the present result. It can be seen that the revised reaction rate, calculated using the mean value of Γ_γ (i.e., $\Gamma_\gamma^{\text{mean}}$, and indicated by the grey solid line), is very close to that predicted by the present NON-SMOKER calculation with ETFSI mass model or by the SMOKER calculation.

In summary, a revised thermonuclear reaction rate of the ^{45}V$(p,\gamma)^{46}$Cr reaction, relevant to ^{44}Ti production in core-collapse supernovae, has been presented. This revision is based on the level properties in ^{46}Cr which have been investigated by a consistent application of the TELD formalism. The present work finds a reaction rate that agrees very well with that predicted by the NON-SMOKER statistical calculation with the ETFSI mass model or by the SMOKER calculation. Furthermore, a significant impact of the present work is that the experimental measurement for states in ^{46}Cr via resonant proton elastic scattering (i.e., use of a radioactive ion beam of ^{45}V impinging protons in a CH_2 target) is now clearly precluded, since the proton widths of resonant states in ^{46}Cr are too narrow to be observed.

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